

Book Review

Théorème vivant

Reviewed by Jacques Hurtubise

Théorème vivant

Cédric Villani

Grasset et Fasquelle

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“But what do you actually *do*?” We have all had to answer that question. “What does a research mathematician actually do? Isn’t it already known? Do you just sit down and write things out?” And so on.

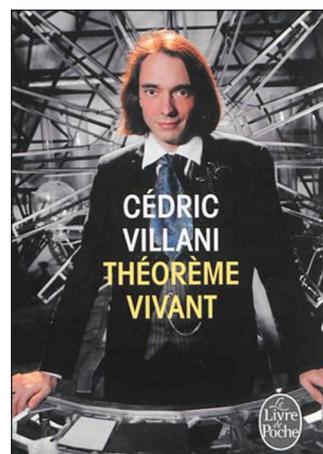
It is a difficult question to answer if one wants to give some idea both of the effort and of the intellectual pleasure involved. This pleasure is quite real and keeps us all going, as arcane as it might seem to the passer-by. To quote André Weil, as does the book under review, “Tout mathématicien digne de ce nom a ressenti, ne serait-ce que quelquefois, l’état d’exaltation lucide dans lequel une pensée succède à une autre comme par miracle.... Contrairement au plaisir sexuel, ce plaisir peut durer plusieurs heures, voire plusieurs jours. (Any mathematician worthy of the name has felt, if only a few times, that state of lucid exaltation in which one thought follows another as if by miracle.... Unlike sexual pleasure, this state can go on for hours, even days.)”

Our world, also, is different. We evolve inside a mathematical culture which is to a great degree alien to the common culture. Our heroes are, by and large, unknown to the public. Though we share to a great degree the values of fellow scientists, even within a faculty of science, we are often strangers, and outliers, doing that strange stuff that is immune to experiment.

Of course, we have explained ourselves in several ways: mathematicians have written autobiographies. Cleaving to the theme of French

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mathematics, appropriately for this review, two come to mind, which, incidentally, illustrate to a certain degree how French mathematical culture has evolved over time. One is André Weil’s *Souvenirs d’apprentissage*: beautifully written, quirky, if, in the end, too preoccupied with the anecdotes of a well-filled life, having carefully avoided any intimacy. The other is *Un mathématicien aux prises avec le siècle*, by Laurent Schwartz, which follows a life of someone who, much more than most of us, was deeply involved in the political and intellectual struggles of his time: it is a beautiful book, following the life of a truly exceptional man. However, neither of these gives any real idea of what mathematical research is actually like and why it involves us so deeply. Nor do many other autobiographies.

The book *Théorème vivant*, by Cédric Villani, explores a different direction and gives a wonderfully living answer to the question. It follows, through a few years of work, a collaboration by Villani and his former student, Clément Boutot, on a difficult and important result: establishing Landau damping for the Vlasov equation, beyond linear perturbation. The Vlasov, or Vlasov-Poisson, equation governs the evolution of plasmas. It displays time reversibility and does not reflect collisions of the particles, the mechanism which would normally govern a rapid convergence to an equilibrium. Landau, in the 1940s, argued that nevertheless the electric field converged exponentially to an equilibrium and computed the rate from the linearized equation: this is Landau damping. The

Boutot-Villani theorem establishes this damping for the full equation.

The core of the book, thus, is about a theorem. It is a very major theorem, not one tossed off in a weekend, and its genesis exhibits all the rebounds that we have felt in our own work. The technique Villani uses to show this is kaleidoscopic, or, rather, by vignette; perhaps a stained-glass window would be a better analogy. One has the initial dialogue, a sort of jam session, between the two collaborators, trying to work out what they are going to do. After a while, the desired theorem crystallizes in their minds, and the hunt is on. One watches, mainly through a series of email transcripts, as their proof takes shape. We are even given the full proofs of a few lemmas, a sampling. There are ups and downs: gaps appear and are filled. The theorem and its proof evolve into something that can be presented to a specialized audience. There follows the first seminar, and in the course of preparation, doubts appear, as is often the case. The audience criticizes, and the authors go back to work. Better approaches evolve; the argument is refined; estimates treated before in a block are attacked individually, sharpened, and the theorem improves. It is submitted to a major journal, and the editors reject it: very good, but too long, not quite there, and so on. The final theorem, hardened and improved by its trial by fire, in the end is accepted. Recognition follows.

The writing is true to life: the emails have the informal style, interspersed with borrowings from English that actual French mathematicians actually use; the descriptive passages are more formal, with a very pleasant prose. The formal mathematics is, well, formal mathematics. The dialogues of the two collaborators as they are beginning to work on their theorem are perhaps a bit artificial and slick, to my mind: a transcript of my own efforts at a blackboard with a collaborator would include a lot more of “Huh? Could you repeat that again?” On the other hand, there is a stylistic challenge of summarizing a five-hour session into something that does not numb the brain. By and large, though, true to the word “living” in the title, the prose reflects life. I presume that there will be an English translation eventually, but one should read the book in French if one can.

Interspersed with the mathematics, there is indeed life, the rest of it, which again resonates with us all: we are not abstract theorem-proving machines, but people, with interests, and families, and various duties. The chapters on the development of the theorem interleave with paragraphs or sections on looking after the children, on travel, on taking a walk in the grounds of the Institute for Advanced Study, on the wonders of tea, or on the difficulties of finding good cheese in the United States. There are several beautiful discourses on music, about which Villani cares very much and which is a constant companion in his life. He is

particularly moved by the songs of Catherine Ribeiro; one of them is reproduced in the book, along with a photograph of her. One has a long and eclectic list, almost like a well-known poem of Prévert, of his musical likes; unlike Prévert’s poem, though, as far as I can tell, it is resolutely raccoon-free. From the animal kingdom, Blake’s Tyger makes an appearance, following a reference to “tyger” phenomena for Burger’s and Euler’s equations.

It is quite naturally in these sections that the author’s personality shines through. There are of course the personal quirks, his clothes for example. (Villani dresses like a nineteenth-century romantic poet.) Mostly, however, through these passages, the strongest sense one gets is of the relentless and sympathetic curiosity, the omnivorous cultural enthusiasm, and the boundless energy of the man, which he deploys with great generosity. The book is interspersed with personal vignettes on other mathematicians, some of them heroes from the past (Malliavin, for example), some of them his colleagues, either in his area, or outside of it, encountered in the course of the few years covered by the book. Each is accompanied by a beautiful line drawing by Claude Gondard of the person in question.

Villani has already had a major impact on the public perception of mathematics in France: he is a born communicator, and he has things to say. The book has been very popular in francophone countries; it was quite visibly displayed in the Montreal bookstore where I bought my copy. The implicit task that Villani had set for himself, of explaining what it is all about, is a difficult one, mostly due to the wide variety of audiences concerned: colleagues in the field, colleagues without, a general scientific reader, and of course members of that vast and rather undefined set—the general public. To give each a sense of what is going on, without pandering, so that each goes away with a good sense of the mathematical process, or of that mathematical process, is not easy. To my mind the book succeeds wonderfully.