

AMERICAN MATHEMATICAL SOCIETY



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June 1975 Issue No. 162

Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Cholicas*) was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
		<u></u> .	
726	August 18–22, 1975 (79th Summer Meeting)	Kalamazoo, Michigan	June 17, 1975
727	October 25, 1975	Cambridge, Massachusetts	Sept. 2, 1975
728	November 1, 1975	Chicago, Illinois	Sept. 2, 1975
729	November 7-8, 1975	Blacksburg, Virginia	Sept. 23, 1975
730	November 15, 1975	Los Angeles, California	Sept. 23, 1975
731	January 22-26, 1976 (82nd Annual Meeting)	San Antonio, Texas	Nov. 5, 1975
732	March 4-5, 1976	Tallahassee, Florida	
733	March 15-20, 1976	Urbana, Illinois	
734	April 23-24, 1976	Reno, Nevada	
735	June 18-19, 1976	Portland, Oregon	
	November 19-20, 1976	Columbia, South Carolina	
	November 26-27, 1976	Albuquerque, New Mexico	
	January 27-31, 1977 (83rd Annual Meeting)	St. Louis, Missouri	

*Deadline for abstracts not presented at a meeting (by title)

August 1975 issue: June 10 October 1975 issues: August 26 November 1975 issue: September 16

OTHER EVENTS

August 16-17, 1975 Short Course on Applied Combinatorics, Kalamazoo, Michigan

Please affix the peel-off label on these \mathcal{N} to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

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Notices)

OF THE

AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors Hans Samelson, Associate Editor

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The Seven Hundred Twenty-Fifth Meeting Washington State University Pullman, Washington June 21, 1975

The seven hundred twenty-fifth meeting of the American Mathematical Society will be held at Washington State University in Pullman, Washington, on Saturday, June 21, 1975. The Mathematical Association of America and the Society for Industrial and Applied Mathematics will hold Northwest Sectional Meetings in conjunction with this meeting of the Society. The Association will have sessions on Friday and Saturday, June 20 and 21; featured addresses will be given by Professor Roy Dubisch, University of Washington, and Professor Ivan Niven, University of Oregon. The main speaker for the SIAM meeting will be Professor Victor Klee of the University of Washington. Professor Sidney G. Hacker of Washington State University will be the speaker at a banquet on Friday evening.

Washington State University has a proposal pending with the NSF to host an NSF Regional Conference the following week, June 23–27. Professor Solomon W. Golomb of the University of Southern California would present a series of lectures on "Practical applications of finite mathematics."

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. Professor Theodore E. Harris of the University of Southern California will lecture at 11:00 a.m. on Saturday on "Some results on Markov interaction processes". Professor David W.Barnette of the University of California at Davis will lecture at 2:00 p.m. on Saturday. The title of his address is "From convex polytopes to manifolds." Both addresses will be given in Room 16 of the Physical Sciences Building, Sessions of contributed papers will be scheduled on Saturday. Late papers will be accepted for presentation at the meeting, but they will not be listed in the printed program of the meeting.

The registration desk will be located in the auditorium foyer of the Physical Sciences Building, and will be open during the following periods: 8:00 a.m. to noon and 1:00 p.m. to 4:00 p.m. on Friday; 8:00 a.m. to noon and 1:00 p.m. to 2:30 p.m. on Saturday. There will be a registration fee of \$2.00.

Dormitory housing will be available on campus. The rates are \$5.00 per person per night on a double occupancy basis, \$7.00 per person per night on a single occupancy basis, and \$3.50 per person for college students. Reservations should have been made prior to May 16.

The following motels are located in Pullman (zip code 99163). Reservations should be made directly with them.

THUNDERBIRD LODGE SE 915 Main Street, Box 255 Phone (509) 332-2646 Single Double	\$ 13 up \$ 18 up
ROYAL MOTOR INN W. 120 Main Phone (509) 564-1254 Single Double	\$ 14 up \$ 17 up
TRAVELODGE S. 515 Grand Phone (509) 564-1143 Single Double	\$ 13 up \$ 18 up

MANOR LODGE MOTEL

Double

Main and Paradise	
Phone (509) 564-1245	
Single	

		\$10.	50	up
		\$12.	50	up

AL KIRCHER'S HILLTOP MOTEL AND STEAK-

HOUSE	
P.O. Box 296	
Phone (509) 564-1195	
Single	\$ 8.40 up
Double	\$ 9.98 up

WILSON COMPTON UNION (on cal	mpus)
Box 2100, College Station	
Phone (509) 335-3578 or 335-3548	
Single	\$10.50
Double	\$13.65

Meals can be taken at the local Pullman restaurants; a list will be provided at the registration desk. The Compton Union Building cafeteria will be open Friday only, for breakfast and lunch. The Association is sponsoring a banquet Friday evening at Austin's Steakhouse.

Pullman is located in southeast Washington at the junction of U.S. Highway 195 and State Highway 270. Moscow, Idaho, is eight miles to the east on Highway 270. The main campus entrance to Washington State University is also on Highway 270, with the dormitory complex on the immediate right at this entrance. The Pullman-Moscow Airport, three miles east of Pullman, is served by Cascade Airways. Limousine service into Pullman is available.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 9:30 A.M.

SATURDAY, 9:30 A.M.			
Session on Algebra and	Number Theory, Room 322, Physical Sciences Building		
9:30 - 9:40 (1)	Axiomatic systems for complex numbers, quaternions and octonions. Professor CARLOS A. INFANTOZZI, Universidad de la República, Montevideo, Uruguay (725-E1)		
9:45 - 9:55 (2)	Semi-prime generalized right alternative rings. Professor IRVIN ROY HENTZEL, Iowa State University, and Dr. GIULIA MARIA PIACENTINI CATTANEO*, University of Rome, Italy and Iowa State University (725-A4)		
10:00 - 10:10 (3)	Fermat's last theorem. I. Some interesting observations for the first case. Professor J. M. GANDHI, Western Illinois University (725-A2)		
10:15 - 10:25 (4)	On the unsolvability of $k/n = 1/x + 1/y + 1/z$. Preliminary report. WILLIAM A. WEBB, Washington State University (725-A1)		
10:30 - 10:40 (5)	Algebraic independence of constants connected with the functions of analysis. Preliminary report. G.V. CHOODNOVSKY, Kiev, U.S.S.R. (725-A3)		
	SATURDAY, 9:30 A.M.		
	d Geometry, Room 328, Physical Sciences Building		
9:30 - 9:40 (6)	Semi-field planes with autotophism groups having large orbits. Professor MICHAEL J. KALLAHER, Washington State University (725-D2)		
9:45 - 9:55 (7)	Ergodic measures for compact group extensions. Professor HARVEY B. KEYNES*, University of Minnesota, and Professor DAN NEWTON, University of Sussex, England (725-G1)		
10:00 - 10:10 (8)	T-regular-closed convergence spaces. Professor R. GAZIK, Arkansas State University, Professor G. RICHARDSON, East Carolina University, and Pro- fessor D.C. KENT*, Washington State University (725-G4)		
10:15 - 10:25 (9)	Sequential convergence in C(X). II. Preliminary report. Professor ROMAN FRIČ, University of Transport Engineering, Žilina, Czechoslovakia, Pro- fessor KELLY McKENNON*, Washington State University, and Professor GARY D. RICHARDSON, East Carolina University (725-G3)		
10:30 - 10:40 (10)	Identities for conjugation in the Steenrod algebra. Professor PHILIP D. STRAFFIN, Beloit College (725-G2)		
	SATURDAY, 9:30 A.M.		
Session on Analysis B	oom 334, Physical Sciences Building		
$\frac{9:30 - 9:40}{11}$ (11)	A symmetric two-body problem in Whitehead's theory of relativity. Dr. ARNOLD A. JOHANSON, University of Toledo (725-C1) (Introduced by Dr. L. Bentley)		
9:45 - 9:55 (12)	Boundary value problems for second order ordinary differential equations and applications to singular perturbation problems. Preliminary report. Dr. TAI-CHI LEE, University of Utah (725-B3)		
10:00 - 10:10 (13)	A decomposition theorem for product measures. Professor ROY A. JOHNSON, Washington State University (725-B4)		
10:15 - 10:25 (14)	Sharpness in the Hausdorff-Young theorem on unimodular groups. Preliminary report. Professor JOHN J.F. FOURNIER, University of British Columbia (725-B2)		
10:30 - 10:40 (15)	On invariant sets. ANDREW M. BRUCKNER, JACK G. CEDER*, and MELVIN ROSENFELD, University of California, Santa Barbara (725-B1) SATURDAY, 11:00 A.M.		
Invited Address, Room (16)	16, Physical Sciences Building Some results on Markov interaction processes. Professor THEODORE E. HARRIS, University of Southern California (725–F1)		
	SATURDAY, 2:00 P.M.		
Invited Address, Room (17)	16, Physical Sciences Building From convex polytopes to manifolds. Professor DAVID W. BARNETTE, University of California, Davis (725-D1)		
Fugeno Orogon	Kenneth A. Ross		

Eugene, Oregon

Kenneth A. Ross Associate Secretary

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Seventy-Ninth Summer Meeting Western Michigan University Kalamazoo, Michigan August 18-22, 1975

SHORT COURSE ON APPLIED COMBINATORICS, August 16 and 17

On the recommendation of its Committee on Employment and Educational Policy, the American Mathematical Society will present a one and one-half day Short Course on Applied Combinatorics on Saturday and Sunday, August 16 and 17, in Room 1104 of Rood Hall at Western Michigan University. The course is designed to give substantial introductions to several important areas of application of combinatorics and graph theory. It is intended to present both mathematically challenging aspects and connections with problems encountered in other disciplines, industrial practice, or work of government agencies. This short course, which is open to all who wish to participate, will be similar in format to the short courses on computing and operations research recently given at Society meetings (August 1973, January 1974, January 1975).

Fulkerson, Department of Operations Research and Center for Applied Mathematics, Cornell University. The current members of the AMS Committee on Employment and Educational Policy are Michael Artin, Charles W. Curtis, Wendell H. Fleming, Calvin C. Moore, Martha K. Smith, and Daniel H. Wagner.

The program will consist of six seventy-five minute lectures on the following topics: A survey of algebraic coding theory, I and II, Elwyn R. Berlekamp, Department of Mathematics and Department of Electrical Engineering and Computer Science, University of California, Berkeley; Some problems involving graphs, D. R. Fulkerson; Combinatorial scheduling theory, I and II, Ronald L. Graham, Bell Laboratories, Murray Hill, New Jersey; and Integer programming, Ellis J. Johnson, Mathematical Sciences Department, IBM T. J. Watson Research Center.

The program is under the direction of D.R.

SEVENTY-NINTH SUMMER MEETING, August 18-22

The seventy-ninth summer meeting of the American Mathematical Society will be held at Western Michigan University, Kalamazoo, Michigan, from Monday, August 18, through Friday, August 22, 1975. All sessions of the meeting will take place on the campus of the university.

Two sets of Colloquium Lectures are scheduled. Ellis R. Kolchin of Columbia University will lecture on "Differential algebraic groups". The other set of lectures will be given by Elias M. Stein of Princeton University; his title is "Singular integrals, old and new." The first lecture in each series will be given in Miller Auditorium on Tuesday afternoon, August 19, with Professor Kolchin speaking at 1:00 p.m. and Professor Stein speaking at 2:15 p.m. The second, third, and fourth lectures in each series will be held at 8:30 a.m. on Wednesday, Thursday, and Friday mornings.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings there will be seven invited one-hour addresses. The names of the speakers, the titles of their addresses, and the times of presentation are as follows: Roy L. Adler, IBM T.J. Watson Research Center, "Ergodic properties of elementary mappings of the unit interval," 11:00 a.m. Friday; Everett C. Dade, University of Illinois at Urbana-Champaign, "Nearly trivial outer automorphisms of finite groups," (tentative title) 9:45 a.m. Wednesday; Bernard Maskit, State University of New York at Stony Brook, "On the classification of Kleinian groups," 11:00 a.m. Wednesday; David Mumford, Harvard University, (title to be announced in the August cNoticca)), 9:45 a.m. Friday; Jack H. Silver, University of California, Berkeley, "The singular cardinals problem," 9:45 a.m. Thursday; James D. Stasheff, Temple University, "The continuous cohomology of groups and classifying spaces," 1:30 p.m. Thursday; Wilhelm F. Stoll, University of Notre Dame," Aspects of value distribution theory in several complex variables," 11:00 a.m. Thursday.

The 1975 Leroy P. Steele Prizes in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein and the Norbert Wiener Prize in Applied Mathematics will be awarded at 3:15 p.m. on Thursday, August 21.

There will be fourteen special sessions of selected twenty-minute papers. Janos D. Aczel of the University of Waterloo is organizing a special session on Functional Equations, to be held Thursday afternoon and all day Friday; the speakers will be announced in the August issue of these cNoticaD. Amassa C. Fauntleroy of the

University of Illinois and Andy R. Magid of the University of Oklahoma are organizing a special session on Affine Algebraic Groups, to be held Thursday and Friday afternoons; the tentative list of speakers includes William Haboush, William F. Hammond, Melvin Hochster, James E. Humphreys, T. Kambayashi, Andy R. Magid, Robert A. Morris, and Joel L. Roberts. George Fix of the University of Michigan is organizing a special session on Scientific Computing, to be held Thursday afternoon and Friday morning; the tentative list of speakers includes Abdul K. Aziz, James Barnes or Allan Lomax, Melvin Clement, James Duderstadt or William Martin, and Dianne M. Prost. Casper Goffman of Purdue University is organizing a special session on Aspects of Real Analysis, to be held all day Thursday; the tentative list of speakers includes Daniel Waterman, Clifford E. Weil, and Robert E. Zink. W. Charles Holland of Bowling Green State University is organizing a special session on Ordered Groups, to be held Wednesday and Thursday mornings; the tentative list of speakers includes Richard N. Ball, Richard D. Byrd, Andrew M.W. Glass, Stanley P. Gudder, Herbert A. Hollister, Justin T. Lloyd, Jorge Martinez, Norman Reilly, Akbar H. Rhemtulla, and J. Roger Teller. Robert E. Huff of Pennsylvania State University is organizing a special session on Banach Spaces with the Radon-Nikodym Property, to be held all day Thursday and Friday; the tentative list of speakers includes William J. Davis, Joseph Diestel, Michael Edelstein, Gerald A. Edgar, Daniel R. Lewis, Heinrich P. Lotz, Peter D. Morris, Terry J. Morrison, Haskell P. Rosenthal, Francis Sullivan, and J. Jerry Uhl, Jr. Paul C. Kainen of Case Western Reserve University is organizing a special session on Topological and Chromatic Graph Theory to be held Tuesday afternoon and Wednesday morning; the tentative list of speakers includes Michael O. Albertson, Seth R. Alpert, Frank R. Bernhart, Jonathan L. Gross, Pavol Hell, Richard D. Ringeisen, Saul Stahl, and Arthur T. White II. Leroy M. Kelly of Michigan State University is organizing a special session on Geometry of Metric Spaces, to be held Tuesday afternoon and Wednesday morning; the tentative list of speakers includes J. Ralph Alexander, Jr., Leonard M. Blumenthal, David C. Kay, Leroy M. Kelly, William A. Kirk, Clinton M. Petty, Kenneth B. Stolarsky, and Hans S. Witsenhausen. Pierre J. Malraison, Jr. of Carleton College is organizing a special session on Categorical Methods in Algebraic Topology, to be held all day Friday; the tentative list of speakers includes John M. Boardman, Martin Fuchs, Peter V.Z. Cobb, Dana May Latch, Pierre J. Malraison, Jr. (speaking on a paper of Jonathan M. Beck), Marvin V. Mielke, and R. Neil Vance. Tilla K. Milnor of Rutgers University is organizing a special session on Riemannian Geometry, to be held Wednesday and Thursday mornings; the tentative list of speakers includes Bang-Yen Chen, Harold G. Donnelly, Patrick B. Eberlein, Robert B. Gardner, Peter B. Gilkey, Herman R. Gluck, N. Gromov, Ravi Kulkarni, and Ann K. Stehney. David E. Muller of the University of Illinois is organizing a special session on Theoretical Computer Science, to be held all day Thursday; the tentative list of speak-

ers includes Michael A. Harrison, Chung Laung Liu, Franco P. Preparata, and Shmuel Winograd. Peter J. Nyikos of the University of Illinois is organizing a special session on General Topology, to be held Tuesday afternoon, Wednesday morning, Thursday morning, and all day Friday; the tentative list of speakers includes Raymond F. Dickman, Jr., Gary F. Gruenhage, R.F. Levy, Minakshisundaram Rajagopalan, Mary Ellen Rudin, J.C. Smith, Jr., Jerry E. Vaughan, and Howard H. Wicke. Hans Schneider of the University of Wisconsin is organizing a special session on Numerical Ranges for Matrices and Other Operators on Normed Spaces, to be held Tuesday afternoon and Wednesday morning, the tentative list of speakers includes Earl R. Berkson, Moshe Goldberg, Charles R. Johnson, Marvin Marcus, B. David Saunders, and Joseph G. Stampfli. Peter J. Weinberger of the University of Michigan is organizing a special session on Efficient Algorithms for Exact Computation, to be held all day Friday. The speakers will be announced in the August issue of these *cNotices*). Most of the papers presented at these fourteen sessions will be by invitation.

Contributors of abstracts for the meeting who felt that their papers would be particularly appropriate for one of these special sessions were requested to indicate this conspicuously on the abstract and submit it by May 27, 1975, three weeks earlier than the deadline for abstracts for contributed ten-minute papers, in order to allow time for the additional handling necessary.

There will be sessions for contributed tenminute papers on Tuesday afternoon, Wednesday morning, and all day Thursday and Friday. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940; the deadline for receipt of abstracts is June 17, 1975. There is no limit on the number of papers that will be accepted for presentation. Those individuals having time preferences for the presentation of their papers should so indicate clearly on their abstracts. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

There will be a Poster Session for contributed papers from 3:30 p.m. to 5:00 p.m. on Tuesday, August 19. (Please note that the date and time are different from those announced in the April issue of these cNoticea).) Poster Sessions represent an alternative method for presenting papers. At the session individuals will display their papers either on an easel or a bulletin board, and remain in the room set aside for this purpose to expand on the material and answer questions during the one and one-half hour session. Individuals who wish their papers considered for the poster session should so indicate on their abstracts, clearly in large block letters.

Rooms 1110 and 1111 in Brown Hall have been set aside as informal discussion rooms, and will be open daily from 8:00 a.m. to 6:00 p.m. to small groups desiring a quiet room with blackboard space to discuss mathematics. Room 1110 is available on a first-come, firstserved basis; Room 1111 is available for onehour periods only, and must be reserved in advance. A reservation form will be posted on the door to Room 1111 for individuals to sign up for use of this room. It is requested that discussion groups not be planned to conflict with business meetings or major lectures.

The AMS Committee on Employment and Educational Policy (CEEP) will sponsor a panel discussion on "The role of applications in Ph.D. programs in mathematics" on Thursday evening, August 21, at 8:00 p.m. Members of the panel will include Richard D. Anderson, former chairman of CEEP; Lipman Bers, president of the American Mathematical Society; Henry O. Pollak, president of the Mathematical Association of America; and Wendell H. Fleming, current chairman of CEEP, who will serve as moderator. The same committee is planning an open meeting at 4:30 p.m. on Monday, August 18, consisting of a brief report on the state of the job market, followed by an open discussion with comments and suggestions welcomed from the audience.

This meeting of the Society will be held in conjunction with the annual meetings of the Mathematical Association of American and Pi Mu Epsilon. The Mathematical Association of America will meet from Monday, August 18, through Wednesday, August 20. The twenty-third series of the Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Frederick J. Almgren, Jr., Princeton University. The title of his lectures is "Geometric measure theory and the calculus of variations." At the Business Meeting of the Association at 10:00 a.m. on Tuesday, August 19, the Lester R. Ford Awards will be presented.

J. Sutherland Frame, Michigan State University, will address Pi Mu Epsilon on Tuesday, August 19, at 8:00 p.m.; the title of his lecture will be "Matrix functions: a powerful tool."

The Association for Women in Mathematics will hold a panel discussion on "Noether to nowthe woman mathematician" on Tuesday, August 19, at 3:30 p.m. The Mathematicians Action Group will hold a discussion on "Unemployment: an exchange of experiences" at 4:30 p.m. on Wednesday, August 20. All mathematicians who have recently experienced unemployment or who anticipate being unemployed in the near future are especially welcome.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 5:00 p.m. on Tuesday, August 19, in the Green Room of Miller Auditorium (not at the University Student Center as previously announced).

The Business Meeting of the Society will be held in Miller Auditorium at 4:00 p.m. on Thursday, August 21. The secretary notes the following resolution of the Council: Each person who attends a Business Meeting of the Society shall be willing and able to identify himself as a member of the Society. In further explanation, it is noted that "each person who is to vote at a meeting is thereby identifying himself as and claiming to be a member of the American Mathematical Society."

In accord with Article X, Section 1, of the bylaws of the Society, the Business Meeting of January 24, 1975, in Washington, D.C. has directed that each of the following motions be placed on the agenda of the Business Meeting of August 21, 1975, in Kalamazoo:

Motion I. In the face of the deepening economic crisis, the rapidly rising unemployment from which mathematicians are not exempt, the ominous nature of massive budget cuts for education and other social services, and the Presidential request for an immediate increase of 300 million for military aid to the Thieu regime augmenting the already huge military budget, it is the sense of this meeting that the federal government must: (1) Fund a national open admissions program at institutions of higher education (2) Fund a massive public works program which will use the skills of the presently and soonto-be unemployed (including mathematicians) for sorely needed socially useful tasks (3) Transfer massive funds from the military budget to accomplish these aims. We call upon the officers of the Society to work towards effecting the implementation of the above.

<u>Motion II.</u> Resolved that the AMS will not cooperate with UNESCO until such time as the ruling removing Israel from any regional grouping is rescinded.

<u>Motion III.</u> That the officers of the Society be requested to arrange that an early meeting of the Society schedule an open session intended to discuss the possible establishment of an independent ASSOCIATION OF MATHEMATICIANS FOR SOCIAL ACTION.

This announcement constitutes the notice to the full membership required in the bylaws. At the meeting in August, each motion is subject to substantive changes, such as germane amendment or substitution, and is subject to subsidiary resolutions concerning the disposition of the main motion.

Panel discussions have been scheduled on Motions I and II. Motion I will be discussed at 9:30 p.m. on Monday, August 18. Motion II will be discussed at 3:30 p.m. on Tuesday, August 19. Time has been allotted for audience participation in both cases.

AMENDMENT TO THE BYLAWS

The Council at its meeting of April 11, 1975, recommended the following amendment to Article XI, Section 2, of the bylaws:

The editorial management of the *Notices*) shall be in the hands of a committee [consisting of the executive director and the secretary] chosen in a manner established by the *Council.*

Here the words in brackets are to be deleted and the words in italics added. The intent is to make it possible to replace the two existing editors with a larger and more ecumenical committee to consider, in particular, articles and Letters to the Editor. The amendment becomes effective if adopted by a two-thirds vote of the members present at the Business Meeting. This paragraph constitutes the required notice of action.

MEETING PREREGISTRATION AND REGISTRATION

Registration for the short course <u>only</u> will begin on Friday, August 15. Lecture notes and other short course material will be distributed before the first session at the short course registration desk. Those individuals who do not preregister for the short course are strongly urged to register and pick up their material Friday evening in the dormitory, so as not to miss the start of the lecture on Saturday morning. General meeting registration will commence on Sunday, August 17, at 2:00 p.m. Participants who are not attending the short course are advised that no general meeting information (or registration material) will be available prior to the time listed below for the Joint Mathematics Meetings registration. Upon arrival at the Western Michigan University campus, participants should proceed directly to the reception desk, Harvey Garneau Hall, Goldsworth Valley Residence Hall Complex #2, in order to check in to their accommodations and purchase meal tickets, if desired.

Following are the hours that the desks will be open as well as the respective locations:

Applied Combinatorics Short Course

Date and Time Friday, August 15	Location
4:30 p.m7:30 p.m.	Reception Desk Lobby, Harvey Garneau Hall, Goldsworth Valley Resi- dence Hall Complex #2
Saturday, August 16 8:00 a.m4:00 p.m. Sunday, August 17	1104 Rood Hall Lobby
12:00 noon-2:00 p.m.	1104 Rood Hall Lobby
Joint Mathemati	ice Monting

Joint Mathematics Meetings

Date and Time Sunday, August 17	Location
2:00 p.m8:00 p.m.	Miller Auditorium Lobby, 2nd level
Monday, August 18	
8:00 a.m5:00 p.m.	Miller Auditorium Lobby, 2nd level
Tuesday, August 19 to	
Thursday, August 21	
8:30 a.m4:30 p.m.	Miller Auditorium Lobby, 2nd level
Friday, August 22	
8:30 a.m1:30 p.m.	Miller Auditorium Lobby, 2nd level

Participants who wish to preregister for the meetings should complete the Meeting Preregistration Form on the last page of these *cNoticas*. Those who preregister will pay a lower registration fee than those who register at the meeting, as indicated in the schedule below. Preregistrants will be able to pick up their badges and programs when they arrive at the meeting after 2:00 p.m. on Sunday, August 17, at the Joint Mathematics Meetings registration desk. Complete instructions on procedures for making hotel, motel, or dormitory reservations are given in the sections entitled RESIDENCE HALL HOUSING and HOTELS AND MOTELS.

Meeting registration and preregistration fees partially cover expenses of holding the meetings. The preregistration fee does <u>not</u> represent an advance deposit for lodgings.

Please note that a separate registration fee is required for the Short Course and the Joint Meetings. These fees are as follows:

Applied Combinatorics Short Co	urse
Preregistration (by mail prior to 8/1) All participants \$12	At Meeting \$15
Joint Mathematics Meeting	<u>çs</u>
Preregistration	At

Å

	Preregistration	At
	(by mail prior to $8/1$)	Meeting
Member	\$10	\$12
Student or un	employed	
member	\$ 1	\$2
Nonmember	\$16	\$20

There will be no extra charge for members of the families of registered participants except that all professional mathematicians who wish to attend sessions must register independently.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position. Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than \$7,000 from employment, fellowships, and scholarships.

Checks for the preregistration fee(s) should be mailed to arrive in Providence not later than August 1, 1975. Participants should make their own reservations directly with hotels or motels in the area (cf. section titled HOTELS AND MOTELS.) It is essential, however, to complete the Meeting Preregistration Form on the last page of these *cNolica* to take advantage of the lower meeting registration fee(s).

A fifty percent refund of the preregistration fee will be made for all cancellations received in Providence prior to August 16. There will be no refunds granted for cancellations received after that date or to persons who do not attend the meetings.

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

An experimental variant of the open register will be operated on a limited basis during the meeting, providing an opportunity for applicants and employers to arrange interviews at their mutual convenience.

Employment Register headquarters, located in the Green Room of Miller Auditorium, will be open on Tuesday (August 19) from 8:30 a.m. to 4:00 p.m., and on Wednesday and Thursday (August 20 and 21) from 8:30 a.m. to 4:30 p.m. The room will be closed from noon to 1:15 p.m.

There will be no interviews scheduled by the staff. Instead, facilities will be provided for applicants and employers to display resumes and listings. Message boxes will be set up for individuals to leave messages for one another requesting interviews. Tables and chairs will be provided in the room for interviews. Employers are encouraged to attend the meetings and participate if possible. Applicants should recognize that the Mathematical Sciences Employment Register cannot guarantee that any employers will in fact attend the meeting or be able to participate. The AMS-MAA-SIAM Committee on Employment Opportunities will, however, request employers listing in the June and July 1975 issues of <u>Employment Information for Mathematicians</u> to signify in their listing their intention of participating in the open register at the summer meeting.

EXHIBITS

The book and educational media exhibits will be displayed on the second lobby level of Miller Auditorium at the following times: August 18 (Monday), noon to 4:30 p.m.; August 19-20 (Tuesday and Wednesday), 8:30 a.m. to 4:30 p.m.; and August 21 (Thursday), 8:30 a.m. to noon. All participants are encouraged to visit the exhibits sometime during the meeting.

RESIDENCE HALL HOUSING

The Goldsworth Valley Residence Hall Complex #2 has been set aside for the exclusive use of the Mathematics Meetings participants and for participants of the Applied Combinatorics Short Course. The dormitories are within a ten to fifteen minute walk from the Miller Auditorium and the meeting rooms which will be used during the meetings. Frequent shuttle bus service will be provided between the dormitories and the meeting area. There is a twenty-five cent (\$0.25) fare for each one-way trip. The cash fare will be collected by the bus driver; it would be appreciated if exact fares are paid by participants to avoid the necessity of having drivers carry excess money.

Most of the sleeping rooms are in suites of two double rooms which share a bath. Linens, towels, and daily maid service (beds made only) are provided with all rooms. Each room contains the following furniture: two twin beds, a chest of drawers, a lounge chair, a desk, and two study chairs. Keys, curtains, glasses, and soap are also provided. Each dormitory has a fully equipped laundry room with coin-operated washers and dryers. Ironing facilities are also available. A limited number of irons are available at dormitory desks.

Residence hall rooms can be occupied from noon on Friday, August 15, to noon on Sunday, August 17, for participants in the Applied Combinatorics Short Course; and from noon on Sunday, August 17, to noon on Saturday, August 23, for Joint Meeting participants. Clerks will be available on call in each housing unit twenty-four hours a day. Parking for residents will be available free of charge in lots near the dormitories. The daily rate per person is as follows:

Singles	\$8.	00	per	person	per	day
Doubles	\$5.	75	per	person	per	day

Children will be housed at the regular rates in rooms adjacent to their parents. Cribs and cots are not available from the university and sleeping bags are not permitted. An infant may occupy the parents' room at no extra charge if the parents supply a crib and bedding. A limited number of cribs are available for rent by writing in advance to United Rent-All, 403 Balch Street, Kalamazoo, Michigan 49003. Pets are not permitted in the dormitories.

To be assured of a room, guests should register in advance. Please use the Room Reservation Form provided on the last page of these c*Notices*). Residence hall reservation requests will be acknowledged by the University.

HOTELS AND MOTELS

There are a number of hotels and motels in the area which are listed below. All prices are subject to change without notice; a six percent tax should be added to the room rates listed. It should be noted that this area is a popular vacation resort area so that early reservations are recommended. Participants should make their own reservations. The following codes apply: FP - Free Parking; SP - Swimming Pool; AC -Air Conditioned; TV - Television; CL - Cocktail Lounge; RT - Restaurant.

HOLIDAY INN - CROSSTOWN (616) 349-6711 220 E. Crosstown (8 blocks south of downtown Kalamazoo)

147 rooms				
Singles			\$14.00	
Doubles	1 Bed	2 Persons	18.00	
	2 Beds	2 Persons	19.00	
Studios		1 Person	15.00	
		2 Persons	19.00	
Twins	2 Beds	2 Persons	19.00	
Extra person in room			4.00	
Code: FP-SP-AC-TV-CL-RT				
1-1/2 mile	s from camp	us		

HOLIDAY INN - EXPRESSWAY (616-381-7070) 3522 Sprinkle Road (Use Exit 80 at I-94, turn south).

~~~~	/•
146	rooms
1-10	1001113

Singles	1 Bed	1 Person	\$16.00	
	1 Bed	2 Persons	20.00	
Doubles	2 Beds	2 Persons	21.00	
	$2  \operatorname{Beds}$	3 Persons	25.00	
Twins	2 Beds	4 Persons	29.00	
Suites	1 Person		20.00	
(two available only)				
	2 Persons		24.00	
Code: FP-SP-AC-TV-CL-RT				
7 milos fr	0.000 000 001 0			

⁷ miles from campus

HOLIDAY INN - WEST (616) 375-6000

2747 - 11th Street (Use Oshtemo Exit at U.S. 131, turn west)

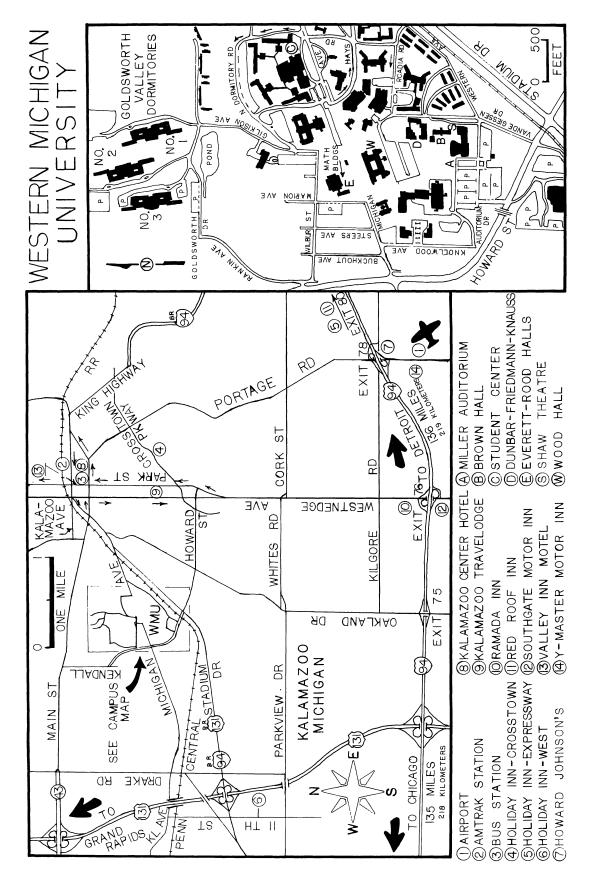
118 rooms		
Singles	1 person	\$17.00
	2 persons	21.00
Doubles	1 person	17.00
	2 persons	22.00
Twins	Same as a double	
Suites	1 person	18.00
	2 persons	23.00
Extra pers	son in room	4.00
Code: FP-	SP-AC-TV-CL-RT	
2 milos fre	om compute	

3 miles from campus

HOWARD JOHNSON'S (616) 382-2303

1912 East Kilgore (Exit 78 off I-94; turn south) 70 rooms Singles \$15.00

		φ±0.00
Doubles		19.00
Twins	1 person	16.00
	2 persons	21.50
Extra per	son in room	4.00
Code: FP-	-SP-AC-TV-CL-RT	
6 miles fr	om campus	



KALAMAZOO CENTER HOTEL ( 100 West Michigan (Downtown Kal 288 rooms	
Singles	\$22,00
Doubles	30.00
Suites	30.00- 76.00
Extra person in room	6.00
(12 years and older)	
Code: FP-SP-AC-TV-CL-RT	
2 miles from campus	

KALAMAZOO TRAVELODGE (616) 381-5000

(Toll free number 1-800-255-3050)

1211 S. Westnedge (Exit 76-B off I-92; turnnorth) 57 rooms Singles ¢12 00

Diligics	ψ10.00
Doubles	15.00
Twins	17.00
Extra person in room	2.00
Rollaway bed	3.00
Code: FP-AC-TV	
1-1/2 miles from campus	

RAMADA INN (616) 382-1000

12111111111 (010) 001 1000	
5300 S. Westnedge (Exit 76 off I-94;	turn north)
102 rooms	
Singles	\$15.50
Doubles	18.50
Twins	20.50
Suites (one available only)	24.50
Extra person in room	3.00
Code: FP-SP-AC-TV-CL-RT	

5 miles from campus

DED DOOR DDI (C1C) 909 COFA

RED ROOF INN (616) 382-6350	
3701 E. Cork (Exit 80 off I-94,	Sprinkle Road)
79 rooms	
Singles	\$ 9.50
Doubles	12.50
Twins	13.50
Extra person in room	3.00
Code: FP-AC-TV	

4-1/2 miles from campus

#### SOUTHGATE MOTOR INN (616) 343-6143 5630 S. Westnedge (Exit 76 off I-94) 125 rooms

Singles	\$14.00
Doubles	16.50
Twins	18.00
Extra person in room	3.00
Code: FP-SP-AC-TV-CL-RT	
5 miles from campus	

VALLEY INN MOTEL (616) 349-9736 200 N. Park (Downtown Kalamazoo) 107 rooms Singles Doubles Twins

		<b>H0</b> .00
Suites	25.00 and	30.00
Extra person in room		4.00
Special group rates available		
Code: FP-SP-AC-TV-CL-RT		
2 miles from campus		

Y-MASTER MOTOR INN (616) 345-8603 2333 Helen (1/4 mile south of I-94, off Portage Road opposite airport) 50 rooms Singles \$10.50 Doubles 14.50 Twins 15.50

Studios	\$20.00
Suites	25.00
Extra person in room	3.00
Code: FP-SP-AC-TV	
10 miles from campus	

#### FOOD SERVICES

The Goldsworth Valley #2 Residence Hall Cafeteria (not #3 as previously announced) will be open starting with breakfast on Monday, August 18. The cafeteria will continue serving through lunch on Friday, August 22, with the exception of the evening meal on Wednesday, August 20. Hours of service and prices for individual meals are:

-		Children	
			Age 6
		Ages	$\operatorname{and}$
Hours	Adults	7-12	Under
Breakfast			
7:15 a.m 8:30 a.m.	\$2.00	\$1.50	\$1.00
Lunch			
11:30 a.m12:45 p.m.	2.50	2.00	1.25
Dinner			
5:00 p.m 6:30 p.m.	4.50	3.25	2.25

A package plan including all meals served from breakfast on Monday, August 18, to lunch on Friday, August 22, (with the exception of the evening meal on Wednesday, August 20) will be available at the dormitory check-in desk at a price of \$32 for adults, \$24.75 for children from seven to twelve years of age, and \$16 for children six years of age and under.

Light snacks and beverages will be available at the snack bar in the University Student Center; the hours of operation will be posted in appropriate areas during the meeting.

For participants in the Short Course, the University Student Center Cafeteria and/or the snack bar will be open for meals. (Please note that this is a change from the information which appeared in the April issue of these cNotices.) The hours of operation will be posted in the dormitories.

#### PARKING

Parking permits will be required for parking on all areas of the campus, with the exception of metered lots. Parking will be free of charge in the lots near the dormitories for persons residing there, and in the lots near Miller Auditorium. Maps showing the location of the various college parking lots will be available at the Local Information Desk along with the permits. There is no charge for the permits.

#### CAMPING

The following campgrounds are located approximately one-half hour or less from Western Michigan University:

1. Klines Resort, Route 2, Box 257, Three Rivers, 49093 Telephone: 616-649-2514 Travel Time: 35 minutes from W.M.U. Facilities: Electric hook-ups, bathrooms, lakeside swimming, boat rentals, 40 sites Cost: \$4.00 per night, \$24.00 per week, reservations accepted

\$14.50

20.00

20.00

- Oak Shores Resort, Route 3, 28th Street, Vicksburg, 49007 Telephone: 616-649-1310 Travel Time: 30-25 minutes from W. M. U. Facilities: 80 acre lake, swimming, boating, water and electricity at sites, also sewer hook-up available, club area, 93 sites available Cost: Water and electricity \$4.00/day,
- Schnable Lake Campground, 11th Street, Martin, 49070 Telephone: 616-672-7524 Travel Time: 30 minutes from W. M. U. Facilities: 45 acre lake, swimming, canoeing, water and electricity are provided, 93 camp sites

sewer hook-up \$4.50/day

Cost: \$4.00 per night, deposits required

- 4. <u>Shady Bend Park</u>, 15320 Augusta Drive, Augusta, 49012 Telephone: 616-731-4503 Travel Time: 25 minutes from W. M. U. Facilities: The pavillion houses showers and toilets, spring fed pond, canoeing, 62 sites available Cost: \$3.50 per night
- 5. <u>Willow Lake Campground</u>, Box 295, Three Rivers, 49093 Telephone: 616-279-7920 Travel Time: 20 minutes from W. M. U. Facilities: Tent camping, water, campfires, toilet facilities, private fishing lake on 110 acres, 51 sites available Cost: \$3.00 per day

#### BOOKSTORES

The Campus Bookstore is located in the University Student Center. Its hours of operation are from 8:00 a.m. to 5:00 p.m., Monday through Friday. The University Bookstore (private) at 2529 W. Michigan is open from 9:00 a.m. to 5:00 p.m. Monday through Friday, and from 10:00 a.m. to 3:00 p.m. on Saturday. The Book Raft (private) at 2624 W. Michigan is open from 10:00 a.m. to 7:30 p.m. Monday through Thursday, from 10:00 a.m. to 9:30 p.m. on Friday and Saturday, and from 10:00 a.m. to 5:00 p.m. on Sunday. The latter two are also situated near the campus area.

#### LIBRARIES

The mathematics library, including current mathematical journals and books, is part of the Physical Sciences Library on the third floor of Rood Hall, and is open from 8:00 a.m. to midnight Monday through Thursday, from 8:00 a.m. to 5:00 p.m. on Friday, from 10:00 a.m. to 5:00 p.m. on Saturday, and from 1:00 p.m. to midnight on Sunday. Information can also be obtained in the mathematics library regarding the location of books in other areas. The main collection of other books is in the Waldo Library, which is open from 8:00 a.m. to 11:00 p.m., Monday through Thursday, 8:00 a.m. to 5:00 p.m. on Friday, 10:00 a.m. to 5:00 p.m. on Saturday, and from 1:00 p.m. to 11:00 p.m. on Sunday.

The Kalamazoo Public Library has its main

branch located at 315 S. Rose. The hours of operation are from 9:00 a.m. to 9:00 p.m. Monday through Friday, from 9:00 a.m. to 6:00 p.m. on Saturday, and from 2:00 p.m. to 6:00 p.m. on Sunday.

#### MEDICAL SERVICES

Kalamazoo is served by Borgess Hospital and Bronson Methodist Hospital. The emergency rooms there are staffed around the clock. The Kalamazoo Academy of Medicine can also make referrals Monday through Friday, from 8:00 a.m. to noon, and from 1:00 p.m. to 5:00 p.m. (Telephone: 342-8502). Referrals to dentists can be made by calling 381-0400 during usual office hours. For dental emergency service, the dentist on call may be reached through the Bronson Methodist Hospital. Additional information will be available at the Mathematics Meetings Registration Desk.

#### ENTERTAINMENT AND RECREATION

Western Michigan University has planned a program of recreation and entertainment for mathematicians and their families. It is hoped that many people as possible will take advantage of these activities.

A picnic is planned for Wednesday, August 20, at 6:00 p.m. The menu will be barbecued chicken and baked ham; no alcoholic beverages will be served. The cost will be \$5 per adult, \$4 for children between the ages of seven and twelve, and \$2.50 for children six years of age and under.

A beer party has been arranged to follow the picnic from 9:00 p.m. to midnight at the Holiday Im-West. Soft drinks in cans will also be available for those who prefer them. Potato chips and similar snacks will be served. The cost is \$2.50 per person. Bus service from the campus to the Holiday Im-West and back will be available; the fare will be \$0.25 each way.

Tickets to both the picnic and beer party will be sold in advance at the Local Information Desk in the Miller Auditorium Lobby, but can also be purchased at each event.

A tour of the Upjohn pharmaceutical plant, whose home office is in Kalamazoo, is planned for Tuesday morning, August 19. Space on this tour is limited, and it will be necessary to make reservations in advance at the Local Information Desk. Bus service to the plant will be available for a nominal fee.

Michigan is the third largest wine producing state (after California and New York). The wineries are centered in an area 15-20 miles west of Kalamazoo, and all welcome visitors. A tour of several of the wineries, with ample tasting privileges, is planned for Tuesday afternoon, August 19. Again, transportation will be available at a moderate cost, and interested parties can obtain further information and sign up for the tour at the Local Information Desk.

Several other local industries, such as the Kellogg cereal company in Battle Creek, offer tours of their facilities. Information about these tours will be available at the Local Information Desk.

Nearby Colon, Michigan, is the "Magic (Text continued on page 178)

The purpose of this summary is to provide assistance to registrants in the selection of arrival and departure dates. The program, as outlined below, is based on information available at press time.

SATURDAY, August 16			
8:00 a.m 4:00 p.m.	REGISTRATION	(Short Course Only)	
9:00 a.m 10:15 a.m.	Some problems inv		
	D. R. Fulkerso		
10:45 a.m 12:00 noon	Combinatorial scheduling theory I Ronald L. Graham		
2:00 p.m 3:15 p.m.	A survey of algebraic coding theory, I Elwyn R. Berlekamp		
3:45 p.m 5:00 p.m.	Combinatorial scheduling theory II Ronald L. Graham		
SUNDAY, August 17			
12:00 noon - 2:00 p.m.	REGISTRATION	(Short Course Only)	
2:00 p.m 3:15 p.m.	A survey of algebr	aic coding theory, II	
3:45 p.m 5:00 p.m.	Elwyn R. Berlekamp Integer programming Ellis J. Johnson		
	AMS — MAA SUMMER MEETINGS		
SUNDAY, August 17	American Mathematical Society	Mathematical Association of America	
9:00 a.m 4:00 p.m.	MAA Board of Governors Meeting		
2:00 p.m 8:00 p.m.	REGI	STRATION	
7:00 p.m 9:30 p.m.		MAA - Film Program	
7:00 p.m 7:25 p.m.	Logic II: a B.B.C. Broadcast as par Open University Foundation Course in matics (black and white)		
		Films of the College Geometry Project	
7:30 p.m 7:43 p.m.	Dihedral kaleidoscopes		
7:45 p.m 8:01 p.m.	Curves of constant width		
8:05 p.m 8:21 p.m.		Projective generation of conics	
		Films of the MAA Calculus Film Series	
8:25 p.m 8:45 p.m.	What is area?		
8:50 p.m 9:04 p.m.	Area under a curve		
9:08 p.m 9:30 p.m.		Shapes of the future I: Some unsolved prob- lems in geometry—two dimensions with Victor Klee, a film of the MAA Individual Lectures Film Project	
MONDAY, August 18	AMS	МАА	
8:00 a.m 5:00 p.m.	REGISTRATION		
9:00 a.m 9:15 a.m.	WELCOME ADDRESS John Bernhard, President Western Michigan University		
9:15 a.m 10:15 a.m.	THE EARLE RAYMOND HEDRICK LECTURES I: Geometric measure theor and the calculus of variations Frederick J. Almgren, Jr.		
10:30 a.m 11:00 a.m.	AMS-MAA Panel discussion: The mathematics accreditation question Wade Ellis Daniel T. Finkbeiner II (moderator) Shirley A. Hill		
	Calvin T. Long General discussion by panel and audience		
11:30 a.m 12:00 noon			

MONDAY, August 18	American Mathematical Society	Mathematical Association of America	
12:00 noon - 4:30 p.m.	EXI	HIBITS	
1:30 p.m 2:30 p.m.	EXHIBITS THE EARLE RAYMOND HEDRICK LECTURES II Frederick J. Almgren, Jr.		
3:00 p.m 4:00 p.m.		INVITED ADDRESS Mathematical models in cancer research James R. Thompson	
4:30 p.m 5:30 p.m.	AMS Committee on Employment and Educational Policy Open meeting: The state of the job market Richard D. Anderson Charles W. Curtis Wendell H. Fleming (moderator)		
7:00 p.m 9:30 p.m.		MAA - Film Program	
7:00 p.m 8:03 p.m.		NIM and other oriented graph games (Andrew Gleason), a film of the MAA Mathematics Today Series (black and white)	
8:10 p.m 8:26 p.m.		Space filling curves, a film of the Topology Film Project	
8:30 p.m 8:45 p.m.		The theorem of the mean, a film of the MAA Calculus Film Series	
8:50 p.m 9:00 p.m.		Linear programming	
9:05 p.m 9:30 p.m.		Fourier Series—a B. B. C. Broadcast as part of the Open University's course Linear Mathe- matics	
9:30 p.m 10:30 p.m.	AMS Panel discussion: Motion I		
TUESDAY, August 19	AMS	Other Organizations	
8:30 a.m 4:30 p.m.	REGISTRATION		
8:30 a.m 4:30 p.m.	EX	HIBITS	
8:30 a.m 4:00 p.m.	EMPLOYME	ENT REGISTER	
9:00 a.m 10:00 a.m.	THE EARLE RAYMOND HEDRICK LECTURES III Frederick J. Almgren, Jr.		
10:15 a.m 11:15 a.m.		MAA Business Meeting Presentation of Lester R. Ford Awards	
11:30 a.m 12:30 p.m.		MAA INVITED ADDRESS Some recent applications of graph theory Ronald L. Graham	
12:00 noon		Pi Mu Epsilon Council luncheon	
12:00 noon - 1:00 p.m.		Mathematicians Action Group Steering Committee - Open meeting	
1:00 p.m 2:00 p.m.	COLLOQUIUM LECTURE I Differential algebraic groups Ellis R. Kolchin		
2:15 p.m 3:15 p.m.	COLLOQUIUM LECTURE I Singular integrals, old and new Elias M. Stein		
3:00 p.m 5:00 p.m.		Pi Mu Epsilon Contributed Paper Sessions	
3:30 p.m 4:30 p.m.	AMS Panel discussion: Motion II John W. Milnor (moderator)		
3:30 p.m 5:00 p.m.	Poster Sessions		
3:30 p.m 5:00 p.m.		Association for Women in Mathematics Panel discussion: Noether to now-the woman mathematician Lenore Blum Vivienne Mays M. Susan Montgomery Mary Ellen Rudin Jane Cronin Scanlon Alice T. Schafer (moderator)	
3:30 p.m 5:30 p.m.	Sessions for Contributed Papers and Special Sessions	AWM Business Meeting	
5:00 p.m.	AMS Council Meeting		
6:30 p.m.		IME Banquet	

TUESDAY, August 19	American Mathematical Society	Other Organizations
7:00 p.m 9:15 p.m.		MAA - Film Program
		Films produced by C. B. Allendoerfer
7:00 p.m 7:25 p.m.		Gauss-Bonnet theorem
7:30 p.m 7:52 p.m.		Cycloidal curves or Tales from the Wanklen- berg Woods
8:00 p.m.		Allendoerfer Films of the MAA Arithmetic Films
8:00 p.m 8:10 p.m.		Area and pi
8:13 p.m 8:20 p.m.		Associative property
8:25 p.m 8:35 p.m.		Binary operations and commutative property
8:37 p.m 8:45 p.m.		Distributive property
8:50 p.m 9:00 p.m.		Geometric concepts
9:05 p.m 9:15 p.m.		Geometric transformations
8:00 p.m.		IME INVITED LECTURE Matrix functions: a powerful tool J. Sutherland Frame
WEDNESDAY, August 20	AMS	Other Organizations
8:00 a.m.		IME Dutch Treat Breakfast
8:00 a.m 12:00 noon	Sessions for Contributed Papers and Special Sessions	INE DUCH Treat Breaklast
8:30 a.m 4:30 p.m.		I STRATION
8:30 a.m 4:30 p.m.		THIBITS
8:30 a.m 4:30 p.m.		ENT REGISTER
8:30 a.m 9:30 a.m.	COLLOQUIUM LECTURES II	
	Ellis R. Kolchin	
8:30 a.m 9:30 a.m.	COLLOQUIUM LECTURES II Elias M. Stein	
9:45 a.m 10:45 a.m.	INVITED ADDRESS Nearly trivial outer automorphisms of finite groups (title tentative) Everett C. Dade	
10:00 a.m.		MAG - Business Meeting
10:00 a.m 12:00 noon		IME Session for Contributed Papers
11:00 a.m 12:00 noon	INVITED ADDRESS On the classification of Kleinian groups Bernard Maskit	
1:30 p.m 2:30 p.m.		MAA INVITED ADDRESS Error correcting codes: Practical origins and mathematical implications Vera T. Pless
2:45 p.m 3:45 p.m.		MAA Panel discussion: The training of non- doctoral mathematics students for nonacademi employment David C. Bossard Alan Karr Dale W. Lick (moderator) Werner Ulrich
3:45 p.m 4:15 p.m.		General discussion by panel and audience
4:30 p.m 6:00 p.m.	MAG - Discussion: Unemployment: an exchange of experiences	
		Paul Green (moderator)
6:00 p.m.	រជ	Paul Green (moderator) CNIC

THURSDAY, August 21	American Mathematical Society	Other Organizations	
8:00 a.m 3:00 p.m.	Sessions for Contributed Papers and Special Sessions		
8:30 a.m 4:30 p.m.	REGISTRATION		
8:30 a.m 12:00 noon	EX	HIBITS	
8:30 a.m 4:30 p.m.	EMPLOYM	ENT REGISTER	
8:30 a.m 9:30 a.m.	COLLOQUIUM LECTURES III Ellis R. Kolchin		
8:30 a.m 9:30 a.m.	COLLOQUIUM LECTURES III Elias M. Stein		
9:45 a.m 10:45 a.m.	INVITED ADDRESS The singular cardinals problem Jack H. Silver		
10:00 a.m 4:00 p.m.		Conference Board of the Mathematical Science Council Meeting	
11:00 a.m 12:00 noon	INVITED ADDRESS Aspects of value distribution theory in several complex variables Wilhelm F. Stoll		
1:30 p.m 2:30 p.m.	INVITED ADDRESS The continuous cohomology of groups and classifying spaces James D. Stasheff		
3:15 p.m 4:00 p.m.	Steele Prize Session Wiener Prize Session		
4:00 p.m.	AMS Business Meeting		
7:30 p.m 10:00 p.m.		CBMS - Council Meeting	
8:00 p.m.	AMS Committee on Employment and Educational Policy Panel discussion: The role of applications in Ph. D. programs in mathematics Richard D. Anderson Lipman Bers Wendell H. Fleming (moderator) Henry O. Pollak		
FRIDAY, August 22	AMS		
8:00 a.m 5:00 p.m.	Sessions for Contributed Papers and Special Sessions		
8:30 a.m 1:30 p.m.	REGI	STRATION	
8:30 a.m 9:30 a.m.	COLLOQUIUM LECTURES IV Ellis R. Kolchin		
8:30 a.m 9:30 a.m.	COLLOQUIUM LECTURES IV Elias M. Stein		
9:45 a.m 10:45 a.m.	INVITED ADDRESS Title to be announced David Mumford		
11:00 a.m 12:00 noon	INVITED ADDRESS Ergodic properties of elementary mappings of the unit interval Roy L. Adler		

Capital of the World." It is the home of the major sources of professional magicians' supplies. From August 20-23 magicians from all over the continent will be convening in Colon. Magic demonstrations by professionals for professionals, and for the general public, will be held every evening and on Saturday afternoon. Further information about this unique event will be available at the Local Information Desk.

The recreational and athletic facilities of Western Michigan University will be available to all participants. Among these are badminton, basketball, billiards, bowling, handball, paddleball, indoor and outdoor track, ice skating (rental skates available), softball, swimming (no rental suits available), table tennis, tennis, and volleyball. In addition, the bathing beaches of Lake Michigan are within an hour's drive from the campus. There are several smaller lakes in and around Kalamazoo which have public bathing and picnicking facilities. Interested parties should inquire at the Local Information Desk.

A block of seats has been reserved for the Thursday evening (August 21) performance of "Oh, Calcutta" at the nearby Augusta Barn Summer Stock Theatre. Reservations to attend the performance and other theatre information can be obtained at the Local Information Desk.

A tour of the Kalamazoo Nature Center, suitable for children of all ages will be scheduled. Further information about this tour and the facilities at the Nature Center will be available at the Local Information Desk. In addition, there will be information about several other attractions in the area which are of special interest to young people.

There will be daily supervised arts and crafts for elementary age children at the dormitory complex. Information about the class hours and location will be available at the Local Information Desk.

There are also several commerical day care centers in Kalamazoo. Listed below are two which will be in operation during the meetings. They will take children on a short-term basis. Interested parties should write directly for further information and/or registration forms.

- Child Development Center Western Michigan University 1401 Cherry Street Kalamazoo, Michigan 49008 Telephone: (616) 383-4076
- (Will take children two and one-half year to five year of age Hours of operation: 6:30 a.m. to 6:30 p.m.)
- 2. Michigan Young World 110 W. Cork Street Kalamazoo, Michigan 49001 Telephone: (616) 349-2445
- (Will take children two and one-half years to eight years of age Hours of operation: 6:30 a.m. to 6:30 p.m.)

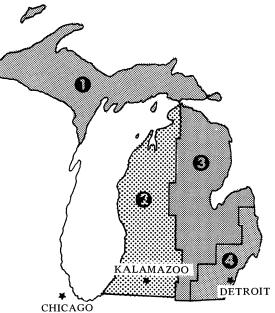
#### TRAVEL AND LOCAL INFORMATION

Kalamazoo is situated on two major expressways, I-94 (east-west), and US-131 (north-south), and is approximately halfway between Chicago and Detroit. It is also served by North Central Airlines, Greyhound and Indian Trails Bus Lines, and Amtrak. Car rentals from Avis, Hertz and National are available at the airport, but prior reservations are advisable. The airport is within the city limits, and regular cab service is available for transportation between the airport and campus. An information desk will be maintained at the airport at the times of the most frequently used incoming flights to assist arrivals. (During the summer, Michigan is on Eastern Daylight Time.)

The second largest industry in Michigan is vacationing and tourism. If you are interested in a vacation in Michigan you may obtain free vacation information by writing to: Michigan Tourist Council, Suite 102, 300 S. Capitol Avenue, Lansing, Michigan 48926. More specific information is available free from the four Regional Tourist Centers, indicated on the map below.

- Michigan's Upper Peninsula Travel and Recreation Association
   P.O. Box 400
   Iron Mountain, Michigan 49801
- 2. West Michigan Tourist Association 136 Fulton Street East Grand Rapids, Michigan 49502
- East Michigan Tourist Association The Log Office Bay City, Michigan 48706
- Southeast Michigan Travel and Tourist Association 1200 6th Avenue

Detroit, Michigan 48228



A free copy of the Official Michigan Transportation (Highway) Map can be obtained by writing the State Highway Commission, Lansing, Michigan 48926.

#### WEATHER

The normal daytime high temperature during this period is 84° F. Normal nighttime low is 60° F. Rainfall in August averages 2.78 inches, with a twenty percent to thirty percent probability of precipitation each day. Humidity normally ranges from a daytime high of eightytwo percent to a nightime low of fifty-five percent. The record high and low temperatures for August are  $101^{\circ}$  F and  $41^{\circ}$  F respectively. Light jackets and sweaters are advised for evening wear.

#### MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Western Michigan University, Kalamazoo, Michigan 49003. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located at the registration area in the lobby area of Miller Auditorium.

A message center will be located in the same area to receive incoming calls for regis-

trants during the hours the registration desk is open, cf. the section entitled MEETING PRE-REGISTRATION AND REGISTRATION, on a previous page. Messages will be taken down, and the name of any member for whom a message has been received will be posted until the message is picked up at the Message Center. The telephone number of the Message Center will be listed in the August issue of these *CNotica*.

#### LOCAL ARRANGEMENTS COMMITTEE

Yousef Alavi (chairman), Paul T. Bateman (ex officio), Jean M. Calloway, Gary Chartrand, A. Bruce Clarke, Florence M. Clarke, S. F. Kapoor, Don R. Lick, John W. Petro, James H. Powell, David P. Roselle (ex officio), Gordon L. Walker (ex officio), and Alden H. Wright.

Urbana, Illinois

Paul T. Bateman Associate Secretary

# The Seven Hundred Twenty-Eighth Meeting University of Illinois at Chicago Circle Chicago, Illinois November 1, 1975

The seven hundred twenty-eighth meeting of the American Mathematical Society will be held at the University of Illinois at Chicago Circle, Chicago, Illinois, on Saturday, November 1, 1975. All sessions of the meeting will be held in the Lecture Center of the university. The university is located approximately one mile west and one-half mile south of the intersection of State and Madison Streets, the origin of coordinates in the Chicago street numbering system.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Jonathan L. Alperin of the University of Chicago will speak at 11:00 a.m. on the topic "Finite groups viewed locally." Professor R. O. Wells, Jr. of Rice University will speak at 1:45 p.m. on "Poincaré's equivalence problem for real hypersurfaces in  $\mathbb{C}^{n}$ ."

There will be sessions for contributed tenminute papers both morning and afternoon. Abstracts should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of September 2, 1975. Those having time preferences for the presentation of their paper should indicate them clearly on their abstracts. There will be a session for late papers if needed, but late papers will not be listed in the printed program of the meeting.

There will be eight special sessions of selected twenty-minute papers. The subjects of these special sessions and the names of the mathematicians arranging them are as follows: Number Theory, Professor Bruce C. Berndt, University of Illinois at Urbana-Champaign; Lattice Theory, Professor Philip Dwinger, University of Illinois at Chicago Circle; Finite Groups, Professor Paul Fong, University of 11linois at Chicago Circle; Category Theory, Professor Saunders Mac Lane, University of Chicago; Stochastic Analysis, Professor Mark A. Pinsky, Northwestern University; Global Analysis, Professor R. Clark Robinson, Northwestern University; Harmonic Analysis on Locally Compact Groups, Professor Paul J. Sally, Jr., University of Chicago; Algebraic Geometry, Professor Philip D. Wagreich, University of 11linois at Chicago Circle. Most of the papers presented at these eight sessions will be by invitation. Anyone contributing an abstract for the meeting, however, who feels that his paper would be particularly appropriate for one of these special sessions should indicate this conspicuously on his abstract and submit it three weeks earlier than the above deadline, namely by August 12, 1975, in order to allow time for the additional handling necessary.

Further information will appear in the August issue of these CNoticea; the final program of the meeting will appear in the October CNoticea.

Urbana, Illinois

Paul T. Bateman Associate Secretary

# ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to Providence by the deadlines given below and should be clearly marked "For consideration for special session on <u>(title of special session)</u>." Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

Kalamazoo, Michigan, August 1975 expired	
Janos D. Aczel, Functional Equations	
Amassa C. Fauntleroy and Andy R. Magid, Affine Algebraic Groups	
George Fix, Scientific Computing	
Casper Goffman, Aspects of Real Analysis	
W. Charles Holland, Ordered Groups	
Robert E. Huff, Banach Spaces with the Radon-Nikodym Property	
Paul C. Kainen, Topological and Chromatic Graph Theory	
Leroy M. Kelly, Geometry of Metric Spaces	
Pierre J. Malraison, Jr., Categorical Methods in Algebraic Topology	
Tilla K. Milnor, Riemannian Geometry	
David E. Muller, Theoretical Computer Science	
Peter J. Nyikos, General Topology	
Hans Schneider, Numerical Ranges for Matrices and Other Operators on Normed Spaces	
Peter J. Weinberger, Efficient Algorithms for Exact Computation	

#### Chicago, Illinois, November 1975

Bruce C. Berndt, Number Theory Philip Dwinger, Lattice Theory Paul Fong, Finite Groups Saunders Mac Lane, Category Theory Mark A. Pinsky, Stochastic Analysis R. Clark Robinson, Global Analysis Paul J. Sally, Jr., Harmonic Analysis on Locally Compact Groups Philip D. Wagreich, Algebraic Geometry

#### Los Angeles, California, November 1975

September 2, 1975

August 12, 1975

Deadline

Theodore W. Gamelin, Function Algebras (title tentative) Nathaniel Grossman, Differential Geometry (title tentative)

# INVITED SPEAKERS AT AMS MEETINGS

This section of these  $\mathcal{N}$  lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

Pullman, Washington, June 1975		Chicago, Illinois, November 1975		
David W. Barnette	Theodore E. Harris	Jonathan L. Alperin	R.O. Wells, Jr.	
Kalamazoo, Michigan, August 1975		Blacksburg, Virginia, November 1975		
Roy L. Adler Everett C. Dade Ellis R. Kolchin	Jack H. Silver James D. Stasheff Elias M. Stein	J. D. Buckholtz Robert J. Daverman	William Jaco	
(Colloquium Lecturer)	(Colloquium Lecturer)	Los Angeles, Califor	nia, November 1975	
Bernard Maskit David Mumford	Wilhelm F. Stoll	Jerry Kazdan	Robert Osserman	

### CASE STUDIES

### Some Mathematicians with Nonacademic Employment

The Society's Committee on Employment and Educational Policy has assembled several case studies of mathematicians with nonacademic employment (see these cNotica), November 1974 and February 1975). Four such persons participated in a panel discussion sponsored by the Committee at the annual meeting of the Society in Washington on January 24, 1975. The panel members were Robert L. Anderson, Deering Milliken, Inc.; Steve Bravy, Analytic Services; John Matherne, U.S. Air Force Aerospace Defense Command; and Michael Weiss, Ketron, Inc. Their case studies appear below, followed by a summary of comments, arranged loosely by topic, made during the discussion but not appearing in the case studies. Professor Martha K. Smith, University of Texas, Austin, a member of the Society's Committee on Employment and Educational Policy served as a moderator for the panel discussion.

ROBERT L. ANDERSON Operations Research Department Deering Milliken, Inc. Spartanburg, South Carolina

Since July 1973 I have served as an internal consultant in the Operations Research Department at Deering Milliken Research and Management Information Center in Spartanburg, South Carolina. Deering Milliken is one of the largest privately owned industrial companies in the U.S. and a leader in textiles and textile research. I received my Ph.D. (differential equations) from the University of Tennessee in 1973.

In addition to typical consulting and basic operations research problems, I have been involved with design, programming and installation of a large forecast system (for marketing), a production planning system (for manufacturing), and a massive accounting system to replace the manual chores previously tended to by accountants and business analysts.

The particular department in which I work is made up of specialists from several different fields with a common knowledge of basic mathematics. We work very effectively by pooling our knowledge and consulting with each other on projects when necessary. Although some knowledge of textiles is essential for productive work, this is usually superfluous to the basic prerequisites for such a job. As is true with any type of manufacturing organization, one must, and can, acquire the knowledge of the product being manufactured in a short time after his induction.

I feel that positions such as mine have in the past not been given due consideration by mathematicians. In the handling of large manufacturing systems, multi-million dollar inventories and their allocations, very complicated mathematical problems arise. Such mathematical tools as linear programming and optimal control theory are useful in solving problems of production scheduling, inventory control, allocation procedures, and now the conservation of energy. Basic statistics and design of experiments are essential in assessing the characteristics of certain processes and predicting the patterns and behavior of the market. To most efficiently use these tools in solving problems, one must (i) be able to communicate effectively with people in the manufacturing plants to understand their problems, (ii) formulate these problems in a

language (mathematics) where the problems can be solved, (iii) employ computers to do the voluminous and time-consuming tasks of handling the information, (iv) interpret the solution of the problem and measure its reliability and finally (v) communicate the solution to the person who has the problem. Obviously, the first and last steps are as important as being able to solve the problem. (For more on points (ii), (iv) and (v), the following two sources are recommended: (1) Stacy Rowley, "Fantasies in Operations Research", presented at a Region II meeting of AIIE. (2) Robert Woolsey, "New Management Science Tools", Proceedings of the Fifth Annual Conference of the Society for Management Information Systems, September 1973, pp. 41-53.)

Many large businesses are beginning to rotate young professional people from research and information areas into marketing, product planning, and manufacturing. Their philosophy is that by building a two-way communication route from line management to the problem solvers they will alleviate one of the biggest problems of all, the communication gap. Once in a line management position, a professional mathematician can readily recognize and classify problems, their nature, and most important of all, where to take them for help.

A prospective employer of a mathematician in the business world will most likely emphasize the importance of the ability to work and communicate effectively with other people in the business as well as the significance of a good foundation in the basic tools for solving business related problems. A business is not searching for a noncommunicant to sit at a desk and prove irrelevant theorems forty hours a week (they will tell you so in the interview). They are looking for a logical innovative person with a substantial bag of various mathematical tools and the skill to use them. The most important concern of a business is to make money and, if possible, to benefit society. The best way to do this is to develop, manufacture, and market a higher quality product cheaper and more efficiently than their competitors. The technical qualifications may be met by one trained in operations research, mathematics, statistics, or management science, but to be competitive for such a position it is most helpful to have experience or some basic course work in statistics, modeling, simulation,

linear programming, computer science, economics, forecasting, control theory, general operations research or management science, with at least one of these as a specialty.

In conclusion, I believe that one with a fairly general mathematics background-some statistics, computer programming, economics, etc. should find working for industry very satisfying and rewarding, provided he has the ability to work well with others. One's rewards are immediate and recognition for accomplishments are frequent.

My advice to someone searching for such a position would be to obtain a list of Fortune Magazine's "top 500" industries, a college placement directory (or some other such list of possible industrial employers), select those in a suitable location and send a short one-page resume indicating his interests and qualifications. I believe the type of product being manufactured should be secondary to such things as location and character of the prospective employer. I also believe that if a suitable position is available, a short resume would initiate a sufficient inquiry. It is important that you don't overlook a large business because of the product they manufacture. There are many happy professional mathematicians in meat packing, pharmaceuticals, forestry, city planning, etc. And as for me, I never dreamed that I would become a "man of the cloth".

STEVE BRAVY Analytic Services Inc. Falls Church, Virginia

1. Job Description. I am employed as an associate analyst, in the Strategic Division of our company. We primarily assist the Air Force by providing studies and recommendations of the utility of various strategic weapon and logistic systems.

The techniques used include modeling (programming and concepts both) and optimization techniques such as linear programming and Lagrangian multiplier methods. Probability theory also is occasionally useful.

While the problems are interesting and quite complex, I have found that "sophisticated" mathematics is seldom of use. The interesting aspects of the job lie more in the problems than in the methods of solution.

2. <u>Mathematics Background</u>. Ph. D. from the University of Maryland. Thesis subject "Generalizing the Grothendieck Prime Spectrum." Courses in Algebra, Analysis, and Topology, including almost no "useful" mathematics.

3. <u>Other Background</u>. Minor in Physics as an undergraduate. What has proved to be much more of a saving grace were two years in realtime programming for IBM and a four-year assistantship at the University of Maryland Computer Center.

4. Job Application. The University of Maryland Placement Service provided the opportunity for the interview. I stressed interest in real problem solving and modeling and expressed (accurately) some distaste for the abstract theological view of mathematics.

5. <u>Nonmathematical Skills</u>. I have found that apart from obtaining mathematical results, it is most important to prepare one's material in a way that is palatable to the viewer-the theorem, proof, theorem approach is generally unsuitable. The results should be carefully discussed and their implications and utility stressed. Proofs are usually relegated to appendices. The most brilliant work, ineptly presented, is likely to be relegated to oblivion.

6. <u>Recommended Training</u>. Modeling, programming, statistics, and operations research are useful areas. Differential equations are also occasionally useful. As important, a problemoriented frame of mind rather than an esthete's is important for job satisfaction. If your life is incomplete without the Yoneda Lemma, you had best not try industry/government.

#### JOHN W. MATHERNE

U.S. Air Force, Aerospace Defense Command Eglin Air Force Base, Florida

Educational Background. I receive a Ph. D. from L.S.U. in 1973 where my dissertation was done in the area of quadratic forms. My course work was essentially pure with the traditional analysis, algebra, and topology courses. As an undergraduate, my major was physics.

Employment Prior to Ph.D. I spent one summer during my graduate study working for a geophysical firm. During this summer, I became acquainted with some data analysis techniques used in geophysical exploration. Additionally, I spent two years on active duty with the U.S. Army, where I was assigned to an engineering R & D laboratory. These two years provided me with some "system analysis" type experience which proved helpful when I began looking for nonacademic employment.

Employment After Ph.D. in June 1973, I accepted a position with Teledyne-Brown Engineering, a firm whose work is primarily in the electronics area. Here, I found myself involved in the analysis and design of algorithms for the operation and control of sophisticated radar tracking systems. A few months ago I left the engineering firm to join a new analysis group being formed at a U.S. Air Force satellite tracking station. The purpose of this group is to provide technical assistance in the area of phased array radar technology. It is perhaps surprising to realize that in a group of this type a mathematician can be on equal ground with the engineers. However, the mathematics needed is sometimes more sophisticated than that normally acquired in engineering programs, and a mathematician with advanced training adds considerable capability to the group.

Obtaining Employment. Much has been said about obtaining nonacademic employment, and I will try to avoid comments which have been made many times before. My initial contact with Teledyne-Brown Engineering was obtained through the placement office at Louisiana State University. It is my feeling, however, that the vehicle for making contact with a prospective employer is not so very important. The thing that does matter is the initial interview, and it is here that a young Ph.D. in mathematics must overcome a big obstacle. First, he must do away with the idea that the university is the only place for him. Secondly, he must convince the prospective employer that he is sincerely interested in applying his mathematical training in an environment that is

substantially different from academia. It is important that he project a positive attitude toward a nonacademic job; if he does not really want a nonacademic position, it will be very difficult to convince a prospective employer that he does. It should be emphasized that the Ph.D. mathematician does have a place in industry (nonacademia) and a vital function to fill. Through his mathematical training, he has developed maturity and an ability to think which makes him a valuable asset to any organization. It is this message which must be conveyed to any prospective employer. I must admit, though, that the situation is complicated by an "ivory tower" stigma which some people have attached to the Ph.D. in mathematics. This stigma can often generate prejudices which make communication difficult. These attitude problems are unfortunate, but real obstacles, and perhaps the most difficult ones to overcome. It is probably true that success in this venture will require certain personality traits that we do not all possess.

# MICHAEL D. WEISS Ketron, Inc.

My early academic background was typically pure-mathematical. I had always wanted to be a scientist and, by the age of sixteen, was determined to obtain a doctorate and become a mathematics professor. I majored in mathematics at Brandeis, did my doctoral work at Brown, and, upon its completion in 1969, joined the Wayne State faculty as an assistant professor.

In the ensuing years, I experienced a growing awareness that my mathematical and other interests were not being satisfied. I wanted to be involved with mathematics, but in a way that really mattered—not simply as a game. Furthermore, I wanted a broader contact with the "real world." Accordingly, I began thinking about nonacademic positions.

I embarked on a deliberate program, over several years, of strengthening my expertise in an applied direction. My dissertation had been in ergodic theory, and I chose to concentrate my reorientation in the allied field of probability. I arranged to teach a number of graduate and undergraduate courses in probability and became active in departmental committee work involving applications of mathematics to other disciplines. At the same time, I joined SIAM, thus gaining an excellent source of information about the role of mathematics and mathematicians in a nonacademic setting. Finally, as an added tool, I began to learn computing.

The field of operations research seemed to offer the ideal opportunity to apply mathematics to an extensive range of real-world problems. During 1973-1974, I sent inquiries to a number of government, nonprofit, and private organizations involved with operations research. I received several positive replies and ultimately accepted a position with Ketron, Inc., an impressively dynamic operations research firm headquartered near Washington, D.C.

The transition from an academic to a nonacademic environment presented no difficulty. As a result of the high level of experience and education characteristic of Ketron's staff (including Ph.D.'s in a number of disciplines), there is a good deal of advanced knowledge (both mathematical and otherwise) and intellectual sophistication in the air. There is, moreover, much crossfertilization between disciplines. In this sense, organizations such as Ketron are perhaps more truly "universal" than universities.

The change to a nine-to-five life style (more accurately, if not literally, nine-to-six) was, also, entirely uneventful. In lieu of the apparent academic pattern of alternating periods of frenzy and vacation (vacation serving as the time to catch up on what wasn't completed earlier), one has a more measured coupling of solid workdays with "mini-vacations" of essentially free evenings and weekends.

Ketron's Washington office is primarily concerned with defense studies. My own activities have centered around the design of a computerassisted war game. There have been several opportunities to apply my mathematical background. Subjects such as Markov chains, generating functions, and even Egorov's Theorem, have arisen naturally in the course of my modelling efforts. In the future, I hope to identify areas of application in the field of arms control for certain of my past research. Of course, in both my own activities and Ketron's work generally, mathematics is only one of the tools employed; nevertheless, it is an indispensable and highly valued tool.

The mathematician interested in job-hunting in the nonacademic sector may wish to consider, among other possible techniques, the following:

(1) Pick a city and "blitz" it. Write to many organizations in the area and indicate that you expect to visit the city at a designated time. Your personal availability (at no cost to them) might persuade some organizations to indulge their curiosity about you.

(2) To identify organizations which might be able to use your services, consult: the College Placement Annual (available at college placement offices); the Commerce Business Daily (a government publication, available at many libraries, which announces the awarding of government contracts); directories of consulting organizations (available in the business and finance sections of libraries); internal telephone directories of government agencies (available at government bookstores); those issues of the mathematics societies' Combined Membership List which have geographical listings; the editors' and authors' affiliations which appear in relevant journals; and the Yellow Pages of out-of-town telephone directories (available at many libraries). (Warning: Ferreting out the desired tidbits from these sources is not easy; I only claim that it is possible.)

Two more comments: I know from personal experience that an organization not actively seeking a mathematician may nevertheless be sufficiently intrigued when it hears from one to <u>create</u> a job-opening where none previously existed. Therefore, don't fret about whether there is a limited social <u>need</u> for mathematicians; it isn't necessary for you to be necessary-it's sufficient that you be beneficial.

And finally: As a group, private consulting firms tend to suffer from a "boom or bust" syndrome. Several-even large ones-to which I applied during 1973-1974 have already "busted." Hence, if possible, learn about a firm's management and future prospects (perhaps from outside sources) before accepting a position.

#### EXCERPTS FROM OPEN DISCUSSION

#### Usefulness of a Mathematician in Industry

<u>Robert L. Anderson</u>: Many problems are just ignored if the right mathematician is not available to solve them. When my boss commented that he didn't think differential equations and numerical analysis were of much use to us, he really meant that we hadn't used them much in the past. There was no one who was interested in such problems. But since I've been there, I've been able to help people who were working on physics of materials, for example, by using differential equations to model heat transfer.

<u>Daniel Wagner</u> (a member of CEEP): The presence of a mathematician on the scene should also lead to discovery of new problems. A mathematician working in industry should have the ability to recognize good problems in complex, physical, applied situations.

<u>John Matherne</u>: You don't necessarily have to prepare yourself in great detail in an applied discipline. In engineering firms, for example, I assure you that you will be able to handle all the engineering problems in a month or so, a lot better than most of the engineers. You may not know the exact terminology, but you can learn it quickly.

#### Mathematics in the University

Steve Bravy: Because of the way educational institutions are now set up, other departments are considered to be in Siberia. This is an unhealthy intellectual attitude. The problems in the world do not fall into the category of mathematical problems or computer problems or problems done by wretched souls in applied mathematics; they often require several disciplines. Mixed programs in several disciplines should be encouraged. Placement officers and other job availability sources should be consulted in drawing up a program. In addition, more cooperation between universities and industry would be mutually beneficial. The mathematical community should make some arrangements to allow people outside universities to consult university libraries. Co-operative courses with industry, where a course may be taught at the industrial site, offer several advantages. The subject matter would be guaranteed to be applied, students would be exposed to problems in the real world and to the industry, and the industry would provide a source of practical problems.

Edward Ordman (a member of the audience): Universities are among the organizations that don't realize they need mathematicians until someone sells them one. I'm one of at least three people who, two years ago, were untenured members of the mathematics department at the University of Kentucky and who are presently employed elsewhere in the University. [Note: Dr. Ordman's case study appeared in the February issue of the *CNotices*.]

#### Applying for Nonacademic Positions

<u>Michael Weiss</u>: There is no general algorithm for getting a government job. It's much more effective to talk to specific people in the agencies who may be identified by consulting the telephone books of government agencies. (The Yellow Pages is vague here.) I'm told they are available at government bookstores. There is something called the Defense Documentation Center which lists defense contracts. There is also the National Technical Information Service.

Bravy: For government jobs, Civil Service has to give you a rating. To get a rating you submit a standard form 171 to the government Civil Service. Sometimes there are auxiliary forms, depending on the particular category in which you apply.

<u>Member of the Audience</u>: I am in Civil Service. I spent some time in uniform, which is an advantage in Civil Service. I was hired as a physicist, but I do mathematics. If you know you want to work for a particular agency, do not send the rating application to the Civil Service Commission unless they tell you to. Send it to the agency. They can push it through. [Note: Articles on employment by federal agencies have appeared in the April 1974 and 1975 issues of <u>Employment Information for Mathematicians</u>. Other sources of information are listed in the article on nonacademic employment by Wendell Fleming in the April CNoticea.]

Matherne: There is a sort of superficial preparation that you need to make for the initial interview. You have to educate yourself to what I might call a conversational level. You have to be able to go into the interview and talk in language the interviewer understands. You don't need a lot of time, course work and detailed textbooks to do this. You can learn the language by talking with engineers, chemists, physicists, statisticians, biologists. You can do it by reading Scientific American. Going in as a con man is the worst thing you can do, but you want to be able to spark a little bit of interest. Your own natural curiosity (which you all have, or you wouldn't be in mathematics) will carry you on and get you through that initial interview.

<u>Member of Audience</u>: It seems that government and industry are hiring for jobs open now; in June they will be hiring for jobs open in June. Will those of us committed through May have to become unemployed and available before we go looking?

<u>Matherne</u>: It's not wasting your time to make your contacts now. It's not unusual for industry to find those people it wants in advance.

Anderson: We interview and hire people before they have actually finished school. As a matter of fact I was interviewed and hired two months before I finished school.

<u>Weiss</u>: Iwrote to Ketron knowing that Iwould want to come to them a year later. This was OK with them. Sometimes they hire in a long range sense and other times shorter range. A different company said that they wanted a certain post filled "yesterday".

#### Editor, the Notices

Many mathematics departments are reacting to the shortage of tenure positions by offering two or three year terminating appointments rather than regular tenure – track assistant professorships. It is not yet clear what the job market will be for those who will have completed such appointments. Apparently the market is now poor for mathematicians four or more years past the Ph. D., and probably we already have the absurd situation that some mathematicians who are dropping out of academic careers are of higher quality than the average of the new Ph. D's entering the market.

It is important that mathematics departments should give equal opportunities to mathematicians with several years' experience. One reason this has not always been done is that higher salaries have been required for people with more experience. However, if the extra money is not available, perhaps an experienced person would prefer an offer at a lower salary to no offer at all.

If more people with experience are hired, there will be fewer jobs for new Ph.D.'s. I believe such a squeeze, painful as it would be, is the only kind of pressure that will adequately cut down the production of new Ph.D's.

Theodore E. Harris

#### Editor, the Notices)

A letter from Jonathan S. Golan in the Notices) (volume 22, no. 2, p. 106) urges the importance of publishing accounts of methods by which results were obtained and suggests that the American Mathematical Society take the initiative to establish a depository for examples of "mathematics in action" and arrange for possible publication. I heartily agree with his sentiments and proposals. However, I should like to call the attention of mathematicians to the fact that historians of mathematics and their publications are interested in material of this kind. In particular, the international journal HISTORIA MATHEMATICA (University of Toronto, Toronto, Canada M5S 1A1) invites mathematicians to submit for possible publication case histories of heuristic activity. These are not only valuable sources for the historian, but may be of practical assistance to the creative mathematician.

Kenneth O. May

#### Editor, the Notices)

In his letter to the Editor (this Noticea), 22 (2), February, 1975, 107-108) Mr. E. Brieskorn makes a number of assertions to which I must take exception. However, in order to keep this letter as brief as possible, I shall refer only to the following two statements: 1) "In 1959, the year of the revolution, there was practically no mathematics in Cuba"; 2) "There exists a library which has a very small but reasonable good selection of text-books-most of them American and donated by visiting mathematicians".

Concerning the first statement the facts are as follows: a) In addition to the usual courses in Higher Algebra, Analytic Geometry Calculus, etc. the following courses were taught at Havana University on a regular basis: Ordinary and Partial Differential Equations, Real or Complex Analysis, Theory of Groups, Higher Geometry, Projective Geometry, Vector Analysis, Rational Mechanics; and on an occasional basis: Set Theory, Modern Algebra, Operational Calculus, Fourier Series and Boundary Value Problems, Number Theory, History of Mathematics, and a few others. b) During several years a seminar course in Analysis was offered in which I discussed: Continuous groups of transformations, Elliptic functions, Integral equations, Conformal Representation, and Topics in Normed Spaces and Functional Analysis. c) A number of doctoral dissertations were written by our students in the fields of Analysis, Geometry, Modern Algebra and Rational Mechanics. d) Several foreign mathematicians visited Havana and offered lectures on various topics. To mention a few: Professor Pedro Pi Calleja (University of Barcelona): Theory of the Integral, 1942; Professor Marshall Stone (University of Chicago): The spectral theorem in Hilbert space, 1948; Professor M. Krasner (Centre National de la Recherche Scientifique, Paris): Abstract Galois Theory, 1950. e) The Sociedad Cubana de Ciencias Físicas y Matemáticas was established in 1942. The Society maintained a journal in which a good number of papers by Cuban and some foreign mathematicians were published. Papers and notes by several Cuban mathematicians appeared also (before 1959) in Revista Universidad de la Habana, Revista de la Sociedad Cubana de Ingenieros, Revista Ingenieria Eléctrica, and Revista Cubana de Filosofia, as well as in several journals published elsewhere (United States, England, Spain, Argentina, Perú, etc.). f) A large number of books on elementary and advanced mathematics were published in Cuba well before 1959, too many to be listed here. For practically every regular course taught at the University there was a text-book or a set of lectures notes written by the professor in charge of the course. I will only mention the books published by one of my teachers, Dr. Pablo Miquel: "Elementos de Algebra Superior" (Havana, 1914, about 650 pages), "Cálculo Diferencial" (Havana, 1941, 490 pages), and "Calculo Integral" (Havana, 1942, 411 pages). All three books ran through several printings and compare favourably in content and typographical presentation with any

similar books published anywhere in the world. g) Cuba was a charter member of the International Mathematical Union. I was one of the Cuban representatives at the organizational meeting held at Columbia University, New York, in August, 1950. h) I contributed papers to the International Congress of Mathematicians of Cambridge, Mass. (1950), Amsterdam (1954), and Edinburgh (1958). i) I also read papers in two Symposia sponsored by UNESCO (Punta del Este, Uruguay, 1951, and Mendoza, Argentina, 1954). j) References to the work of Cuban mathematicians done before 1951 can be found in "Mathematics: Latin America Contribution to Scientific Progress", UNESCO Publication. Montevideo, 1951.

As to the second statement in Brieskorn's letter, I would like to point out that the University of Havana Main Library contained a fairly good collection of classical and modern books in advanced mathematics purchased throughout the years from its own allocations (not obtained by donations). Those books were mostly in Spanish, English, French, Italian and German. The Library also subscribed to a number of first-rate journals. Either Mr. Brieskorn saw only a collection of text-books in the Mathematics Department, or those in existence at the Main Library were pillaged as happened to many extensive and valuable private collections.

Dr. Mario O. Gonzalez

#### Editor, the *Notices*)

At the January 24 business meeting, a resolution asking the Federal government to "transfer massive funds from the military budget" to "fund a national open admissions program at institutions of higher education" and "a massive public works program" generated considerable heat. In this connection the following datum may be of interest: The amount of the graduate fellowships funding cut in 1973, \$175 million, is the cost of one nuclear aircraft carrier. (Source: "Getting the Biggest Bang for the Buck", by Seymour Melman of Columbia University, in the New York Times, December 4, 1974.) It may be true, as some argued at the meeting, that in practice it is administratively impossible to transfer funds from one budget to another. It is also true that since total resources are finite, a decision for one additional nuclear aircraft carrier in 1973 was literally a decision to cut support for graduate fellowships by \$175 million.

Charles Small

## NEWS ITEMS AND ANNOUNCEMENTS

#### AMS RESEARCH FELLOWSHIPS AWARDED

AMS Research Fellowships for 1975–1976 have been awarded to the following individuals: Terence J. Gaffney, who will receive the Ph.D. in June from Brandeis University; Paul Nèvai, who received the Ph.D. from the University of Szeged in 1973 and who is currently at the Université de Paris-Sud; and George M. Reed, who received the Ph.D. from Auburn University in 1971 and who is now at Ohio University. Each of the fellowships carries a stipend of \$10,000 with an additional \$500 allowance for travel expenses, of which \$3,600 and the \$500 for travel are tax deductible.

The AMS Research Fellowship Fund was established two years ago because of the scarcity of funds for postdoctoral fellowships. The fellowships are awarded to individuals who have recently received the Ph.D. degree and who show unusual promise in mathematical research. Serving on the Committee which administered the Fund this year were C. B. Bell, Walter Feit, Leonard Gillman, Peter J. Hilton, Mark Kac and Alice T. Schafer, Chairman.

#### AMS RESEARCH FELLOWSHIP PROGRAM FOR 1976–1977

The Council of the Society endorsed the Committee on Postdoctoral Fellowship's recommendation that the Research Fellowship Program be continued on the same terms as at present. The fellowships are to be awarded to individuals who have recently received the Ph.D. degree and who show unusual promise in mathematical research. Monies for the fellowship will come from the AMS Research Fellowship Fund, a fund raised from contributions. The Society will contribute each year an amount, between \$9,000 and \$20,000 equal to half the funds raised from other sources.

The survival of the Research Fellowship Program depends on the contributions the Society receives. It is hoped that every member of the Society will be willing to contribute to the Fund. All members will have the opportunity to designate their contribution on the next dues billing from the Society. A contribution of at least \$3.00 from each employed member would make this program a very successful one. Contributions are, of course, tax deductible. Checks should be made payable to the American Mathematical Society, clearly marked "AMS Research Fellowship Fund" and sent to the American Mathematical Society, P.O. Box 1571, Annex Station, Providence, Rhode Island 02901.

#### NATO POSTDOCTORAL FELLOWSHIPS

The National Science Foundation and the Department of State announced the award of fifty North Atlantic Treaty Organization Postdoctoral Fellowships in Science: Nineteen in the life sciences, twenty-five in the physical sciences, including mathematics and engineering and six in the social sciences. Charles M. Newman of Indiana University, Ronald L. Lipsman of the University of Maryland, Joseph C. Ecker of Rensselaer Polytechnic Institute and Joseph L. Gerver of the University of California, Berkeley received awards to continue their work in Mathematical Physics, Mathematical Analysis or Applied Mathematics. They will attend institutions in Israel, Belgium and France. U.S. citizens offered awards were selected on merit out of 272 applicants evaluated by scientists selected by NSF. NATO Fellows will receive a stipend of \$9,600 for twelve months, or \$7,200 for nine months. In addition, dependency allowances and limited allowances for round-trip travel will be provided. For application and information write to Fellowship Section, Division of Higher Education in Science, National Science Foundation, Washington, D.C. 20550.

#### NATIONAL SCIENCE FOUNDATION GRADUATE FELLOWSHIPS

Of 550 National Science Foundation Graduate Fellowships awarded to students of outstanding ability in the sciences, fifty-nine awards were made to students majoring in mathematics. More than 5,770 students competed for the NSF Fellowships which were awarded on the basis of merit. This year NSF awarded twenty-five more Graduate Fellowships than awarded in 1974. Panels of scientists, appointed by the National Research Council, reviewed each application; selections were made by NSF. In addition to the fellowships awarded. NSF accorded honorable mention to 2,078 applicants. All of the fellowships carry a stipend of \$3,600 per year for fulltime study. Each fellowship was awarded to beginning graduate students for three years of graduate study. The fellowships may be used over a five-year period to permit students to fit into their education other valuable experiences such as teaching or research assistantships while not drawing their fellowship stipend. NSF Graduate Fellows may attend any appropriate nonprofit U.S. or foreign institution of higher education.

#### NATIONAL ACADEMY OF ENGINEERING ELECTS NEW MEMBERS

Among the eighty-six new members recently elected to the National Academy of Engineering are: Rutherford Aris (University of Minnesota), Wilbur B. Davenport, Jr. (Massachusetts Institute of Technology) and Ralph E. Gomory (IBM), all members of the American Mathematical Society. Thirty-seven of the new members are associated with business, forty-three are affiliated with universities and colleges and six are with government agencies.

# NEW MASTER OF SCIENCE PROGRAM AT RENSSELAER POLYTECHNIC INSTITUTE

Rensselaer Polytechnic Institute, with funds provided by a four-year, \$176,000 grant from the National Science Foundation, is developing a new Master of Science in Applied Mathematics program designed to prepare students specifically for work in industry and government. The program will provide graduates with a strong background in the formulation of problems and in the variety of methods useful in solving applied problems.

Project coordinators are Dr. William E. Boyce, professor of mathematics, and Dr. Richard C. DiPrima, chairman of the department of mathematical sciences. The program is scheduled to be implemented in September 1976, and its preparation will include development of text materials and the creation of a course on mathematical modeling. "Since many mathe-maticians are employed in nonacademic organizations, and at levels below the doctorate," explains Professor Boyce, "it is appropriate to provide a master's degree program organized with their needs primarily in mind. The program will be a professional one in the sense that graduates will be prepared to function as practicing mathematicians in nonacademic surroundings, such as industry and government." Individuals interested in applying to the program should contact Dr. Boyce or Dr. DiPrima at Rensselaer Polytechnic Institute for further information.

#### 1975-1976 GUGGENHEIM FELLOWS

The John Simon Guggenheim Memorial Foundation has awarded fellowship grants totaling \$4,138,500 to 308 scholars, scientists and artists, ten of whom are mathematicians. Of the ten (1975-1976) Guggenheim mathematicians, three, Bernard M. Dwork (Theory of differential equations). Wu-chung Hsiang (Topology) and Nicholas M. Katz (Arithmetic algebraic geometry) are affiliated with Princeton University and two others, Kai-Lai Chung (Probability methods in potential theory) and E. H. Lee (Aspects of plasticity) are affiliated with Stanford University. Guggenheim mathematicians at other institutions include: Herbert Federer (Geometric measure theory) and Joseph P. LaSalle (Stability of dynamical systems) both at Brown University, and David R. Brillinger (Theory and applications of point processes), Louis Nirenberg (Theory of partial differential equations) and Isadore M. Singer (Geometric and spectral invariants) professors at the University of California, Berkeley, New York University and Massachusetts Institute of Technology, respectively.

Selected from 2,819 applicants on the basis of "demonstrated accomplishment in the past and strong promise for the future," the fellows, most of whom are affiliated with colleges or universities, represent 88 such institutions. For the second consecutive year, the University of California at Berkeley leads in the number of fellows with 15; Harvard and Stanford Universities each have 14.

#### MATHEMATICIANS NAMED TO NATIONAL ACADEMY OF SCIENCES

The National Academy of Sciences has announced the election of the following mathematicians to its membership: Herbert Federer, Brown University; Paul R. Garabedian, The Courant Institute of Mathematical Sciences; Jack C. Kiefer, Cornell University; Donald E. Knuth, Stanford University; David Mumford, Harvard University; Hirsh Z. Griliches, Harvard University; Roy Radner, University of California, Berkeley; Charles Stein, Stanford University; Gertrude Cox, North Carolina State University; and Max Mathews, Bell Laboratories. Of the ten mathematicians listed above, the first five are members of the Society.

#### $\pounds$ 200 ESSAY PRIZE

The Bertrand Russell Memorial Logic Conference is offering a prize of  $\pounds 200$  for an essay which examines in detail some aspect of the relationship between mathematics and the development of social or economic conditions. The essay should be of general interest to mathematicians and should include a consideration of current mathematical practice.

Essays should be submitted by February 1, 1976. Prospective entrants should read the further particulars which are available from Dr. A. Slomson, School of Mathematics, The University, Leeds LS2 9JT, England.

#### THE UNIVERSITY OF TEXAS AT AUSTIN PRIZE IN MATHEMATICS

A prize of \$3,000 for a worthy book, monograph, or article in mathematics published during the period between January 1, 1970 and June 30, 1975, will be made in January of 1976. The award is being made in honor of Professors Robert Lee Moore and Hubert S. Wall. It is funded by a gift to The University of Texas by Professor Emeritus H. J. Ettlinger.

Any ex-student, faculty, or former faculty in mathematics of The University of Texas at Austin is eligible for the prize. The Selection Committee to recommend the recipient of the award consists of Hyman Bass, R. H. Bing (Chairman), Joe Diaz, G. G. Lorentz, Deane Montgomery, C. B. Morrey, and Stan Ulam.

Nominations for the award are invited and could be sent to R. H. Bing, Chairman of the Selection Committee. The award is for a single publication rather than a series of them. It will be helpful to this Committee if along with your nomination you send comments on the proposed article.

#### ANNOUNCEMENT OF 1975–1976 PROGRAM OF NSF CHAUTAUQUA-TYPE SHORT COURSES

There will be places for nearly 3,000 college teachers of the natural and social sciences, mathematics and engineering in the 1975-1976 program of NSF Chautauqua-Type Short Courses. The program is administered by the American Association for the Advancement of Science. Each of the thirty-eight courses will be offered at either one or two of fourteen locations at colleges and universities across the United States.

The objective of the program is to bring to college teachers new information and educational approaches that will be directly useful in their current teaching. There will be a wide range of courses, with emphasis on the relationships between science, technology, and society and other complex problems of an interdisciplinary nature. Of particular interest to teachers of mathematics will be the following courses (Course Director's name in parentheses): Operations Research and the Systems Approach (Thomas L. Saaty); Teaching Calculus by Computers (Philip J. Davis); Analysis and Evaluation of Biological Data (William Hunter and Gunther Schlager); Calculus: Intuition Based, Problem Oriented Approach (William Walton); Uses of Mathematics in Political Science (William Lucas); Patterns of Problem Solving (Moshe F. Rubenstein); Mathematical Modeling in the Life Sciences (H. T. Banks).

Each class will meet in two two-day sessions, the first in the late fall of 1975 and again in the early spring of 1976. During the interim period, participants will work individually or in small groups on projects related to the courses, and will have an opportunity to exchange results at the spring session.

The NSF provides four nights of lodging on a double-occupancy basis for each noncommuting participant, and an allowance to each Course Director for the procurement of instructional materials. Participants or their institutions pay for travel and meals.

To obtain a copy of the program announcement containing course descriptions, class schedules, and locations where the classes will be held, write to NSF Chautauqua-Type Short Courses, Box J, AAAS, 1776 Massachusetts Avenue, N.W., Washington, D.C. 20036.

#### TRANSACTIONS EDITORIAL COMMITTEE ANNOUNCEMENT

In an effort to improve the quality of the <u>Transactions of the American Mathematical Society</u>, we, as editors, have recommended to the <u>Trustees a substantial reduction in the number</u> of pages to be printed in the <u>Transactions</u>. We intend to notify our referees of sharply increased standards for acceptance, and we plan to strictly enforce these standards which include originality, wide readership and careful presentation.

Authors are encouraged to delete routine material and to shorten their papers wherever possible without affecting their readability. Special attention should be directed towards excising repetetive proofs.

> A. H. Lachlan Chairman

#### RECENT AMS APPOINTMENTS

<u>Committee to Select Hour Speakers for</u> <u>Eastern Sectional Meetings</u>. George B. Seligman has accepted appointment to the Committee to Select Hour Speakers for Eastern Sectional Meetings. The Chairman of the committee is Walter Gottschalk and the continuing member is Jack K. Hale.

<u>Committee to Select Hour Speakers for Far</u> <u>Western Sectional Meetings</u>. President Lipman Bers has appointed Shoshichi Kobayashi to the Committee to Select Hour Speakers for the Far Western Sectional Meetings for a two-year term. Kenneth Ross is the Chairman and Michael Crandall is its continuing member.

<u>Proceedings Editorial Committee</u>. The Proceedings Editorial Committee has elected David Lutzer as associate editor in topology. He will begin his four-year term on January 1, 1976.

In addition, Reinhardt Schultz has been elected to a four-term (beginning January 1, 1976) as an associate editor in algebraic and differential topology by the Proceedings Editorial Committee.

<u>Mathematics of Computation Editorial Com-</u> <u>mittee</u>. Morris Newman has accepted appointment as an associate editor of the Mathematics of Computation for a three-year term (1975–1977).

<u>Committee on Science Policy</u>. President Lipman Bers has decided to consolidate two existing committees, the Committee on Relations with Government and the ad hoc Committee on Science Policy; this newly combined committee will assume the charges of its predecessors. William LeVeque will serve as the Chairman. Its members are : R. H. Bing, Garrett Birkhoff, Felix Browder, Leon Cohen, John Jewett, Anil Nerode and Elias M. Stein.

<u>Bicentennial Program Committee</u>. President Lipman Bers has authorized the formation of a Bicentennial Program Committee for the January 1976 Meeting in San Antonio. Its Chairman will be Joseph J. Kohn and its members are Felix Browder, Leonard Gillman and Henry O. Pollak.

Committee on Graduate Education. President Lipman Bers recently appointed the following individuals to serve on the Society's Continuing Committee on Graduate Education: William M. Boothby, Lewis A. Coburn, Ronald G. Douglas, Karl H. Hofmann and Murray Protter, Chairman.

# NOMINATIONS FOR VICE-PRESIDENT OR MEMBER-AT-LARGE

Two positions of vice-president of the Society and member of the Council  $\underline{ex officio}$  for a term of two years are to be filled in the election of October 1975. The Council has nominated four candidates for the positions, namely

> Stephen C. Kleene George D. Mostow Louis Nirenberg Max M. Schiffer

Additional nominations by petition in the manner described below are acceptable.

Five positions of member-at-large of the Council for a term of three years are to be filled in the same election. The Council has nominated seven candidates for these positions, namely

> William K. Allard Joan S. Birman Edwin E. Floyd Joachim Lambek Hugo Rossi Barry Simon Guido L. Weiss

Additional nominations by petition in the manner described below are acceptable. The Council intends that there shall be at least ten candidates for the five positions and will bring the number up to ten if the number of nominations by petition is less than three.

Names of these candidates are published to assist those who may wish to make nominations by petition.

The name of a candidate for the position of vice-president or of member-at-large of the Council may be placed on the ballot by a petition that conforms to several rules and operational considerations, as follows:

1. To be considered, petitions must be addressed to Everett Pitcher, Secretary, Box 6248, Providence, Rhode Island 02940, and must arrive by August 11, 1975. 2. The name of the candidate must be given as it appears in the <u>Combined Membership List</u>. If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of these *NoticeD*.

3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.

4. On the facing page is a sample form for petitions. Copies may be obtained from the Secretary; however, petitioners may make and use photocopies or reasonable facsimiles.

5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column. At least fifty valid signatures are required for a petition to be considered further.

6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the <u>Combined</u> <u>Membership List</u> and the mailing lists. No attempt will be made to match variants of names with the form of name in the CML. A name not in the CML or on the mailing lists is not that of a member. (Example: The name Everett Pitcher is that of a member. The name E. Pitcher appears not to be. Note that the current mailing label of these *CNotical* can be peeled off and affixed to the petition as a convenient way of presenting the printed name correctly.)

7. When a petition meeting these various requirements appears, the Secretary will ask the candidate whether he is willing to have his name on the ballot. His assent is the only other condition of placing it there. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving his consent.

#### NOMINATION PETITION FOR 1975 ELECTION

Signature

*

The undersigned members of the American Mathematical Society propose the name of

Printed or typed name and address or cNotices mailing label

*Specify "vice-president" or "member-at-large of the Council".

## THE NOMINATING COMMITTEE FOR 1976

It is possible that the Committee on Committees will recommend and the Council adopt a change in procedure for selection of the Nominating Committee. The proposal under consideration includes the election of some of the members of the Nominating Committee by the membership of the Society. Moreover, the admissibility of nomination by petition of candidates for the Nominating Committee is being considered.

If such a proposal were adopted at the Council of August 18, 1975, it would be too late subsequently to circulate petitions for places on a ballot which the bylaws require to be distributed before October 10. With this in mind, the Committee on Committees has proposed that the possibility of such nominations be announced now, with the understanding that finally they may not be admissible.

Subject to Council approval of the entire procedure, the name of a candidate for member of the Nominating Committee may be placed on the ballot by a petition that conforms to several rules and operational considerations, as follows:

1. To be considered, petitions must be addressed to Everett Pitcher, Secretary, Box 6248, Providence, Rhode Island 02940, and must arrive by August 11, 1975.

2. The name of the candidate must be given as it appears in the <u>Combined Membership List</u>. If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of these *cNotices*).

3. The petition for a single candidate may consist of several sheets each bearing the state-

ment of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.

4. On the facing page is a sample form for petitions. Copies may be obtained from the Secretary; however, petitioners may make and use photocopies or reasonable facsimiles.

5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column. At least 100 valid signatures are required for a petition to be considered further.

6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the <u>Combined</u> <u>Membership List</u> and the mailing lists. No attempt will be made to match variants of names with the form of name in the CML. A name not in the CML or on the mailing lists is not that of a member. (Example: The name Everett Pitcher is that of a member. The name E. Pitcher appears not to be. Note that the mailing label of these <u>CNotices</u> can be peeled off and affixed to the petition as a convenient way of presenting the printed name correctly.)

7. When a petition meeting these various requirements appears, the Secretary will ask the candidate whether he is willing to have his name on the ballot. His assent is the only other condition of placing it there. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving his consent.

#### NOMINATION PETITION FOR 1975 ELECTION (Nominating Committee of 1976)

The undersigned members of the American Mathematical Society propose the name of

as a candidate for the position of Member of the Nominating Committee of the American Mathematical Society for the year 1976.

Printed or typed name and address or  $\mathcal{N}olices$ ) mailing label

Signature

### SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these Notices) if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced.

In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the Chotices are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

June 9-12, 1975

MICHIGAN STATE UNIVERSITY CONFERENCE ON FUNCTIONAL ANALYSIS AND NONLINEAR DIFFEREN -TIAL EQUATIONS

Michigan State University, East Lansing, Michigan

Principal Speaker: Lamberto Cesari, University of Michigan

Information: R. Kannan or J. Schuur, Department of Mathematics, Michigan State University, East Lansing, Michigan 48824.

#### June 9-12, 1975

SEMINAR ON FIXED POINT THEORY AND ITS APPLI-CATIONS

Dalhousie University, Halifax, Nova Scotia, Canada Program: The seminar will be primarily concerned with geometric and analytical aspects of fixed point theory and its applications. There will be several one-hour addresses given and a series of half-hour talks is also being planned.

Speakers: F. E. Browder (Chicago), M. M. Day (Urbana), M. Edelstein (Dalhousie), A. Granas (Montreal), Les Karlovitz (Maryland) and W. A. Kirk (Iowa/Vancouver). Sponsors: The Mathematics Department of Dalhousie University.

Support: The Canadian Mathematical Congress (so as to form part of its Summer Research Institute, Eastern Branch) and Dalhousie University.

Information: S. Swaminathan, Mathematics Department, Dalhousie University, Halifax, Nova Scotia, Canada B3H 3J5.

June 17-20, 1975

COLLOQUE ANALYSE ET TOPOLOGIE

Université de Paris-Sud, Orsay, France Program: This colloquium is dedicated to Professor Henri Cartan.

Speakers: J.F. Adams, M.F. Atiyah, A. Borel, R. Bott, H. Grauert, F. Hirzebruch, J. N. Mather, J.C. Moore, R. Narasimhan and D. Sullivan.

Information: Département de Mathématiques, Bâtiment 425, 91405 Orsay, France.

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June 23-27, 1975
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NSF REGIONAL RESEARCH CONFERENCE ON NEW DI-RECTIONS IN SINGULAR PERTURBATIONS: THEORY AND APPLICATIONS

Flagstaff, Arizona

Program: There will be a series of ten one-hour lectures which will include an introductory discussion of what a singular perturbation problem is and an in-depth study of singular perturbation problems for nonlinear twopoint boundary value problems for ordinary differential equations. Special topics will include diagonalization techniques, use of differential inequalities and/or approximate solutions, problems on large (expanding) intervals, multiple time scales, turning points, bifurcation theory and elementary control theory.

Principle Lecturer: William A. Harris, Jr. of the University of Southern California.

Guest Lecturers: J.V. Breakwell (Stanford University), D.S. Cohen (California Institute of Technology), P.V. Kokotovic (University of Illinois-Urbana) and G.C. Papanicolaou (Courant Institute, New York University). They will lecture on applications of singular perturbations that they are currently investigating in celestial mechanics, biochemical reactors, optimal controls and probabilistic problems.

Support: There is a limited amount of support for participants available under a grant from the National Science Foundation.

Information: Lawrence M. Perko, Department of Mathematics, Northern Arizona University, Flagstaff, Arizona 86001.

#### June 23-27, 1975

REGIONAL CONFERENCE ON PRACTICAL APPLICA-TIONS OF FINITE MATHEMATICS Washington State University, Pullman, Washington Program: S.W. Golomb, University of Southern California, will deliver a series of lectures. There will also be informal discussions and seminars. Support: (Subject to final approval by the NSF) Travel and subsistence allowance for 25 invited participants. Invited Participants: Persons interested in being invited should include a brief vita and indications of research interest, if any, in combinatorics, finite fields, finite groups, statistical designs, network theory, error correcting codes and number theory. Information: T.G. Ostrom, Department of Pure and Applied Mathematics, Washington State University, Pullman, Washington 99163.

#### July 7-11, 1975

CONFERENCE ON COMMUTATIVE ALGEBRA Queen's University, Kingston, Ontario, Canada Program: Five one-hour lectures by R.G. Swan and invited talks by H. Bass, D. Eisenbud, R. Fossum, R. Gilmer, N. Heerema, M. Hochster, M. P. Murthy and W. V. Vasconcelos will be given. Support: National Research Council of Canada Information: Anthony V. Geramita, Department of Mathematics, Queen's University, Kingston, Ontario K7L 3N6, Canada.

August 3-16, 1975

NATO ADVANCED STUDY INSTITUTE ON LONG-TIME PREDICTION IN DYNAMICS

Cortina d'Ampezzo, Italy

Program: Stabilization of numerical integration techni-

ques, error propagation and estimation, nonlinear resonances, stability in the solar system, many-body gravitational problem, effect of singularities, strongly perturbed systems, and others. The lecture series are interspersed with seminars and special addresses.

Principal Lecturers: (Tentative) Aarseth (Cambridge), Baumgarte (Braunschweig), Colombo (Padua), Contopoulos (Thessaloniki), Message (Liverpool), Nahon (Paris),

Ovenden (Vancouver), Stiefel (Zürich), Szebehely (Austin), and Vicente (Lisbon).

Sponsor: NATO.

Support: National Research Council of Italy and The University of Texas at Austin.

Information: Victor Szebehely, The University of Texas, Austin, Texas 78712.

#### August 11-13, 1975

CONFERENCE ON THE THEORY OF APPROXIMATION University of Calgary, Alberta, Canada

Invited Speakers: G. Birkhoff, Harvard University; E.W. Cheney, University of Texas; C. de Boor, Purdue University; G. Freud, Ohio State University; J.R. Rice, Purdue University; R.S. Varga, Kent State University. Program: The program includes a number of submitted talks as well as invited addresses and contributed papers. Full conference proceedings will be published. Information: B. N. Sahney, Mathematics Department, The

University of Calgary, Calgary T2N 1N4, Alberta, Canada; or A.G. Law, Mathematics Department, University of Regina, Regina S4S 0A2, Saskatchewan, Canada.

#### August 11-15, 1975

REGIONAL CONFERENCE ON THE STABILITY OF DY-NAMICAL SYSTEMS: THEORY AND APPLICATIONS Mississippi State University

Program: A series of ten lectures on generalizations of classical Liapunov theory and their applications to problems in engineering, economics and biomedicine; discussion sessions will also be scheduled.

Lecturer: J. P. LaSalle, Lefschetz Center for Dynamical Systems, Division of Applied Mathematics, Brown University.

Contributed Papers: A session for a limited number of contributed papers is planned.

Support: National Science Foundation; travel and subsistence allowance for 25 participants; applications for support should be received by July 1, 1975.

Information: John R. Graef, Conference Director, Department of Mathematics, Mississippi State University, Mississippi State, Mississippi 39762.

#### August 12-14, 1975

SYMPOSIUM ON THE SIMULATION OF COMPUTER SY-STEMS

Boulder, Colorado

Program: An exposition of the latest developments for the manager and analyst with technical knowledge and experience in the field will be offered. A tutorial program will also be provided for attendees with an interest but no prior background in one or more aspects of the modeling and simulation process. There will be informal discussions/workshop sessions on computer system simulation packages, simulation of minicomputers, techniques for large computer centers and data collection, analysis, and validation.

Sponsors: National Bureau of Standards of the Commerce Department and the Special Interest Group on Simulation (SIGSIM) of the Association for Computing Machinery. Information: John Caron, FEDSIM/NA, Washington, D. C. 20330.

#### August 25-29, 1975

INTERNATIONAL SUMMER SCHOOL ON PROGRAM ANALYSIS AND OPTIMIZATION

Technion City, Haifa, Israel

Program: Program optimization; Control and data flow analysis; Program diagnosis and verification; Interprocedural analysis; SETL and its implementation; Type determination and data structure choice in SETL and APL; Storage and register allocation; Code generation. Speakers: F.E. Allen (IBM Yorktown Heights), J. Cocke (IBM Yorktown Heights), Z. Manna (Weizmann Institute, Rehovot, Israel), H.J. Saal (IBM S.C., Haifa, Israel), and J.T. Schwartz (Courant Institute, New York). Sponsors: Technion-Israel Institute of Technology, IBM-Israel Scientific Center.

Information: J. Raviv, IBM Israel Scientific Center, Technion City, Haifa, Israel.

August 25-September 2, 1975 THIRD SESSION OF C.I. M. E. ON DIFFERENTIAL OP-ERATORS ON MANIFOLDS

Varenna, Como, Italy

Program and Speakers: M. F. Atiyah (Oxford University, England) will speak on classical groups and classical differential operators on manifolds; R. Bott (Harvard University) will speak on topics in invariant theory and E. M. Stein (Princeton University) will speak on singular integral operators and nilpotent groups.

Deadline for Applications: June 30, 1975.

Information: A. Moro, Secretary, C. I. M. E. -Istituto Matematico "U. Dini", Viale Morgagni 67/A, 50134 Firenze, Italy.

#### September 1-2, 1975

SYMPOSIUM ON SIGNAL PROCESSING FOR ARRAYS Salisbury, South Australia

Information: H.A. d'Assumpcao, Principal Officer, Underwater Detection Group, Weapons Research Establish-ment, Box 2151, G.P.O., Adelaide, South Australia 5001.

September 2-13, 1975

DURHAM SYMPOSIUM ON L-FUNCTIONS AND GALOIS PROPERTIES OF NUMBER FIELDS

Durham, England

Program: The program consists of a number of short lecture courses and seminar talks.

Speakers: J. Coates, A. Frölich, J. Martinet, J.-P. Serre and J. Tate.

Participants: Those who have already indicated they are likely to participate are M. F. Atiyah, J. Birch, W. Casselman, J.W.S. Cassels, K. Iwasawa, R. Langlands

and H. Stark. There is a limited number of places available for people who have not yet been sent invitations. Information: S. M. J. Wilson, Department of Mathematics, University of Durham, Science Laboratories, South Road, Durham DH1 3LE, England.

#### September 3-9, 1975

SEVENTEENTH POLISH SOLID MECHANICS CONFER-ENCE

Szczyrk, Poland

Program: The conference will cover all aspects of solid mechanics, general continuum theory and theory of structures; both theoretical and experimental papers will be presented. There will be an entertainment program, including excursions to Cracow and other places of interest. Information: M. Matczynski, Secretary of the Seventeenth Polish Solid Mechanics Conference, Institute of Fundamental Technological Research, Swietokrzyska 21,00-049 Warsaw, Poland.

#### September 9-11, 1975

SYMPOSIUM ON SPARSE MATRIX COMPUTATIONS Argonne National Laboratory, Argonne, Illinois Program: There will be invited papers reviewing the state-of-the-art and reporting current research and applications.

Sponsors: Energy Research and Development Administration, Office of Naval Research, Department of the Navy, and the Society for Industrial and Applied Mathematics.

Invited Speakers: R.E. Bank (The University of Chicago), A. Björck (University of Linköping, Sweden), J. R. Bunch (University of California, San Diego), B. L. Buzbee (Los Alamos Laboratory), A. K. Cline (ICASE, NASA Langley Research Center), S.C. Eisenstat (Yale University), W.M. Gentleman and J.A. George (University of Waterloo, Can-ada), P.E. Gill and W. Murray (National Physical Laboratory, England), D. Goldfarb (City University of New York), F. Gustavson (IBM Yorktown Heights), G. Hachtel

(IBM Yorktown Heights), J. E. Hirsh (Harvard University), T. L. Magnanti (Sloan School, Massachusetts Institute of Technology), B. N. Parlett (University of California, Berkeley), T. A. Porsching (University of Pittsburgh), D. J. Rose (Harvard University), M. Saunders (New Zealand and Stanford), A. Sherman (Yale University), G. W. Stewart (University of Maryland), R. E. Tarjan (University of California, Berkeley and Stanford), R. E. Varga (Kent State University) and P. T. Woo (Chevron Research Laboratory). The Symposium will also feature a talk by James H. Wilkinson, FRS following dinner on Tuesday evening, September 9.

Information: James R. Bunch, Department of Mathematics, University of California, San Diego, La Jolla, California, 92037.

September 22-27, 1975

TENTH CONGRESS OF THE UNIONE MATEMATICA ITALIANA

Cagliari, Italy (September 22-26), Alghero, Italy (September 27)

Information: Unione Matematica Italiana, X Congresso, Cagliari, Italy.

October 3-4, 1975

THIRD ANNUAL MATHEMATICS AND STATISTICS CONFERENCE

Miami University, Oxford, Ohio

<u>Program</u>: Two Hundred Years of Mathematics in America. This conference is a part of Miami University's celebration of the American Revolution Bicentennial, and it will emphasize American contributions to mathematics. Emphasis will also be placed on helping college and secondary school teachers use the history of mathematics in their classrooms.

<u>Principal Speakers</u>: Garrett Birkhoff (Harvard University), Morris Kline (New York University), and Dirk Struik (Massachusetts Institute of Technology).

Information: David E. Kullman, Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056.

November 3-7, 1975

CONFERENCE ON THE LATEST RESEARCH RESULTS IN COMMUTATIVE ALGEBRA

Northwestern University, Evanston, Illinois

<u>Information</u>: Eben Matlis or Judith Sally, Mathematics Department, Northwestern University, Evanston, Illinois 60201

November 10-13, 1975

EUROMECH 69: LARGE ELASTIC DEFORMATIONS OF DISCRETE SYSTEMS

Hungarian Academy of Sciences, Matrafured, Hungary <u>Program</u>: The following topics will be considered: numerical analysis of elastic deformations in systems with finite-degree-of-freedom; stability and postbuckling behaviour of finite-degree-of-freedom elastic systems; kinetic analysis of cable nets and membranes.

<u>Participation</u>: Participation in the colloquium is by invitation only and is limited to about 40 specialists. Participants are requested to send a brief summary (about 200 words) stating the domain of interest, main results to be presented and indicating the novel aspects of the proposed contribution.

Information: J. Szebó, Technical University, Budapest, Hungary.

#### November 13-15, 1975

ANNUAL CONFERENCE ON UNDERGRADUATE MATHE-MATICS

Guilford College, Greensboro, North Carolina

<u>Program</u>: Presentation of papers written by students during their undergraduate careers and invited papers presented by professional mathematicians.

<u>Deadline for Papers</u>: Papers submitted before August 29, 1975, for publication in the Journal of Undergraduate Mathematics will be considered for presentation at the Conference. Notification of acceptance will be made by October 1, 1975. Papers may be expository, survey, historical or undergraduate research.

Support: Transportation (up to \$100.00) and room and

board will be paid by the Journal of Undergraduate Mathematics.

<u>Information:</u> J.R. Boyd, Managing Editor, Journal of Undergraduate Mathematics, Department of Mathematics, Guilford College, Greensboro, North Carolina 27410.

November 17-19, 1975

JOINT ORSA/TIMS NATIONAL MEETING

MGM Grand Hotel, Las Vegas, Neveda <u>Program</u>: Approximately 120 sessions are planned, covering OR/MS applications-methodology. A new audienceparticipation forum is being introduced and an organized OR/MS employment program conducted. It is being held in cooperation with the Department of Mathematics of the University of Nevada at Las Vegas. <u>Speakers:</u> Russell L. Ackoff (The University of Pennsyl-

Vania) and Harvey M. Wagner (Yale University). Information: William B. Widhelm, Division of Management Science-Statistics, College of Business and Management, University of Maryland, College Park, Maryland 20742.

#### December 3-5, 1975

SIAM-SIGNUM 1975 FALL MEETING

San Francisco, California <u>Program</u>: There will be four main symposia centered on: numerical solution of ill-posed and ill-conditioned problems, optimization in science and engineering, applied combinatorics, and applied mathematics and energy. There will also be special lectures on education and applications in applied mathematics.

Invited Speakers: Richard Hanson (Washington State University), William Kahan (University of California, Berkeley), Bert Rust (Union Carbide Corporation), James Varah (University of British Columbia), Richard Cottle (Stanford University), Ben Rosen (University of Minnesota), Michael Held (IBM Systems Research Institute), Donald Rose (Harvard University), Burt Colvin (National Bureau of Standards) and C. C. Lin (Massachusetts Institute of Technologv).

Support: It is anticipated that the Department of the Navy-Office of Naval Research will in part support these symposia sessions and special lectures.

Contributed Papers: Papers on all areas of applied mathematics are solicited. There will be about 20 contributed paper sessions, and so far as possible contributed papers will be grouped according to areas of interest.

Deadline for Abstracts: (Maximum 200 words) September 15, 1975.

<u>Information:</u> Society for Industrial and Applied Mathematics, 33 South 17th Street, Philadelphia, Pennsylvania 19103.

December 29, 1975-January 3, 1976

INTERNATIONAL RESEARCH SYMPOSIUM ON RELATIV-ITY AND UNIFIED FIELD THEORY

Satyendranath Bose Institute of Physical Sciences, Calcutta University, Calcutta, India

<u>Contributed Papers</u>: Deadline for original papers (only a synopsis is required) with an abstract is October 3, 1975. Interested participants should send their name and title of the paper before July 5, 1975.

Information and Abstracts: M. Dutta, Professor-in-Charge, Satyendranath Bose Institute of Physical Sciences, 92 Acharya Prafulla Chandra Road, Calcutta-700009, India.

#### January 1-7, 1976

ANNUAL CONFERENCE OF INTERNATIONAL LEVEL OF INTERNATIONAL CENTRE FOR APPLIED ANALYSTS Calcutta, India

Information: S.K. Ghoshal, Director, Department of Mathematics, Jadavpur University, Calcutta (32), India.

#### January 19-21, 1976

THIRD ACM SIGACT-SIGPLAN SYMPOSIUM ON PRINCI-PLES OF PROGRAMMING LANGUAGES

#### Atlanta, Georgia

<u>Contributed Papers</u>: Papers on significant developments in the principles of programming languages, and on theoretical studies with application to or primarily motivated by programming languages, are being solicited. Six copies of a detailed summary which make clear the significance and originality of the proposed paper, and also include comparisons with and references to relevant literature, should be submitted by August 16, 1975. The submitted summaries should be about 1000-2000 words long. Authors will be notified of acceptance or rejection by September 30, 1975.

Information: Susan L. Graham, Computer Science Division, University of California, Berkeley, California 94720.

June 7-11, 1976

THIRTEENTH YUGOSLAV CONGRESS OF RATIONAL ME-CHANICS

Sarajevo, Yugoslavia

Information: Jugoslovensko Društvo za Mehaniku, Kneza Miloša 9/1, 11000 Beograd, Jugoslavia

August 16-21, 1976

THIRD INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION (ICME)

Program: There will be six main papers by invited speakers devoted to matters of general interest in mathematics and the didactics of mathematics. Panel discussions will take place to discuss particularly topical, or controversial themes, or ones of general interest. Selected projects and study groups working on research and development in mathematics education will be invited for presentations. Publishers and firms producing teaching aids may present their books and material.

Information: E.F. an Huef, Secretary, Third International Congress on Mathematical Education, Hertzstr. 16, D 75 Karlsruhe, Federal Republic of Germany.

# **QUERIES**

#### Edited by Hans Samelson

This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers will be edited into a composite answer and published in a subsequent column. All answers received to questions will ultimately be forwarded to the questioner. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

#### QUERIES

65. S.R. Caradus (Department of Mathematics, Queen's University, Kingston, Canada K7L 3N6). T.T. West (Proc. London Math. Soc. (3) 16(1966), 737-752) showed that Riesz operators on Hilbert space can be decomposed as quasinilpotent plus compact. C. Olsen (Amer. J. Math. 93(1971), 686-698) showed that polynomially compact operators on Hilbert space can be decomposed as algebraic plus compact. Both results are quite difficult to prove. Viewed algebraically they have an interesting reformulation: if K is the compact operators and  $\pi: \mathcal{L}(H) \to \mathcal{L}(H)/K$ , the canonical homomorphism, then West's result states that  $\pi$  maps the class of quasinilpotents onto the class of quasinilpotents, Olsen's result states that # maps the class of algebraics onto the class of algebraics. Does anyone see how one might unify and extend these two surjection theorems (1) using Banach algebra techniques, or (2) using the fact that  $\pi$ has a continuous cross-section?

66. <u>Eugene H. Lehman</u> (Département de Mathématiques, Université du Québec à Trois-Rivières, Case Postale 500, Trois-Rivières, Québec, Canada G9A 5H7). Je cherche une expression explicite pour la convolution f(u) de la fonction Weibull:

où

٦

$$w(x) = ax^{a-1} \exp(-x^{a}), \quad a et x > 0.$$

 $f(u) = \int_{-\infty}^{u} w(u - x)w(x) dx$ 

67. <u>A. H. Zemanian</u> (Department of Applied Mathematics and Statistics, SUNY at Stony Brook, Stony Brook, New York 11794). Let S be the linear space of all infinite

vectors  $[a_1, a_2, \dots]^T$  of complex numbers  $a_j$  having no restriction on the rate of growth of the  $a_i$ . I would

appreciate receiving any information or references on the invertibility of linear operators on S. A sufficient condition is that the infinite matrix representation for such an operator be nearly lower triangular with square blocks along the main diagonal, where each block is nonsingular. Do all invertible linear operators have this form when the rows are suitably permuted?

68. <u>Albert A. Mullin</u> (1500 Ronstan Drive, Killeen, Texas 76541). I would appreciate receiving relatively recent references and new ideas or formal hypotheses on the applications of modern mathematics to ethics. The important references by C.S.S. Peirce (<u>Collected Papers</u>, 2,196-2,200) and by G.D. Birkhoff (<u>A Mathematical</u> Approach to Ethics) are already known to me.

#### RESPONSES TO QUERIES

55. (vol. 22, p. 71, Jan. 1975, Shelupsky):  $x^4 = 1$  is satisfied by the unit and the three involutions in  $S_3$ ; thus the conjecture fails. On the other hand, if there is an element of order d, its powers form a cyclic subgroup of order d; so if  $x^d = 1$  has exactly d solutions, they are given by that subgroup. (Communicated by M. Barr)

56. (vol. 22, p. 71, Jan. 1975, Zwillinger): The number  $5 \cdot 2^{1947} + 1$  (R.M. Robinson, Proc. Amer. Math. Soc. 9 (1958), 673-681), a factor of the Fermat number 2  2  + 1, is perhaps the largest known prime not of the form  $2^{n} \pm 1$ . (Communicated by R.M. Robinson and W.G. Leavitt)

61. (vol. 22, p. 123, Feb. 1975, Wichmann): Let P be the polynomials on R, let A and B be two nonempty compact subsets of R,  $A \neq B$ , say  $A \notin B$ . Define norms  $|\cdot|_A = \sup_A, |\cdot|_B = \sup_B$  on P. Take f continuous on  $A \cup B$  with  $f|B = 0, f|A \neq 0$ . Take  $p_n$ , in P, converging uniformly to f on  $A \cup B$ . Then  $|p_n|_B \rightarrow 0$ , but  $|p_n|_A \neq 0$ ; the two norms are inequivalent. (Communicated by Tsing-jen Ho, and W.A.J. Luxemburg)

62. (vol. 22, p. 123, Feb. 1975, Wilansky): For (1) and (2) let X be a Banach space with an uncomplemented subspace Y. Let  $\pi: X \to X/Y$  be the quotient map. Put  $E = (\bigoplus_{-\infty}^{+\infty} X_n)_{\ell_2}^{-}$ , with  $X_n = X$  for  $n \leq 0$ ,  $X_n = X/Y$  for n > 0. Define  $T: E \to E$  as the obvious shift, using  $\pi$  on  $X_0$ . Clearly T is bounded and surjective; and its nullspace is Y in  $X_0$ . A projection of E onto this Y would induce a projection  $X = X_0$  onto Y. For (3), let  $S: X \to X$  have nonclosed range. Put  $E = X \oplus X$ ; define  $T: E \to E$  by T(x, y) = (Sx + y, 0). Then T is rangeclosed, but  $T^2$  is not. (Communicated by P.G. Casazza) There were many answers for (3), including a reference to R. Bouldin, Tôhoku Math. J. 25(1973), 359-363.

#### PROBLEM LISTS

#### PROBLEMS IN INTERPOLATION OF OPERATORS AND APPLICATIONS. II

This is the second and last list of problems presented at the Special Session on Iteration of Operators and Applications organized by J. E. Gilbert and G. G. Lorentz (University of Texas at Austin, Austin, Texas 78712), at the meeting of the Society at Washington, D. C., January 21-26, 1975. The first list appeared in the February 1975 issue of these *Cholica* (22(1975), 124-126).

7. <u>E.M. Stein</u> (Princeton University, Fine Hall, Box 708, Princeton, New Jersey 08540). Let  $[A, B]_{\hat{\theta}}$  denote the

complex interpolation of spaces A, B. It is known that  $[H_1, L^p]_{\theta} = L^q$ , where  $q^{-1} = \theta + p^{-1}(1 - \theta)$ . This is an improvement of  $[L^1, L^p]_{\theta} = L^q$ . Find all spaces A for which  $[A, L^p]_{\theta} = L^q$ . A similar question for intermediate spaces  $[A, B]_{\theta}$  can be asked in many other situations.

8. Richard A. Hunt (Purdue University, West Lafayette, Indiana 47907). Are there positive constants  $\rm C_1$  and  $\rm C_2$  such that

$$\left|\left|f\right|\right|_{\mathfrak{M}} < \mathbf{y} \Rightarrow \mathbf{m} \{\mathbf{x} \in [0, 2\pi] : \left|\widetilde{\mathbf{f}}(\mathbf{x})\right| > \mathbf{y} \} \leq \mathbf{C}_{1} \int_{0}^{2\pi} \exp(-\mathbf{C}_{2}\mathbf{y} / \left|\mathbf{f}(\mathbf{x})\right|) \, \mathrm{d}\mathbf{x},$$

where  $\hat{f}$  is the conjugate function of f? The result is true when f is restricted to characteristic functions. A positive answer could be used to prove that lacunary partial sums of the Fourier series of f converge a.e. to

f if  $\int_0^{2\pi} |f(x)| \log^+ \log^+ |f(x)| dx < \infty$ . This result would be best possible.

9. <u>Robert C. Sharpley</u> (Oakland University, Rochester, Michigan 48063).

a. Does there exist a separable rearrangement invariant Banach function space whose fundamental and Boyd indices are different? An affirmative answer can be given if either the requirement of separability or of the Fatou property of the space is dropped. (Shimogaki)

b. Corresponding to each Orlicz space X there exist unique Lorentz spaces  $\Lambda(X)$  and M(X) such that  $\Lambda(X) \subset X \subset M(X)$ . Alternatively for each concave function  $\phi_X$ 

consider the Lorentz  $\Lambda$  and M spaces with fundamental function  $\phi_{X^*}$ . There exists an Orlicz space (unique)  $L_X$  whose fundamental function is  $\phi_X$  and so  $\Lambda(X) \subset L_x \subset$ 

M(X). Is  $L_X$  an interpolation space for  $\Lambda(X)$  and M(X)?

 $(L_X = L_{\Phi} \text{ where } \Phi(s) = (\varphi_X^{-1}(\frac{1}{s}))^{-1}$ .) This result would be the first step in considering fractional integration theorems which would generatize close integration

theorems which would generalize classical results for  $L(p,q)\,$  and Orlicz spaces simultaneously.

c. Strong type interpolation theorems (real methods) for rearrangement-invariant spaces. Can various strong type interpolation theorems for Orlicz spaces be put into a Banach function space setting? For example, can the Jodeit-Torchinski (Studia Math. 1972) work on the Fourier transform and Orlicz spaces be extended to give necessary and sufficient conditions for the Fourier transform to be a bounded map from X to Y? C. Bennett has found sufficient conditions by employing the weak type theory and Calderón's operator. By using Jodeit and Torchinski's characterization of operators of strong type  $(1, \infty)$  and (2, 2), perhaps necessary and sufficient conditions can be obtained.

10. <u>A. Torchinsky</u> (Cornell University, White Hall, !thaca, New York 14850). The complex method of intervolation of Calderón can be viewed as the evaluation of the  $\delta$  distribution on certain families of analytic functions. Using this point of view, Schechter generalized some of the results by considering finite sums of  $\delta$ 's centered at different points. Generalize these results by considering distributions supported in [0, 1], and with support as close to the end-points as possible. This approach may lead to obtain new intermediate spaces between the LP-classes, the Orlicz classes perhaps.

11. Colin Bennett. (Caltech, Pasadena, California 91109). a. Let  $1 \leq p < q \leq \infty$ , and say that a rearrangement-invariant (r.i.) space  $L^{\mu}$  belongs to  $SI(L^{p}, L^{q})$  (for STRONG INTERPOLATION) if every linear operator T bounded on  $L^p$  into  $L^p$  and on  $L^q$  into  $L^q$  is bounded also on  $L^{\mu}$  into  $L^{\mu}$ . Show that all spaces in  $SI(L^{p}, L^{q})$ are generated by the  $(\rho;k)$  interpolation method (C. Bennett, J. Functional Anal. 17(1974), 409-440). Specifically, show that  $L^{\mu} \in SI(L^{p}, L^{q})$  if and only if there is a r. i norm  $\rho$  over  $\mathbb{R}^+$  such that  $L^{\mu} = (L^p, L^q)_{\rho;k}$ , with equivalent norms. The conjecture is true in case p=1 and  $q=\infty$  (loc. cit., Sec. 6). The corresponding problem for WEAK-INTERPOLATION is completely settled (C. Bennett, Proc. Confer. Oberwolfach (to appear)): Thus  $L^{\mu} \in WI(L^{p}, L^{q})$  iff there is a r.i. norm  $\rho$ , with indices  $0 < \beta \leq \alpha < 1$ , such that  $\mathbf{L}^{\mu} = (\mathbf{L}^{p}, \mathbf{L}^{q})_{o:k}$ 

b. Prove a "density theorem" for the k-method without appealing to the corresponding j-method. Thus, when is it true that  $X_0 \cap X_1$  is dense in  $(X_0, X_1)_{\rho;k}$ ? Given such a result, set up the corresponding duality theorem:  $(X_0, X_1)_{\rho;k}^* = (X_1^*, X_0^*)_{\rho';j}$ . When is the isomorphism

isometric? See C. Bennett, loc. cit., for existing results in this direction; for instance, such results are possible when the indices of  $\rho$  satisfy  $0 < \beta \leq \alpha < 1$ . It would be nice to remove this restriction.

c. Burgess Davis (Proc. Amer. Math. Soc., to appear) has recently obtained the following best-possible version of Kolmogorov's conjugate function inequality:

$$\mathbf{y}[\{\mathbf{\tilde{f}}| > \mathbf{y}\}] \leq \frac{1^{-2} + 3^{-2} + 5^{-2} + \dots}{1^{-2} - 3^{-2} + 5^{-2} + \dots} ||\mathbf{f}||_{1}, \quad \mathbf{f} \in \mathbf{L}^{1}.$$

The proof uses Brownian motion. Find a proof by "classical" methods (this problem was suggested by D. Burkholder (Harmonic analysis and probability, preprint)).

When the space  $L \log^+ L$  is endowed with the Lorentz norm

$$||f||_{L\log L} = \int_0^1 f^{**}(t) dt = \int_0^1 f^{*}(t) \log^{1/t} dt,$$

the best-possible version of Zygmund's conjugate function inequality is (C. Bennett, Best constants in Zygmund's conjugate function in equality, to appear)

$$\|\widetilde{f}\|_{1} \leq \frac{1^{-2} - 3^{-2} + 5^{-2} - \dots}{1^{-2} + 3^{-2} + 5^{-2} + \dots} \|f\|_{L \log L}, \quad f \in L \log^{+} L.$$

This proof is entirely classical. The ''dual'' of this result is a weak-type  $\ L^{CO}$  estimate

$$\sup_{t} \tilde{f}^{**}(t)(1-\log t)^{-1} \leq \frac{1^{-2}+3^{-2}+5^{-2}+\ldots}{1^{-2}-3^{-2}+5^{-2}-\ldots} ||f||_{\omega}, \quad f \in L^{\infty},$$

with the same constant as in Davis' result. <u>Can Davis'</u> result be "extrapolated" from it, thus solving Burkholder's problem? d. <u>Comment on Lion's problem 5b</u> (vol. 22, p. 126, Feb. 1975): It is well known that  $C^{(1)}(T)$  is not among the interpolation spaces generated by the complex method or by the  $(\theta, q)$  methods of Peetre. Neither is it generated by the  $(\rho_i)$ ) or  $(\rho_i k)$  methods. For then, by reiteration,  $C^{(1)}$  would be an interpolation space between Lip  $\frac{1}{2}$  and Lip  $\frac{3}{2}$ , which cannot be the case because the conjugate operator is bounded on the Lipschitz spaces but not on  $C^{(1)}$ . This was first pointed out to me by R. De Vore and R.C. Sharpley in April 1974. The problem as stated, however, remains open.

 N.M. Riviere (University of Minnesota, School of Mathematics, Minneapolis, Minnesota 55455).
 a. Let

$$H_{\gamma}(f)(x) = p.v. \int_{-\infty}^{\infty} f(x - \gamma(t)) \frac{dt}{t},$$

 $\begin{array}{l} x \in \mathbb{R}^n, \ \gamma(t) = (\mathrm{sgn}(t) \left| t \right|^{\alpha_1}, \ldots, \mathrm{sgn}(t) \left| t \right|^{\alpha_n}), \ 1 \leq \alpha_1 < \alpha_2, \ldots, \\ < \alpha_n \ (n \geq 2). \ Using the complex method of interpolation, \\ Nagel, Riviere and Wainger [3] have shown that H_{\gamma} is \\ \text{bounded in } L^p(\mathbb{R}^n), \ 1 < p < \infty. \ \text{If we extend the notion} \\ \text{of } H^p\text{-spaces (see [2]) to the case of nonisotopic} \\ \text{dilatations, } H^p_{(\alpha)}(\mathbb{R}^n), \ \alpha = (\alpha_1, \ldots, \alpha_n), \ \text{then it seems} \\ \text{reasonable to ask if an extension of the complex method} \\ \text{could yield the boundedness in } H^1_{(\alpha)}(\mathbb{R}^n) \text{ of } H_{\gamma}. \ \text{Note} \\ \text{that the answer is no when all } \alpha_i \ \text{are equal.} \end{array}$ 

b. Let T be a linear operator such that (1)  $T: H^{1}(\mathbb{R}^{n}) \to L^{1,\infty}(\mathbb{R}^{n})$ , (2)  $T: L^{\infty}(\mathbb{R}^{n}) \to B.M.O.(\mathbb{R}^{n})$ . Is T bounded from  $L^{p}(\mathbb{R}^{n})$  into itself, 1 ? More generally, is

 $(L^{1, \infty}, BMO)_{\theta, q} = L^{p, q}$ ? It is known that when T is sublinear,  $T: L^{1} \rightarrow L^{1, \infty}$  and  $T: L^{\infty} \rightarrow BMO$ , then T is bounded from  $L^{p}$  into itself, 1 , see [4].

c. Let S and T be linear (or sublinear) operators mapping  $L^1(R^n)$  into  $L^{1,\,\infty}(R^n)$  and  $L^2(R^n)$  into itself. Is it true that

$$m(\{x, |T \circ S(f)(x)| \geq 1\}) \leq C \int_{R} |f(x)|(1 + \ell_n^+(|f(x)|)) dx?$$

The result is known when  $S: L^{\infty} \to L^{\infty}$ , see [1].

d. <u>Comments on Peetre's problem 1b</u> (vol. 22, p.124, Feb. 1975): The extension of the complex method to quasi-Banach spaces was considered in my thesis, <u>Interpolation theory in s-Banach spaces</u>, University of Chicago, 1966. In particular the complex interpolation for spaces of mixed norms  $L^{p}(L^{q}), 0 < p, q \leq \infty$ , can be

for spaces of mixed norms  $L^{-}(L^{-}), 0 < p, q \equiv \infty$ , can be found there [see problem 6, Muckenhoupt (vol. 22, p. 126, Feb. 1975)].

#### Bibliography

- N. Fava, <u>Weak type inequalities for iterated</u> operators, Dissertation, University of Minnesota, 1969.
- C. Fefferman and E. M. Stein, H^p spaces of several variables, Acta Math. 129(1972), 137-193.
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#### PERSONAL ITEMS

NORMAN BLACKBURN of the University of Illinois at Chicago Circle has been appointed to the recently established additional Chair of Pure Mathematics at Manchester University.

LAWRENCE G. BROWN of Purdue University has been awarded a Sloan Research Fellowship.

DAVID L. COLTON of Indiana University has been appointed to a Chair in Mathematics, University of Strathclyde.

PAUL R. HALMOS of Indiana University has been elected a Fellow of the Royal Society of Edinburgh.

P. N. RATHIE of the Indian Institute of Technology has been appointed to a professorship at the Universidade Estadual de Campinas, Brazil.

ERNEST E. SHULT of the University of Florida is the new Distinguished Regents Professor for the State of Kansas. He is currently at Kansas State University.

VENKATA R.R. UPPULURI of the Computer Sciences Division, Oak Ridge, Tennessee, is spending the 1975 spring quarter as a visiting professor at Kent State University.

#### PROMOTION

To Professor. Johns Hopkins University: ROGER A. HORN; University of New Haven: BERTRAM ROSS. To Associate Professor. Seton Hall University: CHUNGMING AN; Worcester Polytechnic Institute: GORDON C. BRANCHE.

#### DEATHS

Professor RICHARD F. DeMAR of the University of Cincinnati died on February 11. 1975, at the age of 50. He was a member of the Society for 19 years.

Reverend DONALD T. FAUGHT of the University of Windsor died on February 19, 1975, at the age of 59. He was a member of the Society for 17 years.

Professor HAROLD GARABEDIAN of North Andover, Massachusetts, died on August 13, 1974, at the age of 77. He was a member of the Society for 50 years.

Dr. JULES A. LARRIVEE of Corvallis, Oregon, died on September 27, 1974, at the age of 65. He was a member of the Society for 43 years.

Dr. EVELIO T. OKLANDER of the Universidad Nacional del Sur, Bahia Blanca, Argentina, died on August 28, 1974, at the age of 50. He was a member of the Society for 9 years.

Professor Emeritus JOSEPH SPEAR of Northeastern University, died on February 14, 1975, at the age of 83. He was a member of the Society for 43 years.

# CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

#### TOPICS IN THE HOMOLOGICAL THEORY OF MODULES OVER COMMUTATIVE RINGS by Melvin Hochster

Number 24

75 pages; list price \$4.10; member price \$3.08 ISBN 0-8218-1674-8

To order, please specify CBMS/24

This volume contains expository lectures by Melvin Hochster from the CBMS Regional Conference in Mathematics held at the University of Nebraska, June 24–28, 1974. The lectures deal mainly with recent developments and still open questions in the homological theory of modules over commutative (usually, Noetherian) rings. A good deal of attention is given to the role "big" Cohen-Macaulay modules play in clearing up some of the open questions.

§1 develops necessary background material, while §2 attempts to clarify the relationships among some of the many open questions (rigidity, multiplicities, existence of various kinds of Cohen-Macaulay modules, Bass' question, the intersection conjecture, etc.). §3 explores with fairly complete proofs the consequences of the existence of Cohen-Macaulay modules. In particular, it contains proofs of some of the implications of §2. §4 develops the necessary machinery to prove the existence of "big" Cohen-Macaulay modules in characteristic p > 0, while §5 first gives an expository account (without proofs) of the theory of Henselian rings, Artin approximation, and then part of the reduction of the proof of the existence of big Cohen-Macaulav modules from the case where R contains a field of characteristic 0 to the case where R contains a field of characteristic p > 0. §§3,4,5 yield a fairly complete proof of the intersection theorem, Bass' conjecture, the zerodivisor conjecture. etc. for local rings containing a field.

§6 discusses the phenomenon of depthsensitivity including the Buchsbaum-Eisenbud criteria for acyclicity of certain complexes and related results which go in a somewhat different direction. §7 begins with a discussion of the Buchsbaum-Eisenbud structure theorems for finite free acyclic complexes (with a sketchy proof) and then goes on to a discussion of just what it might mean to give a "best possible" structure theorem in terms of describing generic free acyclic complexes with prescribed Betti numbers. The program is carried out in detail for complexes of length 2 (with partial proofs). This leads into a discussion of linear algebraic groups in §8 and their rings of invariants.

§9 is mostly independent. It contains two applications of homological methods to problems (cancellation of indeterminates, the Zariski-Lipman conjecture) which do not a priori involve homological ideas. It also surveys some open questions about the spectrum of a Noetherian ring (as an ordered set) one of which has some connection with the existence of big Cohen-Macaulay modules.

A supplement at the end includes extra material discussed at the conference. An extensive bibliography is also included.

A modest knowledge of commutative rings and familarity with (the long exact sequences for) Tor and Ext should suffice as a background for the reader.

# PROCEEDINGS OF THE STEKLOV INSTITUTE

PROCEEDINGS OF THE INTERNATIONAL CON-FERENCE ON NUMBER THEORY edited by I. M. Vinogradov

Number 132

298 pages; list price \$30.10; member price \$22.58

ISBN 0-8218-3032-5

To order, please specify STEKLO/132

This volume contains the cover-to-cover translation of the proceedings of the International Conference on Number Theory held in Moscow, September 14-18, 1971.

The following papers are included: "Address of the president of the Academy of Sciences of the USSR" by Academician M.V. Keldys, "Exponential sums in the development of number theory" by K. Chandrasekharan, "Recent works of I.M. Vinogradov" by Ju. V. Linnik, "Some exponential sums" by L.J. Mordell, "Artin's conjectures and the law of reciprocity" by A.I. Vinogradov, "The arithmetic of K3 surfaces" by I.I. Pjateckii-Sapiro and I.R. Safarevič, "The Hilbert modular group and some algebraic surfaces" by F. Hirzebruch, "Recent advances in transcendence theory" by A. Baker, "The principle of the theory of nonstandard functional equations for Dirichlet functions, consequences and applications of it" by A.F. Lavrik, "Largest prime factor of the product of k consecutive integers" by K. Ramachandra, "On the uniform boundness of the torsion of elliptic curves over algebraic number fields" by V.A. Dem'janenko, "On the partition function of positive definite matrices" by T. Mitsui, "Über das Normenrestsymbol einer lokalen unverzweigten Erweiterung von 2-Potenzgrad" by H. Koch, "Distribution problems of arithmetic functions" by I.P. Kubilius, "On sums of squares" by J.W.S. Cassels, "On  $(\infty \times p)$ -adic coverings of curves (the simplest example)" by Y. Ihara, "Siegel forms and zetafunctions" by A. N. Andrianov, "Applications of the method of trigonometric sums to the metric theory of diophantine approximation of dependent quantities" by V.G. Sprindžuk, "Reducibility of quadrinomials" by A. Sincel',"Lattice points in moredimensional ellipsoids" by B. Novák. "The geometry of linear algebraic groups" by V.E. Voskresenskii, "The arithmetic theory of

linear algebraic groups and number theory" by V. P. Platonov, "On arithmetic properties of values of analytic functions" by A. B. Šidlovskiĭ, "Dirichlet characters and polynomials" by D. A. Burgess, "On the extended Hecke theta-formula" by T. Tatuzawa, "Modular correspondences, heights and isogenies of abelian varieties" by A. N. Paršin, "Constructive method in the theory of equations over finite fields" by S. A. Stepanov, "On sums of real characters" by M. Jutila, "On large sieve inequalities and their applications" by E. Bombieri, and "On some problems of prime number theory connected with I. M. Vinogradov's method" by A. A. Karacuba.

ON INTEGRAL FUNCTIONALS WITH A VARI-ABLE DOMAIN OF INTEGRATION by I. I. Daniljuk

Number 118(1972) 124 pages; list price \$17.30; member price \$12.98 ISBN 0-8218-3018-X To order, please specify STEKLO/118

Methods and results are stated of an investigation of integral functionals with a variable domain of integration, in connection with nonlinear problems with a free boundary in an arbitrary force field. Analytical tools are developed in the form of a system of nonlinear singular integro-differential equations for the determination of the free boundary. Conditions of local uniqueness of critical curves are clarified, properties "in the large" of the set of critical points are studied, and a generalization of Morse theory is constructed. A separate section is devoted to the existence problem.

PROBLEMS IN THE THEORY OF POINT EXPLOSION IN GASES by V.P. Korobeinikov

Number 119(1973) 288 pages; list price \$33.00; member price \$24.75 ISBN 0-8218-3019-8 To order, please specify STEKLO/119

The theory of point explosion arose from the necessity of describing processes of propagation of explosions from concentrated charges in continuous media.

The monograph is devoted to the development of the theory of point explosion in gases. It contains formulations of newproblems; theoretical models of the motion of the medium are constructed which take account of various physical phenomena and properties of the medium; methods of solving the new problems which arise are developed; a detailed study of problems of the theory which have previously been formulated is presented. Certain applications to physical problems are considered.

This issue is intended for specialists in hydrodynamics and applied mathematics and especially those students in advanced courses who are studying these disciplines.

#### PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

ALGEBRAIC GEOMETRY-ARCATA 1974 edited by Robin Hartshorne

Volume 29 642 pages; list price \$40.10; member price \$30.08

ISBN 0-8218-1429-X To order, please specify PSPUM/29

This volume contains the proceedings of the Summer Institute in Algebraic Geometry held July 29-August 16, 1974, at Humboldt State University, Arcata. California.

The editor has included the texts of almost all the expository lectures series, and those seminar talks which were of a sufficiently broad nature to serve as introductions to their respective areas. He has also included a survey article on algebraic surfaces, not presented at Arcata, by E. Bombieri and D. Husemoller. This volume should therefore provide orientation to the newcomer on the specialist exploring new fields, by surveying the present state of the art, and giving references for further study.

The following papers are included in this volume from the lecture series at the institute: "Some transcendental aspects of algebraic geometry" by Maurizio Cornalba and Phillip A. Griffiths, "Some directions of recent progress in commutative algebra" by David Eisenbud, "Equivalence relations on algebraic cycles and Subvarieties of small codimension" by Robin Hartshorne, "Triangulations of algebraic sets" by Heisuke Hironaka, "Introduction to resolution of singularities" by Joseph Lipman, "Eigenvalues of Frobenius acting on algebraic varieties over finite fields" by B, Mazur, and "Theory of moduli" by C.S. Seshadri,

The seminar talks included in the volume are "Larsen's theorem on the homotopy groups of projective manifolds of small embedding codimension" by Wolf Barth, "Slopes of Frobenius in crystalline cohomology" by Pierre Berthelot, "Classification and embeddings of surfaces" by Enrico Bombieri and Dale Husemoller, "Linear representations of semi-simple algebraic groups" by Armand Borel, "Knot invariants of singularities" by Alan H. Durfee, "Riemann-Roch for singular varieties by William Fulton, "Report on crystalline cohomology" by Luc Illusie, "p-Adic *l*-functions via moduli of elliptic curves" by Nicholas M. Katz, "Topological use of polar curves" by Lê Dung Tráng, "Matsusaka's big theorem" by D. Lieberman and D. Mumford, "Unique factorization in complete local rings" by Joseph Lipman. "Differentials of the first, second and third kinds" by William Messing, "Short sketch of Deligne's proof of the hard Lefschetz theorem" by William Messing, "p-Adic interpolation via Hilbert modular forms" by Kenneth A. Ribet, "Introduction to equisingularity problems" by B. Teissier, and "Algebraic varieties with group action" by Philip Wagreich.

The Organizing Committee for the Institute included Michael Artin, Phillip A. Griffiths, Robin Hartshorne, Heisuke Hironaka, Nicholas Katz, and David Mumford (chairman). The Institute was partially supported by a grant from NSF.

NONLINEAR FUNCTIONAL ANALYSIS by Felix E. Browder

Volume 18, Part 2 312 pages; list price \$25.90; member price \$19.43 ISBN 0-8218-0244-5

To order, please specify PSPUM/18-2

The present volume contains the booklength text of a paper entitled "Nonlinear operators and nonlinear equations of evolution in Banach spaces" composed in its entirety during the calendar year 1968 to be published as part of the Proceedings of the Symposium on Nonlinear Functional Analysis held in connection with the April 1968 meeting in Chicago of the American Mathematical Society. This paper is in fact a detailed treatment in book form of most of the major branches of nonlinear functional analysis as they had developed up to 1968 and no significant alterations or additions have been made since that time except for the correction of errors in detail.

Despite this delay the publication of the text in its present form is still of great value. This is the case both because of the text's presence as an "underground" part of the literature and the many references to it in the papers since 1968 and because it still is the only treatment in systematic form of the field of nonlinear functional analysis as a whole which deals with the major developments of the 1960's.

The chapter headings are: Introduction; Contractive mappings; Locally Lipschitzian mappings;  $\Phi$ -accretive and  $\Phi$ -coaccretive mappings; Covering space methods; Limits of invertible and semi-invertible mappings; Fixed point and mapping theory for compact multivalued mappings; Monotone mappings in Banach spaces; Nonexpansive mappings in Banach spaces; Accretive mappings and nonlinear equations of evolution; Existence theorems involving accretive mappings; Nonlinear interpolation; Generalizations of the topological degree of a mapping; Compact perturbations of nonexpansive, monotone, and accretive mappings; Nonlinear Fredholm mappings; Orientationpreserving and complex analytic mappings: Asymptotic fixed point theorems; A-proper mappings, approximation methods, and related generalizations of topological degree. There is an extensive bibliography, and author and subject indexes.

### LECTURES ON MATHEMATICS IN THE LIFE SCIENCES

SOME MATHEMATICAL QUESTIONS IN BIOLOGY.V edited by Jack D. Cowan

#### Volume 6

141 pages; list price \$13.00; member price \$9.75 ISBN 0-8218-1156-8

To order, please specify LLSCI/6

The papers published in this volume were given at the seventh symposium on Some Mathematical Questions in Biology, held on July 7, 1973, in Mexico City. The symposium was cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, as part of a joint symposium, "Science and Man in the Americas", organized jointly by the American Association for the Advancement of Science, and the Consejo Nacional de Ciencia y Tecnologia, of Mexico.

The six papers included in this volume provide a small but diverse sample of contemporary theoretical biology, linked by their use of nonlinear stability theory. Two distinguished mathematicians, René Thom and Steven Smale, in their respective contributions, show the potential power of modern mathematics, as seen in the theory of dynamical systems, and in catastrophe theory, as applied to model problems taken from cellular and developmental biology. These papers should be compared with those of three developmental biologists, Lewis Wolpert and Anthony Robertson and Morrel H. Cohen, which show how contemporary theoretical work on development has been used in experimental studies of the development of various organs or organisms, such as cellular slime molds, hydroids, and chick limbs. The last two papers, one by the population biologist Robert May, the other by the editor of this volume, show how far the qualitative mathematical analysis of complex systems, species and nerve nets, respectively. has developed in recent years.

The titles of the papers and names of the authors follow: "Gradients in biology, in mathematics, and simultaneous optimization" by René Thom, "A mathematical model of two cells via Turing's equation" by S. Smale, "Positional information and the development of pattern and form" by L. Wolpert, "Quantitative analysis of the development of cellular slime molds. II" by Anthony Robertson and Morrel H. Cohen, "How many species: Some mathematical aspects of the dynamics of populations" by Robert K. May, and "Mathematical models of large-scale nervous activity" by J. D. Cowan. Author and subject indexes are also included.

#### MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

ON THE GROUPS JO(G) by Chung-Nim Lee and Arthur G. Wasserman

Number 159

62 pages; list price \$3.20; member price \$2.40 ISBN 0-8218-1859-7 To order, please specify MEMO/159

The purpose of this Memoir is to classify real representations of a compact Lie group G under stable J-equivalence. Two representations V, W of G are said to be stably Jequivalent if there exist equivariant maps  $S(V \oplus U) \rightarrow S(W \oplus U)$ ,  $S(W \oplus U) \rightarrow S(V \oplus U)$ , both of degree one, for some representation U, where  $S(V \oplus U)$  denotes the unit sphere in  $V \oplus U$ . Denote by JO(G) the quotient group of the representation ring RO(G), modulo stable Jequivalence. It is shown that V, W are stably J-equivalent if and only if dim  $V^C = \dim W^C$  for every cyclic subgroup C of G such that  $C/C \cap G_0$  is a p-group, p a prime where  $V^C$  denotes the subspace of V fixed under C, or equivalently if and only if the difference character,  $\chi_{\rm V}$  -  $\chi_{\rm W}$ , is constant on each connected component of G which has prime power order  $G/G_0$ , and  $\sum_{g \in C} \chi_V(g) = \sum_{g \in C} \chi_W(g)$  for every finite cyclic subgroup C of a prime power order. In addition, the above theorem has a localized version at any collection of primes. The sufficiency of these conditions is shown via a Thom-Pontryagin type construction and the necessity follows from simple equivariant transversality arguments. The crucial fact necessary to handle arbitrary compact Lie groups is that for any x in G the centralizer and the conjugacy class of x meet transversally. Some of the interesting consequences of these results are (1) JO(G) is a free abelian group, (2) JO(G) injects into the product  $\prod_{p} JO(G_{p})$  where  $G_{p}$  is the inverse

image of a Sylow p-subgroup of  $G/G_0$ , (3) if G

is connected, then  $JO(G) \approx RO(G)$ . Moreover, these methods provide another proof of the Atiyah-Tall theorem  $JO(G) \approx RO(G)_{\Gamma}$ .

INVARIANT SUBSPACES OF HARDY CLASSES OF INFINITELY CONNECTED OPEN SURFACES. by Charles W. Neville

Number 160

151 pages; list price \$4.20; member price \$3.15 ISBN 0-8218-1860-0 To order, please specify MEMO/160

This Memoir presents a version of Beurling's theorem on the shift-invariant subspaces of Hardy class  $H^2$  which is valid for certain spaces of analytic functions on certain open Riemann surfaces. The spaces in question are the H^p spaces, which consist of all analytic functions on the surface for which the pth power of the modulus admits a harmonic majorant. The class of surfaces in question is defined rather technically, but basically consists of those surfaces for which a suitable system of bounded multiple-valued analytic functions exists. It is proved that there are many such infinitely connected surfaces. The version of Beurling's theorem presented in this Memoir characterizes the subspaces of H^p which are invariant under multiplication by all bounded analytic functions on the surface. In addition, a version of the Rubel-Shields' theorem, which characterizes the strictly closed ideals in the algebra of bounded analytic functions on the surface, is obtained. A new construction of the Hayashi boundary, which uses the Gelfand-Naimark theorem applied to a natural Banach algebra structure on the bounded harmonic functions, is given. Finally, this Memoir contains a version of the Cauchy integral theorem, and its converse, the theorem of A.H. Read, in which the integration occurs over the Hayashi boundary, or over other abstract boundaries, such as the Martin boundary.

# ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred. Invited addresses are indicated by •

Abstracts for papers presented at	Appear on Page
723rd meeting in St. Louis, April 11–12, 1975	A-483
724th meeting in Monterey, April 19, 1975	A-484
725th meeting in Pullman, June 21, 1975	A-485

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices*) but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

## Algebra & Theory of Numbers

*75T-A104 S. BURRIS, Department of Pure Mathematics, University of Waterloo, Waterloo, Ont. Boolean Powers: <u>Some Algebraic Aspects</u>, Preliminary report.

For  $\mathcal{A}$  an algebra,  $\mathcal{B}$  a Boolean algebra,  $\mathcal{A}[\mathcal{P}]^*$  will denote the <u>bounded Boolean</u> <u>power</u> of  $\mathcal{A}$  by  $\mathcal{P}$ . For F a filter on  $\mathcal{P}$ ,  $\theta_F^*$  is the congruence of  $\mathcal{O}[\mathcal{P}]^*$  induced by F.  $\mathcal{B}_0 * \mathcal{B}_1$ denotes the <u>relatively free product</u> of  $\mathcal{B}_0$  and  $\mathcal{B}_1$ .  $\mathcal{A} = \langle A, F \rangle$  is nontrivial if Card  $A \geq 2$ . Con $\mathcal{A}$  is the lattice of congruences of  $\mathcal{O}$ . <u>Theorem</u> 1.  $(\mathcal{O}[\mathcal{B}_0]^*)[\mathcal{B}_1]^* \cong \mathcal{O}[\mathcal{B}_0 * \mathcal{B}_1]^*$ .  $\Box$  An algebra  $\mathcal{O}$  is B-<u>separating</u> if  $\mathcal{B}_0 \not\cong \mathcal{B}_1$  implies  $\mathcal{A}[\mathcal{B}_0]^* \not\cong \mathcal{A}[\mathcal{B}_1]^*$ . <u>Theorem</u> 2.  $\mathcal{A}$  is B-separating if it satisfies the following: (i)  $\mathcal{O}$  is simple, (ii)  $\operatorname{Con}(\mathcal{O}^n)$  is modular for  $n < \omega$ , and (iii)  $\operatorname{Con}(\mathcal{O}^2) \cong (\operatorname{Con} \mathcal{O})^2$ .  $\Box$  The next result generalizes a paper of B. H. Neumann and S. Yamamuro (on groups). <u>Corollary</u> Let  $\mathcal{O}$  be a nontrivial algebra. Then, for every Boolean algebra  $\mathcal{B}$  we have  $\operatorname{Con} \mathcal{O}[\mathcal{B}]^* = \{\theta_F^*: F \text{ a filter on } \mathcal{B}\}$  iff  $\mathcal{O}$  satisfies (i) - (iii) of Theorem 2.  $\Box$  <u>Corollary</u>. Every nontrivial congruence distributive variety with a countable language has  $2^{\lambda}$  isomorphism types of power  $\lambda$ ,  $\lambda \geq \aleph_0$ , plus the peculiar direct product phenomena of Boolean algebras observed by W. Hanf. $\Box$ (Received January 28, 1975.)

#### *75T-A105 BENJAMIN VOLK, 13-15 Dickens Street, Far Rockaway, N.Y. 11691. Riemann Zeta Function Approximations.

For all  $N \ge 7$ , and  $0 \le r \le 1$ , and  $\tau \ge 2N$ , a series of 2N + 1 terms is found which gives  $f(s)r(s)(1 - 2^{i-5})$  to within an accuracy of  $3.6/(\exp N)$ . Details are available from the author. (Received February 3, 1975.)

*75T-A106 M. BHASKARAN, 9 - 18 Butterick Place, Girrawheen, W. Australia 6064. On the genus field of a Galois extension - 11, Preliminary report.

The earlier result on this topic (these Notices, Feb. 1975) is used to describe the genus field  $\overline{K}$  of an arbitrary Galois extension K. Kronecker-Weber theorem and a result of Furuta on genus number (this number could differ from that of the previous authors by a factor of 2 as Furuta takes K to be unramified over K even at infinite primes too) are also made use of.

over K even at intinite primes too) are also made use of. Theorem 1. Let e' denote the ramification index of the maximal abelian subfield K' of the p-adic completion K p of K. Let  $\Omega_p$  denote the subfield of degree e' of the field of roots of unity admitting only one ramified prime p and let  $\Omega$  be the composite of all these  $\Omega_p$  where p run through ramified primes. Then  $\overline{K} = K \Omega$ . It is possible to prove that when K is tamely ramified, a unit in  $\Omega_p$  is a local norm from K p iff it is an e'p - th power of a rational p-adic unit. (A generalization of this could be used to give a one page proof for the main theorem in Ax's Ph.D. dissertation, Trans. Am. Math. Soc. 105 (1962) 462 - 474, filling also a gap in the theorem.) But, this is not true in general when K is wildly ramified. However, Theorem 1 and class field theory yield: Theorem 2. Let p^e be the local conductor of K'. If q is a prime unramified in K with f and h as the residue class degree and order in the class group of a K-prime divisor, then f h a give the local conductor of K and the second structure of the second

$$q^{f_q h_q} = + x e_{P} \mod p^{Q}$$
, for some  $x \in \mathbb{Z}$ .

The splitting law in a cyclic extension for primes belonging to some congruence classes is also discussed. Theorem 2 and a theorem of Parry seem to throw some light on the structure of any Galois group indicating that not every finite group is a Galois group over Q. (Cf. Gilmer, Queries, June '73.) (Received April 15, 1975.)

75T-ALO7 E.M. WRIGHT, University of Aberdeen, Aberdeen, U.K. Bounds on the minimum degree of a node in almost all graphs on n nodes. Preliminary Report.

We consider all graphs on n labelled nodes where n is large. Our graphs contain no slings; every two different nodes are not joined or joined by just one edge. There are positive numbers  $C_1$ ,  $C_2$ ,  $C_3$ independent of n with the following properties. In almost all graphs, the degree of each of at least  $C_3$  n/log n nodes is less than  $\frac{1}{2}n - C_2n^{\frac{1}{2}}$  and the degree of every node is greater than  $\frac{1}{2}n - C_1(n \log n)^{\frac{1}{2}}$ . (Received February 13, 1975.)

75T-A108 V.E. HOGGATT, Jr., San Jose State University, San Jose, California 95192 and G.L. ALEXANDERSON, University of Santa Clara, California 95053. <u>Sums of partition sets in</u> generalized Pascal triangles. Preliminary report.

Let  $(1 + x + x^2 + ... + x^{r-1})^n = \sum_{j=0}^{n(r-1)} {n \choose j}_r x^j$ ,  $r \ge 2$ , n = 0, 1, 2, ... This defines a generalized Pascal triangle. If one partitions the elements of the rows into k sets  $s_{i,n} = \sum_{j=0}^{N} {n \choose jk+i}_r$ , N = [(n(r-1) - i)/k], i = 0, ..., k - 1, then one can prove <u>Theorem</u>. If  $r \equiv 2,3 \pmod{5}$ , then for k = 5 the sums  $s_{i,n}$  give three distinct numbers such that the differences form three consecutive members of the Fibonacci sequence. Further, the sums themselves form a regular pattern of the type  $(r^n \pm L_m)/5$  or  $(r^n \pm 2L_m)/5$ , where  $L_m$  is the mth Lucas number. Similar results are obtained for further values of k; other well-known sequences appear in the differences as well as in the sums themselves. (Received February 14, 1975.)

*75T-A109 PATRICK COSTELLO, Harvey Mudd College, Claremont, CA 91711, <u>Amicable pairs</u> of Euler's first form. Preliminary report.

A pair of numbers (m,n) is said to be amicable if  $\sigma(m) = m + n = \sigma(n)$  where  $\sigma(x)$ denotes the sum of all the divisors of x. The author has been involved in a study of pairs of the particular form (apq,ar) where p, q and r are distinct primes not dividing the common factor a. What is proven about the common factor a includes: a is not simply a power of a prime  $\geq 3$  (which answers one of Rolf's questions (Math Teacher, 1967, pp. 157-160) for this type of amicable pair), a is not divisible by both 2 and 3 (answers another of Rolf's questions), and a is not divisible by 2, 5 and 7 or by 3, 5, 7, 11 and 13. With these facts in mind an algebraic formula for p, q, r in terms of a and  $\sigma(a)$  was devised. A computer program was then designed to generate all of Euler's pairs of this form. Besides doing this, the computer produced three more new amicable pairs (4 others listed in *Notices*, August 1974, A-483). They are:  $(3^5 \cdot 5 \cdot 11 \cdot 59 \cdot 197 \cdot 1297, 3^5 \cdot 5 \cdot 11 \cdot 59 \cdot 257003)$ ,  $(3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 281 \cdot 263 \cdot 87671, 3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 281 \cdot 231 45407)$ ,  $(3^4 \cdot 5 \cdot 11^3 \cdot 137 \cdot 71 \cdot 542519, 3^4 \cdot 5 \cdot 11^3 \cdot 137 \cdot 39061439)$  (Received February 17, 1975.) (Author introduced by Professor Alvin

M. White.)

*75T-Allo MARTIN R. PETTET, University of Wisconsin, Madison, Wisconsin 53706. <u>On finite groups admitting a fixed-point-free automorphism of prime power order</u>.

It has been conjectured that a finite group admitting a fixed-point-free automorphism is necessarily solvable. Theorem Let p be a prime and G be a finite group which admits a fixed-point-free automorphism of order a power of p. If the symmetric group of degree four is not involved in G, then G is solvable. Corollary A finite group which admits a fixed-point-free automorphism of order  $3^n$  is solvable. The main tools in the proof are a result of Glauberman on S4-free groups and Goldschmidt's characterization of groups with a "strongly closed" abelian 2-subgroup. (Received March 24, 1975.)

#### *75T-A111 JOHN TILLER, Hendrix College, Conway, Arkansas 72032. <u>On Constructing</u> <u>Reflective and Coreflective Hulls with Sinks and Sources</u>. Preliminary report.

In this paper, it is shown how sources determine reflective hulls and sinks determine coreflective hulls. Herrlich and Strecker have determined that the coreflective hull of a mapinvariant subcategory <u>D</u> of <u>Top</u> is the subcategory of all <u>D</u>-generated spaces. In this paper a categorical definition of <u>D</u>-generated objects is given using sinks. This new definition coincides with the usual topological definition for map-invariant subcategories of <u>Top</u> and gives the coreflective hull of subcategories in several categories (including <u>Top</u>). A categorical definition of residually-<u>D</u> objects is also given using sources which is dual to that of <u>D</u>-generated objects. This definition coincides with the usual group-theoretic definition for subcategories of <u>Grp</u> which are closed under subgroups and gives the reflective hull of subcategories in several categories (including <u>Grp</u>). (Received February 20, 1975.)

#### *75T-A112 DR. LAWRENCE J. RISMAN, TECHNION, HAIFA, ISRAEL. On the Order and Degree of Solutions to Pure Equations.

Let K be a field. Let  $\theta$  be an element of a field extention of K. The order of  $\theta$ over K is the smallest positive integer m such that  $\theta^m$  lies in K, or •. We compare the order m of  $\theta$  to the degree h of  $\theta$  over K. Clearly h<m. Theorem: Let K be a field. Let  $\theta$  be an element of degree h and order m over K. Let p be a prime. Let  $p^e$  be the maximum power of p dividing h, and suppose S where

- p^s divides m.
- 1. If the characteristic of K is p, then s<e.
- 2. If s>e and p is odd, then  $K(\theta)$  contains a primitive pth root of unity u not in K. Moreover K(u) contains a primitive  $p^{S-\theta}$  root of unity.
- 3. If s>e and p=2, then -1 is not a square in K and K( $\Theta$ ) contains i= $\sqrt{-1}$ . Moreover -1 is a 2^{S-e} power in K(i).

This paper contains some results on the multinomial degree of an element. (Received February 21, 1975.)

#### 75T-A113 MASAO KISHORE, University of Toledo, Toledo, Ohio 43606, <u>Quasiperfect numbers</u> are divisible by at least six distinct prime factors. Preliminary report.

A positive integer N is called quasiperfect if  $\sigma(N) = 2N + 1$ , and almost perfect if  $\sigma(N) = 2N - 1$ . H.L. Abbot, C.E. Aull, E. Brown, and D. Suryanarayana have shown (Acta Arithmetica, XXII, 1973, 439-447) that if N is quasiperfect, then (1) N >  $10^{20}$ , (2)  $\omega(N) \ge 5$  if 3|N, and (3)  $\omega(N) \ge 8$  if  $3 \ddagger N$ . Using computer we prove that (1) N >  $10^{30}$ , (2)  $\omega(N) \ge 6$  if 3|N, and (3)  $\omega(N) \ge 9$  if  $3 \ddagger N$ . We had used a similar method to obtain the same results on <u>Odd almost perfect numbers</u> (AMS Notices, April, 1975). (Received February 24, 1975.)

*75T-A114 DAVID BALLEW and RONALD WEGER, South Dakota School of Mines and Technology, Rapid City, SD 57701, <u>Triangular Solutions to the</u> <u>Pythagonean Equation</u>, Preliminary Report.

The following theorems are proven: 1) A necessary and sufficient condition that a Pythagorean triple have all members triangular is that the square of the smallest member be the sum of consecutive cubes, 2) For a fixed k, there are only a finite number of sums cr k consecutive nth powers which can be the square of a triangular number, n = 3, 4, 5. (Received February 24, 1975.)

A <u>quaternary</u> group Q is defined to be a set S and an operation  $\oint$  on S ×  $\mathfrak{s}^3$  to S such that the set S³ and the operation  $\oint$  ³ on S³ × S³ to S³ for which  $\oint$  ³ ((A,B,C),(D,E,F))=( $\oint$  (A,(D,E,F)),  $\oint$  (B,(D,E,F)),  $\oint$  (C,(D,E,F))) constitute a group Q³. (S³consists of all ordered triples of distinct elements of S.) Abbreviate  $\oint$  (A,(B,C,D)) to A(B,C,D) and write (0,1, $\infty$ ) for the identity in Q³. It has been shown that a projective plane  $\pi$  is a Pappus plane if and only if, for some quadrangle  $\alpha$  in  $\pi$  and side j of  $\alpha$ , the set S( $\alpha$ ) of points on j and a certain operation  $\oint$ ( $\alpha$ ) on S( $\alpha$ ) × S³( $\alpha$ ) to S( $\alpha$ ) constitute a quaternary group Q( $\alpha$ ). (R. Seall, Connections between projective geometry and superassociative algebra, Jour.of Geom.,4/1, 1974, 11-33.) <u>Theorem 1.</u> If Q is a finite quaternary group and A(A, $\infty$ , 1) = 0 for every A in Q and  $\neq$  1, $\infty$ , then there exists a Pappus plane  $\pi$  and a quadrangle  $\alpha$  in  $\pi$  such that Q is isomorphic to Q( $\alpha$ ). The order of a finite quaternary group is defined to be the number of elements of S. <u>Theorem 2.</u> Every finite quaternary group is of order p^r+1, p a prime. (Received February 24, 1975.)

#### *75T-All6 SURJEET SINGH, Aligarh Muslim University, Aligarh, India, When are rings hereditary noetherian prime rings ? Preliminary report

Hajarnavis [Bull. London Math Soc 5(1973)] and Zaks [Israel J.Math 10(1971)] proved the following result independently of each other: Let R be a noetherian, bounded, prime rings such that every proper homomorphic image of R is self injective. Then R is a Dedekind prime ring. In this paper the following is proved: Let R be a noetherian, bounded prime ring such that every homomorphic image of R is a generalized uniserial ring. Then R is a hereditary noetherian prime ring. (Received February 24, 1975.)

*75T-All7 S. Singh and Musharrafuddin Khan, Aligarh Muslim University, Aligarh 202001, India, Modules over (hpp)-rings III, Preliminary report.

This paper is the continuation to the paper 'Modules over hereditary noetherian prime rings II', Notices AMS, Volume 21(1974)(October issue); it has same notations and terminology as in that paper. Let R be a bounded (hnp)-ring. A submodule N of a torsion R module M is said to be h-pure if  $H_{D}(N) = N \cap H_{D}(M)$  for all  $n \ge 0$ . The following are the main results established. (I) The direct sum N of any family  $(N_i)_{i \in I}$  of torsion R-modules is h-pure in the largest torsion submodule of the direct product  $\prod_{i \in I} N_i$ . (II) For a submodule N of a torsion R-module M, the following are equivalent:

 (1) N is a h-pure submodule of M, (11) For any submodule T of N such that N/T is bounded, N/T is a direct summand of M/T, (111) For any proper ideal A of R, M/NA is a direct summand of M/Na. (Received February 24, 1975.)
 75T-A118 ELIZABETH D. BEHRENS, 728 N. Minnesota Avenue, Hastings, NE 68901. Indicability in Products of Groups. Preliminary report.

A group is <u>indicable</u> if it has an infinite cyclic homomorphic image. We investigate the properties of indicability and local indicability, and, in particular, in generalized free products of two groups and in standard wreath products of groups. G. Baumslag proved that any generalized free product of an indicable group with a finitely-generated torsion-free nilpotent group is indicable, and any generalized free product of two finitelygenerated torsion-free nilpotent groups is locally indicable. We generalize his results to arbitrary finitely-generated nilpotent groups. We define a group C to be <u>almost locally indicable</u> if every finitely-generated subgroup of G which is not periodically-generated is indicable. We prove that any generalized free product of an indicable group with a finitely-generated nilpotent group is indicable, and that the generalized free product of two locally nilpotent groups with a finitely-generated subgroup amalgamated is almost locally indicable. We characterize indicability in the restricted wreath product as well as in unrestricted wreath products in which the "top" group is finite or non-periodic. We also obtain a complete characterization of local indicability in wreath products. (Received March 4, 1975.)

*75T-A119 D. SURYANARAYANA, University of Toledo, Toledo, Ohio, 43606 and P. SUBRAHMANYAM Andhra University, Waltair, India. <u>The Maximal k-free divisor of m which is</u> prime to n.

Let k be a fixed integer  $\geq 2$  and let  $q_k(m)$  be the characteristic function of the set  $Q_k$  of k-free integers. Let  $\gamma_k(m;n)$  denote the greatest divisor d of m such that d e  $Q_k$  and (d,n) = 1. Let  $\phi(m)$  denote the Euler totient function. In this paper we establish asymptotic formulae for the sums  $\sum_{\substack{m \leq x \\ m,n \geq 1}} q_k(m)\phi(m)$  and  $\sum_{\substack{m \leq x \\ m \leq x}} \gamma_k(m;n)$  with uniform 0-estimates for the error terms both equal to  $0 \left( \frac{\psi(n)n}{\psi(n)} x^{1+(1/k)} \delta(x) \right)$ , where  $\theta(n)$  is the number of square-free divisors of n,  $\psi(n)$  is the Dedekind  $\psi$ -function and  $\delta(x) = \exp\{-A \log^{3/5}x (\log \log x)^{-1/5}\}$ , A being a positive absolute constant. We also improve the above 0-estimate to  $0 \left( \frac{\theta(n)n}{\psi(n)} x^{1+(2/2k+1)} \omega(x) \right)$ , where  $\omega(x) = \exp\{A \log x (\log \log x)^{-1}\}$  on the assumption of the Riemann hypothesis. These results not only generalize the asymptotic formulae but also improve the 0-estimates known so far in some particular cases. (Received March 3, 1975.)

*75T-A120 D. SURYANARAYANA, University of Toledo, Toledo, Ohio 43606. On a paper of S. Chowla and H. Walum concerning the divisor problem.

Let  $\psi(x) = x - [x] - \frac{1}{2}$ ,  $\Delta(x) = \sum_{\substack{n \le x \\ n \le x}} \tau(n) - \{x \log x + (2\gamma - 1)x\}$  and  $G(x) = \frac{1}{2}$ 

 $\sum_{n \le x} \Delta(n) - \{\frac{1}{2} \times \log x + (\gamma - \frac{1}{4}) \times\}, \text{ where } \tau(n) \text{ is the number of divisors of } n \text{ and } \gamma \text{ is } n \le x$ Euler's constant. It has been shown by S. Chowla and H. Walum (Proc. Symposia in Pure Math., Amer. Math. Soc., VIII (1965), 138-143) that the sum  $\sum_{n \le \sqrt{x}} n \{\psi^2(\frac{x}{n}) - \frac{1}{12}\} = O(x^{3/4})$  by making use of G. Voronoi's result  $G(x) = O(x^{3/4})$ . In the course of the proof of their result, they also proved in the appendix that  $\sum_{x=1}^{R} \{\psi^2(\sqrt{x}) - \frac{1}{12}\} = O(R^{3/4})$ . In this paper we obtain a direct relationship between G(x) and the sum  $\sum_{n \le \sqrt{x}} n \{\psi^2(\frac{x}{n}) - \frac{1}{12}\}$  and improve the order  $n \le \sqrt{x}$  estimate of their result in the appendix to  $O(R^{1/2})$ . The direct relationship between G(x) and the sum is given, for example, by  $G(x) + \sum_{n \le \sqrt{x}} n \{\psi^2(\frac{x}{n}) - \frac{1}{12}\} = O(\sqrt{x})$ . (Received March 3, 1975.)

*75T-A121 NEIL HINDMAN, California State University, Los Angeles, California, 90032. <u>A</u> note on partitions and sums of integers with repetitions.

The following theorem can be proved by iterated applications of the finite sum theorem, (J. Comb. Theory (A) <u>17</u> (1974), 1-11). <u>Theorem</u> 1. Let  $r, p \in \mathbb{N}$  with  $p \leq r$ . If  $\mathbb{N} = \bigcup_{i=1}^{r} \mathbb{A}_i$ , then there exist a sequence  $\langle x_n \rangle_{n=1}^{\infty}$  and a function f taking  $\{1, 2, \dots, p\}$  to  $\{1, 2, \dots, r\}$  such that  $\sum_{n \in F} ax_n \in \mathbb{A}_{f(a)}$  whenever  $a \leq p$  and  $F \in [\mathbb{N}]^{c \times \delta_0}$  (and  $F \neq \emptyset$ ). Furthermore, if r+1 is prime, the function f can be forced to be one-to-one.

Theorem 1 raises the following natural question, asked for the case p = 2 and r = 3 by P. Erdős (in a personal communication). Is it true that if  $r, p \in \mathbb{N}$  (with r > p) and  $\mathbb{N} = \bigcup_{i=1}^{r} \mathbb{A}_{i}$ , then there exist  $\leq x_{n} \sum_{n=1}^{\infty}$  and  $\{i_{s}\}_{s=1}^{p}$  such that, whenever  $t \in \mathbb{N}$  and

# $\{a_k\}_{k=1}^t \subseteq \{0,1,2,\ldots,p\}$ one has $\sum_{k=1}^t a_k x_k \in \bigcup_{s=1}^p A_i$ ? <u>Theorem</u> 2. The answer to the above question is "no" for every $p \ge 2$ and every r > p. (Received February 28, 1975.)

*75T-A122 ALLAN B. CRUSE, University of San Francisco, San Francisco, California 92117. <u>A note</u> on 1-factors in certain regular multigraphs. Preliminary report.

Let G = (V, E) be a regular graph of degree d on n = 2m vertices, and let  $A = (a_{ij})$  denote the  $n \times n$  adjacency matrix for G. For any subset S of the vertex-set V, let z(S) denote the number of edges in G having an end in S and an end in V - S. Theorem. Either there is some subset S of V having odd cardinality such that z(S) < d, or else any edge in G belongs to some 1-factor of G. Since  $z(S) \equiv d \pmod{2}$  when S is odd, this theorem strengthens and generalizes a well-known result of Petersen (Acta Math. 15(1891), 193-220) which asserts that every bridgeless cubic graph is the sum of a 1-factor and a 2-factor. The Theorem extends also to multigraphs; its proof consists in showing that, unless z(S) < d holds for some odd S, the symmetric doubly-stochastic matrix (1/d)A will be equal to a convex combination of symmetric permutation matrices (see A. Cruse, Abstract 711-05-27, these cNotica) 21(1974), A-33-A-34). Corollary. If d exceeds 2[[(m-1)/2], then each edge in G belongs to some 1-factor. Examples show this bound is best possible. (Received March 3, 1975.)

75T-A123 PETER J: SLATER, Applied Mathematics Division-205.02, National Bureau of Standards, Washington, D.C. 20234. <u>Trees with disjoint maximum matchings</u>. Preliminary report.

Let v and w be specified vertices in graphs F and G, respectively. Then  $H=F(v) \bigoplus G(W)$  will denote the graph obtained from F and G by identifying vertices v of F and w of G. Graph H will be said to be obtained by surgery on F and G. A matching of a graph H is a collection of edges, no two of which are incident with the same vertex.

A constructive characterization of the set  $S_k$  (k>2) of trees (connected, acylcic graphs) which have at least k disjoint maximum matchings is possible. For each k>2 there are three types of surgery such that T is in  $S_k$  if and only if T can be obtained from a star  $K_{1,n}$  (n>k) by a finite sequence of the specified surgical operations. (Received March 6, 1975.)

*75T-A124 JOSEPH J. HEED, Norwich University, Northfield, Vermont 05663. Entry Points of Recursive Sequences.

Let  $F^n(c,q) = \{(c+\sqrt{q})^n - (c-\sqrt{q})^n\}/2\sqrt{q}$ ,  $c^2 \neq q \pmod{4}$ ;  $= \{[(c+\sqrt{q})/2]^n - [(c-\sqrt{q})/\sqrt{2}]^n\}/\sqrt{q}$ ,  $c^2 \equiv q \pmod{4}$ , be the general term of a recursive sequence. Define the entry point of  $\{F(c,q)\} \pmod{p}$ , p, prime, as the smallest exponent, d, such that  $F^d(c,q) \equiv 0 \pmod{p}$ , and say F(c,q) belongs to d modulo p. For q a quadratic residue (nonresidue) of p, there are  $\phi(d_i)$  incongruent values, c, such that F(c,q) belongs to  $d_i$  modulo p, where  $d_i|p-1$ ,  $(d_i|p+1), d_i \neq 1$ . With c fixed,  $c \neq 0 \pmod{p}$ , for each divisor of p-1 and p+1, except 1 and 2, there are  $\phi(d_i)/2$  incongruent values of q such that F(c,q) belongs to  $d_i$  modulo p.  $\{F(1,5)\}$  is the Fibonacci sequence. Based on the above results and probabilistic arguments the number of maximal entry points are predicted. For p < 3000, the predicted value for p = 10n±1 is 74.25 versus 76 observed, and for p = 10n±3, 87.78 versus 88. (Received March 7, 1975.) (Author introduced by Ernest D. True.)

*75T-A125 D. SURYANARAYANA, University of Toledo, Toledo, Ohio 43606 and R. SITA RAMA CHANDRA RAO, Andhra University, Waltair, India. <u>Two arithmetical functions</u> associated with k-free and k-full integers.

Let k be a fixed integer  $\geq 2$ . Let  $Q_k$  and  $L_k$  denote respectively the sets of k-free and k-full integers. By a k-free integer, we mean (as usual), a positive integer all of whose prime factors occur with multiplicity at most k-l and by a k-full integer, we mean, a positive integer all of whose prime factors occur with multiplicity at least k. The integer l is considered to be both k-free and k-full. We define two arithmetical functions  $\alpha(k)$  and  $\beta(k)$  respectively as the greatest integers such that  $k^{\alpha(k)} \in Q_k$ ,  $k^{\alpha(k)+l} \notin Q_k$ 

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and  $k^{\beta}(k) \notin L_k$ ,  $k^{\beta(k)+1} \in L_k$ . In this paper we establish some results about the orders of magnitude of  $\alpha(k)$  and  $\beta(k)$  and asymptotic formulae for  $\sum_{k=2}^{n} \alpha(k)$  and  $\sum_{k=2}^{n} \beta(k)$ .

(Received March 10, 1975.)

*75T-A126 D. D. ANDERSON, University of Iowa, Iowa City, Iowa 52242 Minimally generated prime ideals, Preliminary report.

Let R be a local ring and  $P = (a_1, \ldots, a_n)$  a prime ideal of height n. It is shown

that a1,..., must be an R-sequence. (Received April 21, 1975.)

*75T-A127 ROBERT L. McFARLAND, University of Glasgow, Glasgow G12 8QW, Scotland and BART F. RICE, Dept. of Defense, Ft. George G. Meade, Maryland 20755, <u>Translates and multipliers of abelian difference sets</u>.

Let D be a difference set in a finite abelian group of order v. We prove:

THEOREM 1. D has a translate which is fixed by all of its numerical multipliers.

THEOREM 2. If D has numerical multipliers  $t_1, \ldots, t_m$  such that

 $gcd(t_1-1, \ldots, t_m-1, v) = 1$ , then D has a unique translate which is fixed by all

numerical multipliers and this translate is also fixed by all nonnumerical multipliers.

(Received March 11, 1975.)

75T-A128 DENNIS P. GEOFFROY, University of South Carolina, Columbia, South Carolina 29208. On 1-factors of point determining graphs. Preliminary report.

A graph G is called point determining if distinct points have distinct neighborhoods. The nucleus G⁰ of a point determining graph G is the set of all points p of G such that G-p is a point determining graph. Summer [Journal of Combinatorial Theory 16(1974), 35-41] has shown that if G is a point

Summer [Journal of Combinatorial Theory 16(19/4), 35-41] has shown that if G is a point determining graph and G^o is  $K_{2n}$ ,  $K_2$   $\mathring{U}$   $K_{2n+1}$  minus an edge, or  $K_{1,3}$ , then G has a 1-factor. We have extended the first two of these results to the following:

Theorem. Let G be a point determining graph such that G° is point determining and  $(G^{\circ})^{\circ}=G^{\circ}$ . If G° has a l-factor, then G has a l-factor.

In particular, if G^o is  $\rm K_{2n},$  a finite disjoint unoin of  $\rm K_2$  's, or  $\rm C_{2n},$  then G has a 1-factor . (Received March 13, 1975.)

*75T-A129 ARTHUR M. HOBBS, Department of Mathematics, Texas A&M University, College Station, Texas 77843. <u>Cycle-Minimal Graphs and Vertices of Low Degree</u>.

A graph G is <u>n-cycle-minimal</u> for a positive integer n if G is n-connected and each cycle  $C_i$  of G includes a set  $S_i$  of edges such that  $G - S_i$  is not n-connected. A graph is <u>cycle-minimal</u> if every component of the graph is 1-cyle-minimal. We show that every cycle-minimal graph with at least two vertices has at least two vertices of degree two or less. Thus, every cycle-minimal graph is 3-colorable, has thickness at most two, and has no more edges than three less than twice its number of vertices. We show that every two-connected Eulerian graph G which is not a cycle includes either a cycle C such that G - C is two-connected or includes a subgraph H which meets G - H in exactly two vertices such that every block of H is a cycle and G - H is two-connected. Fianlly, we show that every 2-cycle-minimal Eulerian graph has at least two vertices of degree two. (Received March 26, 1975.)

75T-A130 K. P. SATAGOPAN, Presidency College, Madras-5, India. On the isomorphism of the endomorphism rings of modules, Preliminary report.

In this paper we investigate classes of unital right R-modules for which every R-isomorphism of the endomorphism rings of two R-modules in the class implies the existence of an isomorphism between the modules themselves. It is shown that the following classes satisfy this property: (1) Injective modules over a commutative Noetherian ring. (2) Finite direct sums of a selected class of indecomposable injective modules over an arbitrary commutative ring. (3) Arbitrary direct sums of modules in (2) provided the endomorphism rings are topologically R-isomorphic. (4) Finite direct sums of modules over an arbitrary ring R, each an injective hull of R/P where P is a centrally generated irreducible prime ideal. (Received March 17, 1975.) (Author introduced by Dr. Laszlo Fuchs.)

*75T-A131 ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. <u>Direct</u> Product Decomposition of Alternative Rings without Nilpotent elements.

Let A be an alternative ring without nilpotent elements. Theorem.  $(A, \leq)$ is a partially ordered set where  $x \leq y$  iff  $xy = x^2$ . Also,  $(A, \leq)$  is infinitely distributive, i.e.,  $r(\sup x_i) = \sup rx_i$  and  $(\sup x_i)r = \sup x_ir$ . Definition. A nonzero element a of A is an atom iff a belongs to a unique multiplicative system of A maximal with respect to not containing 0 as an element. Moreover, A is atomic iff for every nonzero element x of A there is an atom a with a  $\leq x$ . Furthermore, A is <u>orthogonally</u> complete iff sup S exists for every subset S of A where the product of every two distinct elements of S is zero. Theorem. A is isomorphic to a direct product of alternative rings without zero divisors iff A is atomic and orthogonally complete. Remark. The results are proved for rings more general than alternative rings (e.g., for rings A such that A is not necessarily associative or commutative, A has no nilpotent elements and if a product of elements of A is equal to zero then it remains equal to zero no matter how its factors are associated). (Received April 21, 1975.)

75T-A132 NAN-HUNG CHEN, Rutgers University, New Brunswick, New Jersey 08903. Global Dimension Under Change of Ring, Preliminary report.

<u>Theorem</u>. Let  $f: R \rightarrow S$  be a ring homomorphism such that  $0 \rightarrow R_R \rightarrow S_R$ is pure exact. If  $_RM$  is flat and  $S \bigotimes M$  is S-projective then  $_RM$  is projective. The proof is based on the work of L.Gruson and M.Raynaud, (Invent. Math. 13 (1971), 1-89) in which the commutative case is considered. Some applications are: i). If R is a Von-Neumann regular ring and S is any ring containing R then 1. (rt.) gld  $R \leq 1.$  (rt.) gld S. ii). Let  $\sigma$  be any endomorphism of R, d be a  $\sigma$ -derivation and  $S = R[x; \sigma, d]$  or  $S = R[[x; \sigma]]$  then 1. gld  $R \leq w.$  gld R+1. gld S. Whether or not  $\sigma$  is 1-1 is irrelevant in this relation and this result extends to more than one variable. iii). Let  $0 \rightarrow R \rightarrow S$ and S be semi-simple Artinian. If the sequence is pure exact on either side, then w. gld R = gld R on that side. (Received March 20, 1975.)

*75T-A133 G.E. SOBCZYK, 109 Hillcrest Avenue, Clemson, South Carolina 29631 and Polish Academy of Sciences, Warsaw, Poland. <u>Geometric structures in a certain Banach algebra</u>.

Let  $\{\pounds(\mathcal{G}), \circ\}$  be the algebra of bounded linear operators on a Banach algebra  $\{\mathcal{G}, *\}$ . By a biderivation in  $\mathcal{B}$  we mean a transformation  $\chi: \pounds(\mathcal{G}) \to \pounds(\mathcal{G})$  which satisfies for all  $F, G \in \pounds(\mathcal{G})$  and all  $A, B \in \mathcal{B}$ : 1.  $\chi[F + G](A) = \chi[F](A) + \chi[G](A)$  2.  $\chi[F \circ G](A) = \chi[F] \circ G(A) + F \circ \chi[G](A)$  3.  $\chi[F(A) + G(B)] = \chi[F](A) + \chi[G](B)$  4.  $\chi[F(A)*G(B)] = \chi[F](A)*G(B) + F(A)*\chi[G](B)$ . This work is primarily a study of biderivations called geometric structures in a certain real Banach algebra called geometric algebra. Based on this new concept, new characterizations are given of such concepts from differential geometry as integrability, Riemannian curvature, fields, and the Lie bracket of fields. In addition, cogent new formulations of the Weyl projective and conformal tensors are given which have rich geometric significance. The introductory part of this work develops geometric algebra (which is an infinite dimensional Clifford algebra) into a powerful new tool for studying multilinear algebra. Examples of the utility of these methods

are provided, including a direct algebraic way of inverting a nonsingular linear operator and a simple new proof of the Cayley-Hamilton theorem. (Received March 24, 1975.)

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134 Dr.SURJEET SINGH and Mr.ASRAR MOHAMMAD, Aligarh Muslim University, Aligarh, India. Rings in which every finitely generated left ideal is quasi-projective.

The following are the main results : Let R be a semiperfect ring and let N, B denote its Jacobson and prime radicals respectively. (1) If R is a local ring (that is, R/N is a division ring) such that every finitely generated left ideal of R is quasi-projective, then : (i) B is the set of all nilpontent elements of R, and (ii) either  $B^2 = (0)$  or R is a left valuation ring. (2) Let R be a local ring with N nil. Then every finitely generated left ideal of R is quasi-projective if and only if either  $N^2 = (0)$  or R is a left valuation ring. (3) Let  $R = Re_1 \oplus \dots \oplus Re_n$ , where  $e_i$  are primitive orthogonal idempotents, and let N be nil. If every finitely generated left ideal of R is quasi-projective, then : (i) All finitely generated left ideals of  $e_i Re_i$  are quasi-projective; (ii) If  $(e_i Ne_i)^2 \neq i Ne_i$ , whenever  $e_i Ne_i \neq (0)$  for all i, then R is a semiprimary ring; if in addition each Ne_i is quasi-projective then N-T  $\oplus A$  where  $T = \oplus \sum_i Re_i Ne_i$ , A is some left ideal, and R/T is left semihereditary (This T is (0) if R is left semihereditary). (Received March 20, 1975.) *75T-A135 IRVING REINER, University of Illinois, Urbana, Illinois 61801 Locally free class groups. Preliminary report.

Let  $\Lambda,\Gamma$  denote R-orders in separable K-algebras A,B, respectively, where R is a Dedekind domain whose quotient field K is an algebraic number field. Let  $\wedge$ ' be a maximal R-order in A containing  $\wedge$ . Denote by Cl  $\wedge$  the class group of locally free left ideals in  $\wedge$ . The "change of rings" map gives a surjection Cl  $\wedge \longrightarrow$  Cl  $\wedge$ ', whose kernel D( $\wedge$ ) is independent of the choice of  $\wedge$ '. It is shown here that D is "functorial", that is, every homomorphism  $\wedge \longrightarrow \Gamma$  of R-orders induces an additive map D( $\wedge$ )  $\longrightarrow$  D( $\Gamma$ ). This result is used to find lower bounds for the order of D(ZG), where G = symmetric group S_n or alternating group A_n. In particular, D(ZS_n)  $\neq$  0 for  $n \geq 6$ , and D(ZA_n)  $\neq$  0 for n = p or p + 1, with p any odd prime  $\geq 7$ . (Received March 21, 1975.) *75T-A136 LOUIS ROWEN. University of Chicago. Chicago. Ill., 60637.

LI36 LOUIS ROWEN, University of Chicago, Chicago, Ill., 60637. <u>Generalized Polynomial Identities III</u>.

An <u>involution</u> is an anti-automorphism of degree  $\leq 2$ . Let (R,*) denote an (associative) ring R with involution (*); let C = Cent(R). In the category of rings with involution, a <u>generalized polynomial</u>  $f(X_1, X_1^*, \dots, X_m, X_m^*)$  is a sum of terms  $r_1Y_1 \cdots r_tY_tr_{t+1}$ ,  $r_i$  in R,  $Y_i$  in  $\{X_1, X_1^*, \dots, X_m, X_m^*\}$ ; if all  $r_i=1$ , f is called a <u>polynomial</u>. Call f a GI (generalized identity) of R if  $f(r_1, r_1^*, \dots, r_m^*, r_m) = 0$  for all  $r_1, \dots, r_m$  in R. An <u>ideal</u> of (R,*) is an ideal of R which is invariant under (*). (R,*) is <u>prime</u> if  $AB \neq 0$  for all ideals A, B of (R,*). For details and some structural results, see "Generalized Polynomial Identities, I and II," to appear in J. Algebra. The study of structure theory is continued here, with the following applications: <u>Theorem 1</u>. If (R,*) is prime and  $[r,f(X_1, X_1^*, \dots, X_m, X_m^*)]$  is a GI of (R,*) for some nontrivial polynomial f of degree d>0, then either  $r \in C$  or R is a PI-ring of degree  $\leq \max(2,d)$ . Corollary (Herstein): If R is simple and has an involution, and if r commutes with all symmetrics, then either  $r \in C$  or R is a quaternion algebra. Theorem 2. Suppose (R,*) is prime, f is a homogeneous generalized polynomial, and g is a polynomial, such that [f,g] is a GI of (R,*). If degf>0, then either f is a GI of R, or R is a PI-ring. (Received March 27, 1975.)

75T-A137 PAMELA A. FERGUSON, University of Miami, Coral Gables, Florida 33124, <u>A</u> Classification Theorem for COO Groups, Preliminary Report.

A group G is a COO group if a Sylow 3 subgroup M of G contains the centralizer of each of its non-identity elements. Theorem. Let G be a simple COO group with Sylow 3 subgroup M such that  $N_{G}(M)=QM$  where  $\overline{Q}$  is cyclic,  $1Q1 \neq 1$ , 1M1 - 1, and M is not cyclic; then G is isomorphic to a simple group PSL  $(2,3^{n})$ ,  $n \geq 2$ . The work of Herzog, Fletcher, and the author have already proved several cases of this theorem. (Received March 28, 1975.)

*75T-A138 G.J.Rieger, Techn.University, D-3Hannover. On a theorem of Heilbronn concerning continued fractions

Let a,n be natural numbers with (a,n) = 1 and  $1 \le a \le n$ . Denote by E(a,n) the number of terms in the ordinary continued fraction for a/n (see Perron, Kettenbrüche, § 9) and by N(a,n) the number of terms in the continued fraction by nearest integers for a/n (see Perron, § 39). According to Vahlen (1895), we have N(a,n)  $\le E(a,n)$  (see Perron, § 39). Denote by  $\varphi$ the Euler function and let  $\sigma_{-1}(n) := \sum_{\substack{d \ d \ d \ n}} d^{-1}$ . An interesting result of Heilbronn (Abhandlungen aus Zahlentheorie und Analysis zur Erinnerung an Edmund Landau; ed. Paul Turán, Berlin 1968) states  $\sum_{\substack{1 \le a \le n, (a,n) = 1}} E(a,n) =$  $1 \le \pi^{-2} (\log 2) \varphi(n) \log n + O(n(\sigma_{-1}(n)^3))$ . By a similar method we prove  $\sum_{\substack{n \le a \le n, (a,n) = 1}} N(a,n) = 12 \pi^{-2} (\log (1 + \sqrt{5})/2)\varphi(n) \log n + O(n(\sigma_{-1}(n))^3)$ . It is remarkable that the number  $12 \pi^{-2} \log 2$  appears also in the work of P. Lévy on continued fractions, (Received March 31, 1975.)

*75T-A139 S. BULMAN-FLEMING, Univ. of Waterloo, Ontario N2L 3G1, H. WERNER, FBY-AG1 Techn. Hochschule, D-61 Darmstadt, Germany. <u>Equational compactness in quasi-primal</u> varieties.

Recently R. BEAZER characterized the equationally compact Post-algebras and łukasiewicz-algebras of order 3. Using W. TAYLOR'S result that in a quasiprimal variety every algebra has a pure embedding into a product of finite algebras we can extend BEAZER'S result to those varieties.

<u>THEOREM</u>: In a variety generated by a weakly independent set  ${f n}$  of quasi-primal

algebras the equationally compact algebras are the boolean extensions of sub-

algebras of members of  ${\cal R}$  by complete boolean algebras and finite products of

those. (Received April 1, 1975.)

*75T-A140 RICHARD MOLLIN, Math. Dept., Queen's Univ., Kingston, Ont., K7L 3N6 Algebras with Uniformly Distributed Invariants

Terminology in the following discussion can be found in (T. Yamada, The Schur subgroup of the Brauer group, Lecture notes in mathematics, No. 397, Springer-Verlag (1974)).

Let K/Q be finite abelian. We define U(K) to be the subgroup of the Brauer group B(K) consisting of those classes which contain an algebra with uniformly distributed invariants.

Theorem 1. If K/Q is finite abelian, p is an odd prime and [A] in U(K) has ind p(A) = m then  $p \equiv 1 \pmod{m}$ . If p = 2 then ind p(A) = 1 or 2.

This generalizes a result by Witt (J. reine angew Math. <u>190</u> (1952), Satz 10 and 11, p.243).

We also prove that the exponent of U(K) equals the order of the group of roots of unity in K . We obtain general properties of U(K) from which new properties of the Schur group S(K) follow, and obtain simpler proofs for some known results for S(K) . (Received April 1, 1975.)

*75T-A141 RICHARD T. BUMBY, Rutgers University, New Brunswick, NJ 08903 Dimensions of sets of continued fractions

A program has been written (in ALGOLW) which analyzes the Cantor set construction of sets of numbers whose continued fractions contain only certain strings of 1's and 2's and uses this information to compute bounds on the Hausdorff dimension of such sets. A portion of the Markoff spectrum of measure zero can be found by producing a corresponding set of continued fractions of dimension less than .5. Although these properties need not be equivalent, all current methods for producing portions of the spectrum of measure zero also establish the corresponding dimension result. The application to the Markoff spectrum gives measure zero below min(2122111.2.122111) = 3.3343 with dimension less than .49997 while the method fails for max(2122111.2.122111) = 3.3344 with a dimension greater than .5001.

In the course of determining the relative extreme values in this portion of the spectrum, several values were found which were not expressible in terms of a single quadratic surd. For example, max(21111.2.12112) involves  $\sqrt{20889}$  and  $\sqrt{1629575423}$ . (Received April 4, 1975.)

75T-A142 JOHN DAVID, University of Campinas, CP1170, 13100 Campinas, SP, BRAZIL. <u>Inert and</u> Strongly <u>Inert Integral Domains</u>. Preliminary report

Definition: Let R^CA be integral domains, such that every solution in AxA of an equation of form  $X_1X_2+r=0$ , ref, is in RxAUAxR. R is called inert in A. Theorem1: Let R^CA be domains, R inert in A, also R retract of A with principle kernel generated by teA and A/R transcendence one. Then R[t]= A^AK(t), K= quotient field of R, and t is transcendental over R. There is a counterexample to the stronger conclusion, R[t]= A; C[x + y + x] in C[x,y]. Definition: Let R^CA be integral domains such that every solution in AxA of an equation of form  $r_{pq} X_{pq}^{A} Y_{pr}$ ;  $x^{A}X_{q}^{J}=0$ ,  $0 \neq r$ ,  $r \in \mathbb{R}$ , is in AxRU RxA. R is called strongly inert in A. Theorem 2:  $pq^{1/2} Y_{qr}$ ;  $x^{A}X_{q}^{J}=0$ ,  $0 \neq r$ ,  $r \in \mathbb{R}$ , is in AxRU RxA. R is called strongly inert in A. Theorem 2:  $pq^{1/2} Y_{qr}$ ;  $x^{A}X_{q}^{J}=0$ ,  $0 \neq r$ ,  $r \in \mathbb{R}$ , is in AxRU RxA. R is called strongly inert in Q get the stronger result that R[t]=A.Theorem 3: The following are all true and no converse: A ring of polynomials over R implies R strongly inert in A implies R relatively algebraically closed in A, for R^CA, domains. Questions: Let K be a field, K<R strongly inert in K[x,y,z]=A. Is R a retract of A? Let R be a finite domain, R strongly inert in A, an arbitrary domain. Is R[X] strongly inert in A[X]? (Received April 7, 1975.)

*75T-A143 Thomas H. Pate, Emory University, Atlanta, Ga. 30322. Lower Bounds for the Norm of the Symmetric Product. Preliminary report.

Suppose each of m, n, and k is a positive integer,  $k \ge n$ , A is a (real valued) symmetric n-linear function on Em, and B is a k-linear symmetric function on Em. The tensor and symmetric products of A and B are denoted, respectively, by A  $\otimes$  B and A·B. The identity  $\| A \cdot B \|^2 = \sum_{\substack{n \\ q=0}}^{n} {\binom{n}{q} \binom{n}{q}} \| A \otimes B \|^2$  was proven by Neuberger (Norm of symmetric product compared with norm of tensor product, J. Linear and Multilinear Algebra 2 (1974), pp. 115-121). An immediate consequence of this identity is the inequality  $\| A \cdot B \|^2 \ge {\binom{n+k}{n}}^{-1} \| A \otimes B \|^2$ . In this paper a necessary and sufficient condition for  $\| A \cdot B \|^2 = {\binom{n+k}{n}}^{-1} \| A \otimes B \|^2$  is given. It is also shown that under certain conditions the inequality can be considerably improved. This improvement results from an analysis of the terms  $\| A \otimes_q B \|$ ,  $1 \le q \le n$ , appearing in the identity. (Received April 7, 1975.)

of Equationally Compact Semilattices, Preliminary Report.

A bounded A-semilattice S is called <u>infinitely n-distributive</u>  $(n \ge 2)$ if it is complete and satisfies  $a \land VX = V\{a \land (x_1 \lor ... \lor x_{n-1}): x_1, ..., x_{n-1} \in X\}$  for all  $a \in S$ ,  $X \subseteq S$ . S has the <u>n-solvability property</u> (nsp) if every set of semilattice equations in the single variable  $v_0$  with constants in S, every ( $\le n$ )-element subset of which is solvable in S, is itself solvable in S.

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<u>Theorem</u>: S satisfies (nsp) iff S is infinitely n-distributive. Note that for n = 2 this describes the injective semilattices, and that every semilattice satisfying some (nsp) is equationally compact. (Received April 7, 1975.)

*75T-A145 GEORGE HUTCHINSON, National Institutes of Health, Bethesda, Maryland 20014. Embedding and unsolvability theorems for modular lattices.

Let R be a nontrivial ring with 1 and  $\delta$  a cardinal. Let  $L(R,\delta)$  denote the lattice of submodules of a free unitary R-module on  $\delta$  generators. Let M be the variety of modular lattices. A lattice is R-representable if embeddable in the lattice of submodules of some R-module; L(R) denotes the quasivariety of all R-representable lattices. Let  $\omega$  denote aleph-null, and let an (m,n) presentation have m generators and n relations, m,n  $\leq \omega$ . Theorem. There exists a (5,1) modular lattice presentation having a recursively unsolvable word problem for any quasivariety V, V  $\subset$  M, such that L(R, $\omega$ ) is in V. Theorem. If L is a denumerable sublattice of L(R, $\delta$ ), then it is embeddable in some sublattice K of L(R, $\delta^*$ ) having 5 generators, where  $\delta^* = \delta$  for infinite  $\delta$ , and  $\delta^* = 4\delta$ (m+1) if  $\delta$  is finite and L has a set of m generators. Theorem. The free L(R)-lattice on  $\omega$  generators is embeddable in the free L(R)-lattice on five generators. Theorem. If L has an (m,n) L(R)-presentation for denumerable m and finite n, then L is embeddable in some K having a (5,1) L(R)-presentation. (Received April 4, 1975.)

#### 75T-Al46 DAVID ZEITLIN,1650 Vincent Ave. North, Minneapolis, MN.,55411. Identities for integer sequences involving the greatest integer function. X. Preliminary report.

$$\begin{split} & \text{M}, \text{Q} = 1, 2, \dots; \ \text{W}_{0}, \text{W}_{1} \text{ are integers; } \ \text{W}_{n+2} = \text{MW}_{n+1} + \text{SQW}_{n} \ , \ \text{S} = \pm 1. \ \text{If } (\text{W}_{0}, \text{W}_{1}) = (0, 1), (2, \text{M}), \text{then} \\ & \text{W}_{n} \equiv \text{U}_{n}, \ \text{W}_{n} \equiv \text{V}_{n}. \ \text{Set } A(x) = \left| x \text{W}_{0} - \text{W}_{1} \right| \ , \ \text{D}(\text{J}) = (1 + \text{V}_{r+p}) \text{U}_{k+j} \ , \ \text{F} = \text{V}_{r+p} / (1 + \text{V}_{r+p}), \ \text{T}^{2} = \text{H}^{2} + 4\text{SQ} \ , \\ & \text{T}^{2}\text{Z}_{1} = \text{W}_{1}^{2} - \text{MW}_{0} \text{W}_{1} - \text{SQW}_{0}^{2}, \ \text{and} \left[ y \right] \ \text{the g.i.f.} \ \underline{\text{Case}} \ 1. \ \text{S} = 1, \ \text{M} \ge 1, \ \text{with} \ 1 \leq \text{Q} < \text{M+1}; \ \text{P} > 0 \ \text{is a root} \\ & \text{of } x^{2} = \text{Mx} + \text{Q}. \underline{\text{Case}} \ 2. \ \text{S} = -1, \ \text{M} \ge 3, \ \text{with} \ 1 \leq \text{Q} < \text{M-1}; \ \text{P} > 1 \ \text{is a root of } x^{2} = \text{Mx} - \text{Q}. \ \underline{\text{Theorem 80}} \ \text{for} \\ & \text{both } \text{cases} \text{).Let } r, p = 1, 2, \dots \text{.Let } J_{k}(n) = \text{PU}_{nk} + \text{SQU}_{nk-1} \cdot \text{If } A^{2}(\text{P}) < (\text{TP}^{(r+p)k}) / (D(((r+p-1)k)\text{Q}^{(r+p)k}), y) \\ & \text{M}_{k} = \frac{1}{2} + \frac{1}{2}$$

 $\begin{bmatrix} P^{(r+p)k} & W_{rn} & W_{pn} + Z_1 (-SQ)^{np} (J_k(r+p)V_{(r-p)n} - (-SQ)^{kp} V_{(r-p)(n+k)}) + F \end{bmatrix} = W_{r(n+k)} & W_{p(n+k)} \text{ for } r \ge p$ and  $n \ge k \ge 0.\underline{Kx}$ . 1.<u>Fibonacci seq</u>., with S=M=Q=1,  $U_n \equiv F_n, V_n \equiv L_n$ , and  $2P = 1 + \sqrt{5}$ . Let  $C = 5^{-\frac{1}{2}}$ , (r,p,k)=(2,1,1). For  $n \ge 1$ ,  $\begin{bmatrix} P^3F_{2n+2}F_{n+2} + (-1)^n(F_{n+1} + CL_n) + .8 \end{bmatrix} = F_{2n+4}F_{n+3}$ ,  $\begin{bmatrix} P^3L_{2n+2}L_{n+2} - 5(-1)^{n}(F_{n+1} + CL_n) + .8 \end{bmatrix} = L_{2n+4}L_{n+3}$ . <u>Ex</u>. 2.  $W_{n+2} = 4W_{n+1} - W_n$ ,  $W_0 = 1, W_1 = 44P = 2 + \sqrt{3}$ , (r,p,k)=(2,1,1). For  $n \ge 1$ ,  $\begin{bmatrix} P^3W_{2n}W_n + (1/12)((15P-4)V_n - V_{n+1}) + (52/53) \end{bmatrix} = W_{2n+2}W_{n+1}$ . <u>Remark</u>. For r=p=1, Theorem 8 gives Theorem 1; see IV, these NOTICES 21(1974), A-367. (Received April 11, 1975.)

*725-A147 J. M. Gandhi, Western Illinois University, Macomb, Illinois 61455 Fermat's Last Theorem II, A New Circulant Condition for the First Case.

In this paper we prove: Theorem 1. The equation  $x^p + y^p + z^p = 0$  with (x,y,z, p) = 1 and p an odd prime has no integral solution if:

$$G_{p-1} = (C_2, C_3, \dots, C_{p-1}, C_1) \ddagger 0 \pmod{p^3}$$
  
Where  $G_{p-1}$  is a circulant and  $C_1 = \frac{1}{p} [\binom{p-1}{1} + (-1)^{1+1}]$ . As  $G_4 = (1,1,0,1) = 3$ 

 $\ddagger 0 \pmod{5^3}$  and  $G_{10} = (4,11,19,23,19,11,4,1,0,1) = 89373 \ddagger 0 \pmod{11^3}$  and hence FLT is verified for the first case for primes p=5 and p=11. However this method is not helpful if p is a prime of the form 6m+1, since then  $G_{p-1} = 0$ . We predict that the results of this paper will lead to several interesting investigations. (Received April 14, 1975.)

# 75T-A148 RONSON J. WARNE, University of Alabama, Birmingham, Alabama 35294. Generalized orthodox bisimple semigroups of type $\omega$ .

Let S be a regular bisimple semigroup s.t. the union T of the maximal subgroup S is an  $\omega$ chain of completely simple semigroups ( $T_n: n \in N$ , the nonnegative integers). If, furthermore,  $\Xi a \in S$  and an inverse  $a^{-1}$  of a s.t.  $(a^{-1})^n a^n \in T_n \forall$  positive integers and  $aa^{-1} > a^{-1}a$ , we term S a generalized orthodox bisimple semigroup of type  $\omega$ . We characterize S in terms of  $(I, \circ)$ , an  $\omega$ -chain of left zero semigroups  $(I_n : n \in N); (J, *)$ , an upper partial  $\omega$ -chain of right groups  $(J_n : n \in N); a$  homomorphism  $(n, k) \rightarrow \alpha_{(n, k)}$  of C, the bicyclic semigroup, into I', a semigroup of partial endomorphisms of  $(I, \circ); a$  homomorphism  $(n, k) \rightarrow \alpha_{(n, k)}$  of C into J'; an antihomomorphism  $j \rightarrow A_j$  of (J, *) into  $T_I$ , the full transformation semigroup on I; a homomorphism  $i \rightarrow B_i$  of  $(I, \circ)$  into  $T_J$ . In fact,  $S \cong I \times J$  under the product: if  $i \in I_n$ ,  $j \in J_k$ ,  $u \in I_r$ ,  $v \in J_s$ ,  $(i, j)(u, v) = (i \circ (uA_j \alpha_{(k, n)}))$ ,  $jB_u B_{(r, s)}^* v$ ). As a special case, we obtain a characterization of orthodox bisimple semigroups S of type  $\omega(E(S)$  is an  $\omega$ -chain of rectangular bands). (Received April 14, 1975.)

*75T-A149 F. RUDOLF BEYL, Math.Institut, 69 Heidelberg 1, Im Neuenheimer Feld 288, W.Germany. Abelian groups with a vanishing homology group. Preliminary report.

A group A is called *absolutely abelian*, if in every central extension  $N \rightarrow G \rightarrow A$  the group G is abelian. Let  $H_nG$  be the *n*-th homology group of the group G in integral coefficients. THEOREM 1. An abelian group A is absolutely abelian, if, and only if, the Schur multiplicator

H_A vanishes.

THEOREM 2. The abelian group A satisfies  $H_{2k}A = 0$  for some k > 0 exactly, if (*i*) A/Tor A is torsionfree of rank less than 2k, (*ii*) for each prime p, the reduced part of the p-torsion subgroup  $T_p$  of A is (at most) cyclic, and (*iii*) for each prime p,  $T_p$  vanishes unless A/Tor A is p-divisible.

The case of  $H_{2k+1}A = 0$  has been solved by J.A.Schafer [*Canad. J. Math.*21 (1969), 406-409]. Theorem 2 follows from a result on the Lyndon-Hochschild-Serre spectral sequence of a pure abelian extension. In this case the differentials  $d^{p}$  vanish for all  $r \leq 2$ , and the filtering exact sequences for  $H_{n}A$  are pure. (Received April 14, 1975.)

#### *75T-A150 WALTER TAYLOR, University of New South Wales, Kensington, N.S.W. 2033, Australia. <u>Varieties obeying group laws in homotopy</u>.

Continuing Abstracts 74T-A224 and 717-A19. Say that a variety V obeys a group law  $\lambda$  in n-homotopy iff  $\pi_n(A, a)$  obeys  $\lambda$  for any topological algebra A in V and any  $a \in A$ . A groupoid in V is a set G given both the structure of a V-algebra and an (Ehresmann) groupoid, with the V-operations of G commuting with the groupoid (partial) operations of G. Say that G obeys  $\lambda$  iff all the automorphism groups of objects of G obey  $\lambda$ , i.e.  $\lambda$  holds wherever defined. Theorem 8. V obeys  $\lambda$  in 1-homotopy iff all groupoids in V obey  $\lambda$ . Theorem 9. For  $n \ge 2$ , V obeys  $\lambda$  in n-homotopy iff all commutative groupoids in V obey  $\lambda$ . Corollary. For  $n \ge 1$ , the class of varieties obeying  $\lambda$  in n-homotopy is Malcev-definable. Theorem 10. If V obeys  $\lambda$  in homotopy, then the idempotent reduct of some variety equivalent to V also obeys  $\lambda$  in homotopy. [Likewise n-homotopy.] (Received April 14, 1975.)

#### 75T-A151 HEPBERT S. GASKILL, Memorial University of Newfoundland, St. John's, Nfld., A Note on Finite Sublattices of a Free Lattice.

A finite sublattice of a free lattice is said to be generated by a distributive lattice  $\mathcal{L}^*$ , provided that (i)  $\mathcal{L}^*$  is weakly representable (see 74T-Al4) and (ii)  $\mathcal{L}$  is isomorphic to the sublattice of  $FL(J(\mathcal{L}^*))$  generated by  $\mathcal{L}^*$ . Theorem. A finite sublattice,  $\mathcal{L}$ , of a free lattice is generated by a distributive lattice, if and only if  $\mathcal{L}$  is generated by its meet prime elements and also generated by its join prime elements. Various other results concerning the structure of finite sublattices of free lattices are proved. It is also shown that sublattices generated by distributive lattices are freely generated by their join or meet primes. (Received April 14, 1975.)

*75T-A152 ROBERT W. QUACKENBUSH, Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada, R3T 2N2. <u>An algebraic proof of a theorem of Chowla, Erdos and</u> Straus.

<u>Theorem:</u> For every positive integer k there are a positive integer  $m_k$ , a variety  $V_k$ , and binary polynomials  $p_1 \dots p_k$  of  $V_k$  such that: 1) for every integer  $m \ge m_k$  there is an  $\mathfrak{A}$  in  $V_k$  with  $|\mathfrak{A}| = m$ , and 2) for every non-trivial  $\mathfrak{A}$  in  $V_k$ ,  $p_1, \dots, p_k$  are mutually orthogonal latin squares on  $\mathfrak{A}$ . The proof uses R. M. Wilson's solution to the Extended Existence Conjecture for Partially Balanced Designs, a theorem of Trevor Evans characterizing polynomials which form m.o.l.s. in every algebra in a variety, and a construction of the author. (Received April 14, 1975.)

75T-A153 JOSEPH MALKEVITCH, City University of New York, York College, Jamaica, New York 11432. Almost pancyclic planar graphs. Preliminary report.

A connected graph G with v vertices will be called <u>almost pancyclic of order</u> m  $(3 \le m < v)$  if G has cycles of all lengths from 3 to v except for m. <u>Theorem</u>. There exists a planar 3-connected 3-valent almost pancyclic graph G of order m for every  $m \ge 3$ ; if m is even, G can be chosen to be a Halin graph. The last part of the theorem shows that a result of A. Bondy and L. Lovasz on cycle lengths in Halin graphs is best possible. (Received April 16, 1975.)

*75T-A154 SAID N. SIDKI, Universidade de Brasilia, 70.000 Brasilia, DF, Brazil. On HK ∩ KH in groups. Preliminary report.

<u>Theorem</u>. Let H,K be finite subgroups of a group G (not necessarily finite) with |H| = |K|. Suppose that  $HK \cap KH$  contains  $\{hf(h) | h \in H\}$  where  $f: H \to K$  is bijective with f(1) = 1. Then  $\langle H, K \rangle$  is finite. Examples of such  $\langle H, K \rangle$ 's are groups of type HK, VH where V is a normal elementary abelian 2-group of order  $2^{|H|-1}$ , PSL(2, pⁿ) where p is odd, some extension of PSL(2, 2ⁿ). (Received April 16, 1975.)

75T-A155 DEREK KONG CHANG, 4A Rua do Coronel Mesquita, 1/F. Macau

Distribution of prime numbers. Preliminary report.

Theorem. Let  $A=p_1^{w_1}p_2^{\omega_2}\cdots p_n^{\omega_n>6}$ , and  $\alpha_k \ge 1$  for all k less than m. Then there is no prime number in the intervals  $[A-p_m+1, A-2]$  and  $[A+2, A+p_m-1]$ . (Received April 16, 1975.)

*75T-A156 JOE W. FISHER, University of Texas, Austin, Texas 78712 and ROBERT L. SNIDER, Virginia Tech, Blacksburg, Virginia 24061. <u>Rings generated by their units</u>.

Let R be an associative ring with unity. In 1958, Skornyakov conjectured that all von Neumann regular rings in which 2 is a unit are generated by their units. In 1974, G. Bergman settled the conjecture in the negative. At about the same time, the authors obtained some preliminary results in proving the conjecture in the affirmative for a large class of regular rings (See Abstract 720-16-3, these <u>Notices</u> 22 (1975), A-84). The precise results of that investigation are announced here in ascending order of generality. <u>Theorem 1</u>. If R is a regular ring with primitive factor rings Artinian, then R is unit regular and hence, if 2 is a unit in R, then each element of R can be expressed as a sum of two units. <u>Theorem 2</u>. Let R be a  $\mathcal{M}$ -regular ring with primitive factor rings Artinian. If Z/2Z is not a homomorphic image of R, then each element of R can be expressed as a sum of two units. <u>Theorem 3</u>. Let R be a strongly  $\mathcal{M}$ -regular ring. If 2 is a unit in R, then each element of R can be expressed as a sum of two units. (Received April 17, 1975.)

*75T-A157 R. W. DEMING, State University of New York, College at Oswego, Oswego, New York 13126. A generalization of Dilworth's theorem to acyclic directed graphs. Preliminary report.

Let G be an arbitrary finite graph with no loops or multiple edges. Assume the edges of G are oriented by a function  $w \in \Omega$ , the set of all acyclic orientations of the edges of G. A <u>chain</u>  $\alpha$  in the directed graph  $\overline{G}$  is a sequence of vertices  $v_1, \ldots, v_{\ell}$  where  $(v_i, v_{i+1})$  is an edge in G and w orients  $v_i \rightarrow v_{i+1}$ . A <u>chain partition</u>  $\beta$  of  $\overline{G}$  is a partition of the vertex set of G into a family of disjoint chains. Let  $C_w$  and  $\chi_w$  be the families of <u>all</u> chains, and all chain partitions, respectively, of  $\overline{G}$ . The two principal results of this investigation are <u>Theorem</u> 1.  $\min_{w \in \Omega} \max_{\alpha \in C_w} |\alpha| = \text{chromatic number } k$ , of G. <u>Theorem</u> 2.  $\max_{w \in \Omega} \min_{\beta \in \mathcal{H}_w} |\beta| = \text{independence, or vertex packing number } i$ , of G. If  $k_w =$ 

 $\max_{\alpha \in C_{W}} |\alpha| \text{ and } i_{W} = \min_{\beta \in \mathcal{H}_{W}} |\beta|, \text{ then } k_{W} \text{ is an upper bound of } k \text{ and } i_{W} \text{ is a lower bound for } i \text{ for any acyclic orientation } w. \text{ In particular, if } w \text{ is a transitive orientation of the edges of } G (v_{i} \rightarrow v_{j}, v_{j} \rightarrow v_{k} \Rightarrow (v_{i}, v_{k}) \text{ is an edge in } G \text{ and } v_{i} \rightarrow v_{k}), \text{ then two vertices of an independent set cannot belong to the same chain } \alpha; \text{ hence } i \leq i_{W}. \text{ Dilworth's theorem follows from this inequality and Theorem 2. (Received April 18, 1975.) }$ 

75T-A158 J.L. BRENNER, 10 Phillips Road, Palo Alto, California. 94303 and JAMES WIEGOLD, University College, Cardiff, U.K. CF1 1XL.

#### Two-generator groups I.

In the alternating group  $A_n$ , every nontrivial element is a member of a generating pair. This answers (in part) a 1962 [Can. J. Math.] question of R. Steinberg. Defn:  $G \in \Gamma_r$  means that for every r nontrivial elements  $x_i \in G$ , there exists  $y \in G \ni \langle x_i, y \rangle = G$  for each i. Thl: If G is nonabelian,  $G \in \Gamma_1$ , then G is monolithic; G' is the monolith.

Th2:  $A_{2n} \in \Gamma_4 \setminus \Gamma_5$  (n > 3). Th3:  $A_{19} \in \Gamma_{t-1}$ , t =  $17!/3^4 6! = 6,098,892,800$ . Th4: If g  $\epsilon$  G = PSL(2,q),  $g^2 \neq 1$ , q > 3, then h exists,  $h^2 = 1 \cdot 3 < g, h > = G$ . Th5: Th4 is false if G = PSL(n,q), n > 2, (n,q)  $\neq$  (3,2), (3,4). Our proof of Th4 uses character sums (and an argument kindly supplied by L. Carlitz). Other proofs use geometric algebra, or theorems concerning permutation groups. This article will be published in Michigan Math. J. (Received April 18, 1975.)

75T-A159 J. M. Gandhi and Mark Stuff. Western Illinois University Macomb, Illinois 61455. On the First Case of Fermat's Last Theorem and the Congruence  $2^{p-1}\equiv 1 \pmod{p^3}$ .

It is well known that if (1)  $a^{p}+b^{p}=c^{p}$  with (abc,p)=1 has an integral solution, then according to Wieferich's Criterion  $2^{p-1}\equiv 1 \pmod{p^2}$ . This is the most important condition for the first case of Fermat's Last Theorem (FLT). Linkovski (Math. Nach. 38 (1968) 314) allegedly proves that if (1) has an integral solution for the first case with p>3 then the congruence  $2^{p-1}\equiv 1 \pmod{p^3}$  must be satisfied. He uses a result due to Grebenuk [Dokl. Akad. Nauk Uz. SSR 1956 No 8, pp 9-11] that every prime factor q of a+b-c,q/abc must divide  $2^{p-1}-1$ . We remark that Grebenuk's result does not appear to be correct. Fistly Grebenuk shows that any integer k satisfying  $a\equiv kb \pmod{q}$ , however  $a\equiv q-k-1 \pmod{q}$  and derives,  $k^{p}+1-(k+1)^{p}\equiv 0 \pmod{q}$ , however  $a\equiv q-k-1 \pmod{q}$  and derives,  $k^{p}+1-(k+1)^{p}\equiv 0 \pmod{q}$ , however  $a\equiv q-k-1 \pmod{q}$  and observes that  $f(\alpha) = f(-\alpha-1)$  and so (3)  $f(k)\equiv f(-k-1) \pmod{q}$ , but  $f(k)\equiv f(-k-1) \pmod{q}$  does not necessarily imply (4) k\equiv-k-1 \pmod{q} and hence his arguments are not correct and therefore Linkovski result does not follow. As a numerical counter-example if we consider k=3 and q=p=5, then both (1) and (3) are satisfied but (2) which he has assumed as well (4) are not satisfied. (Received April 14, 1975.)

75T-AL60 G. GRÄTZER, University of Manitoba, Winnipeg, Canada R3T 2N2.A Note on the Amalgamation Property.

Let  $\underline{K}$  be an equational class of algebras, A,  $B_1$ ,  $B_2 \in \underline{K}$  and let  $\varphi_i: A \rightarrow B_i$  be an embedding for i = 1, 2. We say that  $V = \langle A, B_1, B_2, \varphi_1, \varphi_2 \rangle$  can be amalgamated in  $\underline{K}$  iff there exists a  $C \in \underline{K}$  and embeddings  $\psi_i: B_i \rightarrow C$  (i = 1, 2) such that  $a\varphi_1\psi_1 = a\varphi_2\psi_2$  for all  $a \in A$ . Amal( $\underline{K}$ ) consists of all  $A \in \underline{K}$  such that all  $V = \langle A, B_1, B_2, \varphi_1, \varphi_2 \rangle$  can be amalgamated in  $\underline{K}$ .  $\underline{K}$  has the Amalgamation Property iff Amal( $\underline{K}$ ) =  $\underline{K}$ . The following result is proved without the Axiom of Choice. Theorem. If V cannot be amalgamated in  $\underline{K}$ , then there exist finitely generated subalgebras  $A', B'_1, B'_2$  of  $A, B_1, B_2$ , respectively, such that  $\langle A', B'_1, B'_2, \varphi'_1, \varphi'_2 \rangle$  cannot be amalgamated in  $\underline{K}$ , where  $\varphi'_1$  is the restriction of  $\varphi_i$  to  $B'_i$ , i = 1, 2. We have two applications of this result. 1. The statement that the class of all distributive lattices has the Amalgamation Property is independent of the Axiom

of Choice. 2. We get an algebraic proot of a result of M. Yasuhara (Math. Scand. 34 (1974), 5-36) that Amal( $\underline{K}$ ) is cofinal with  $\underline{K}$ . The original proof uses model theoretic tools. (Received April 18, 1975.)

*75T-Al61 DUNCAN A. BUELL, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Class groups of quadratic fields.

The author has computed the class groups for all complex quadratic number fields  $Q(\sqrt{-D})$  of discriminant -D for  $0 \le D \le 3974800$ , using the IBM 370/158 computer at the University of Illinois at Chicago Circle. In so doing, it was found that the first occurrences of rank three in the 3-Sylow subgroup are D=3321607= prime, class group  $C_3xC_3xC_{63}$  ( $C_n$  a cyclic group of order n), and D=3640387= 421.8647, class group  $C_3xC_3xC_{18}$ . The author has also found polynomials representing discriminants of 3-rank $\ge 2$ , and has found 3-rank three for D=6562327= 367.17881, 8124503, 10676983, 193816927, all prime, 390240895=5.11.7095289, and 503450951=prime.

The smallest examples of noncyclic 13-,17-, and 19-Sylow subgroups have been found, and of groups noncyclic in two odd p-Sylow subgroups. D=119191= prime, class group  $C_{15}xC_{15}$ , had been found by A.O.L. Atkin; the next such D is 2075343=3.17.40693, class group  $C_{30}xC_{30}$ . Finally, D=3561799=prime has class group  $C_{21}xC_{63}$ , the smallest D noncyclic for 3 and 7 together. (Received April 21, 1975.) (Author introduced by Professor A. O. L. Atkin.)

*75T-Al62 MICHAEL RICH, Temple University, Phila., Pa. 19121 and Ben Gurion Univ.of the Negev, Beersheva, Israel. The Levitzki radical in associative and Jordan rings.

The following results extend the earlier results announced by the author (these Notices 22 (1975), Abstract 75T - A5). Let A be an algebra with involution * over a field of characteristic  $\neq$  2 and let S denote the linear Jordan algebra of *-symmetric elements of A. If L denotes the Levitzki radical then: Theorem 1: L(S) = S $\bigwedge$  L(A).

If R is a ring of arbitrary characteristic and if  $R^{q}$  denotes the attached quadratic Jordan ring of R, then: Theorem 2:  $L(R) = L(R^{q})$ . (Received April 21, 1975.)

75T-Al63 SEUNG AHN PARK, University of Illinois, Urbana, Illinois 61801 <u>A characterization of the simple groups</u>  $U_{\mu}(2)$  and  $L_{\mu}(2)$  by a non-central involution.

The four-dimensional projective unimodular unitary group  $U_{4}(q)$  and the fourdimensional projective unimodular linear group  $L_{4}(q)$ , where  $q = 2^{n} > 2$ , have been characterized by the structure of the centralizer of a non-central involution (see Notices of A.M.S. 22 (1975), A-305). The characterization for the case q = 2 is contained in the following theorem: <u>Theorem</u>: Let  $H_{2}$  be the centralizer in  $U_{4}(2)$ of a non-central involution of  $U_{4}(2)$ . If G is any finite group containing an involution whose centralizer H in G is isomorphic to  $H_{2}$ , then one of the following holds: (i) G contains a normal subgroup of index 2, (ii)  $G = O_{2}(G)L$  where  $O_{2}(G)$ is elementary abelian of order  $2^{4}$  and  $L \cong L_{2}(4)$ , (iii) G is isomorphic to  $U_{4}(2)$ , or (iv) G is isomorphic to  $L_{h}(2)$ . (Received April 21, 1975.)

75T-A164 CLIFTON E. CORZATT, St. Olaf College, Northfield, Minnesota 55057. <u>Permutation</u> polynomials over the rational numbers.

Nonlinear polynomials over the rational numbers which permute the integers  $0, 1, \ldots, N$  are investigated. The function  $\nu(N)$  represents the minimum degree of all such polynomials. It is shown that  $\lceil (N+1)/4 \rceil \leq \nu(N) \leq N-1$  for all  $N \geq 13$ . It is also shown that  $\nu(N) \leq N-2$  for N odd and  $N \geq 7$ , that  $\nu(N) \leq N-3$  for  $N=2 \mod 6$ , and that if  $\epsilon > 0$  then  $\nu(N) \geq (N-1)(1-\epsilon)/2$  for N sufficiently large. (Received April 14, 1975.)

# Analysis

75T-B97 SHIMSHON ZIMERING, Ohio State University, Columbus, Ohio 43210. <u>Some properties of</u> regularly varying functions.

We will denote by  $R_{\alpha}$  the set of regularly varying functions of order  $\alpha$ , i.e. the collections of functions  $\underline{f}$ , such that  $\lim_{X\to\infty} (f(\lambda x)/f(x)) = \lambda^{\alpha}$   $(0 < \lambda < \infty)$ . We have proved the two following results: Let f(x) be a strictly positive and differentiable function in  $[0,\infty)$ . Then  $f(x) \in R_{\alpha}$  iff  $\mathbb{Z}$  a function  $g(x) \ge f(x) \cong g(x)$   $(x \to \infty)$ , and (a) g(x) can be written in the form  $g(x) = x^{\alpha + \epsilon(x)}$  where  $\epsilon(x) \to 0$  and  $x \ln x \epsilon^{\epsilon}(x) \to 0$   $(x \to \infty)$  or (b)  $\lim_{x\to\infty} \{(1 + g'(x)/g(x))^{x}\} = e^{\alpha}$ . (Received December 12, 1974.)

#### *75T-B98 ELEMER E. ROSINGER, Technion-Israel Institute of Technology, Haifa, Israel 32000 Non-Symmetry of Dirac Distributions.

In two previous papers: "A Distribution Multiplication Theory" and "A Modified Distribution Multiplication Theory", the author constructed associative, commutative algebras with unit element, containing the distributions in  $D'(R^1)$ , possessing an extension of the distribution derivative and holding relations as  $(x-x_0)^{T} \cdot \delta^{(q)}(x-x_0) = 0$ , for n > q, where  $\delta$  is the Dirac distribution. In the present paper it is shown that in some of the mentioned algebras, the Dirac distributions are not symmetric, that is, relations such as:  $\delta^{(q)}(x_0-x) \neq k \cdot \delta^{(q)}(x-x_0)$ ,  $\forall k \in C^1$ , with given  $x_0 \in R^1$ , q = 0, 1, 2, ..., and variable  $x \in R^1$  can be proved. (Received January 21, 1975.) (Author introduced by Professor Francois Treves.)

75T-B99 DANIEL M. OBERLIN, Florida State University, Tallahassee, Florida 32306. <u>Multipliers</u> of invariant subspaces of L^p(G). Preliminary report.

Let G be a compact abelian group with character group X. For a subset E of X and for  $1 \leq p \leq \infty$ , let  $M_E^p$  be the set of functions in  $\ell^{\infty}(E)$  which multiply  $\widehat{L_E^p(G)}$  into itself. Let  $M^p|_E$  be the set of restrictions to E of functions in  $M_X^p$ . Theorem. If  $1 \leq p < 2$  or if p > 2 is an even integer, there exists  $E \subseteq X$  for which  $M^p|_E$  is a proper subset of  $M_E^p$ . (Received February 12, 1975.)

*75T-B100 D.D. BONAR, Denison Univ., Granville, OH 43023, F.W. CARROLL, Ohio State Univ., Columbus, OH 43210 and PETER COLWELL, Iowa State Univ., Ames IA 50010 <u>Tsuji</u> T₂ <u>functions form a residual set</u>, Preliminary report.

In this note, D is the unit disk, and H(D) is the space of holomorphic functions on D with the topology of almost uniform convergence. "Curve" is a rectifiable Jordan curve contained in D. If f is mercomorphic on D and  $\Gamma$  is a curve, let  $L(f, \Gamma) = \int_{\Gamma} |f'(z)| (1+|f(z)|^2)^{-1} |dz|$ . A Tsuji  $T_2$ -function (W. K. Hayman, Mich. Math. J. 15, 1968) is a meromorphic function f on D for which there exists a sequence  $\{J_n\}$  of curves such that (i)  $J_n \subset int J_{n+1}$ , (ii) min  $\{|z|: z \in J_n\} \neq 1$  as  $n \neq \infty$ , (iii) lim  $\sup_n L(f, J_n) < \infty$ . For p > 0 and  $k = 2,3,\ldots$ , let  $A(p, k) \subset H(D)$  consist of those f for which there is a curve J, containing  $\{z: |z| \leq 1 - \frac{1}{k}\}$  in its interior, such that L(f, J) < p. We show that A(p, k) is dense and open in H(D). <u>Theorem 1</u>  $\cap\{A(\frac{1}{n}, k): n, k = 2,3,\ldots\}$  is residual in H(D). A fortiori, <u>Corollary</u>  $T_2 \cap H(D)$  is residual in H(D). (Received February 12, 1975.)

75T-BLOL R. J. LIBERA and E. J. ZLOTKIEWICZ, University of Delaware, Newark, Delaware, 19711. Loewner-type approximations for convex functions, Preliminary report.

Let  $f(z) = z + a_2 z^2 + ...$  be regular and univalent in the unit disk  $\Delta$  and let f(z) map  $\Delta$  onto a convex domain. For each t > 0, f(z) is subordinate to f(z) + tzf'(z), consequently there is a univalent function

$\omega(z)$ satisfying Schwarz's Lemma such that $f(\omega(z)) + t\omega(z)f'(\omega(z)) = f(z)$ .
The class of functions $\omega(z)$ taken over all admissible $t$ and $f(z)$ is
denoted by $\mathcal{F}$ . It is shown that $\omega(z,t)$ is in $\mathcal{F}$ if and only if it
satisfies the equation $\frac{\partial \omega(z,t)}{\partial t} = -\omega(z,t) [1+tp(\omega(z,t))]^{-1}$ , for $p(z)$ a
function of positive real part. This equation is used to study the behavior
of functions in ${\cal F}$ . (Received February 13, 1975.)
*75T-B102 RICHARD I. LOEBL, Wayne State University, Detroit, Michigan 48202. Contractive Linear Maps on C [*] -Algebras, Preliminary Report.
Let $\mathcal{A}, \mathcal{B}$ be $C^*$ -algebras and $C_k[\mathcal{A}, \mathcal{B}]$ denote the class of linear maps $\varphi: \mathcal{A} \to \mathcal{B}$ such that
$\  \varphi_k \  \leq 1, \text{ where } \varphi_k = \varphi \otimes \mathrm{id}_k \colon \mathcal{A} \otimes \mathrm{M}_n \to \mathcal{B} \otimes \mathrm{M}_n. \text{ Then } \mathrm{C}_1 \supseteq \mathrm{C}_2 \supseteq \mathrm{C}_3 \dots, \text{ and let } \mathrm{C}_{\varpi} = \cap \mathrm{C}_k.$
<u>Theorem 1</u> : If $\beta$ is commutative, $C_1[\alpha, \beta] = C_{\alpha}[\alpha, \beta]$ for all $\alpha$ .
<u>Theorem 2</u> : If $C_1[\mathcal{A}, \mathcal{B}] = C_2[\mathcal{A}, \mathcal{B}]$ then either $\mathcal{A}$ or $\mathcal{B}$ is commutative.
<u>Theorem 3</u> : If $C_1[\mathcal{A}, \mathcal{B}] = C_2[\mathcal{A}, \mathcal{B}]$ for all $\mathcal{B}$ , then $\mathcal{A}$ is the continuous functions on at
most two points. We say $\varphi$ is completely bounded if $\sup_{n} \ \varphi_n\  < \infty$ . Let $\mathbb{B}_{\mathbf{x}}[\mathcal{A}, \mathcal{B}]$
be the class of all completely bounded maps from $\alpha$ to $\beta$ ; and $B_1[\alpha, \beta]$ the bounded
maps.
<u>Theorem 4</u> : $B_1[\mathcal{A}, \mathcal{B}] = B_{\alpha}[\mathcal{A}, \mathcal{B}]$ for all $\mathcal{B}$ if and only if $\mathcal{A}$ is finite-dimensional.

Corollary:	The case $\mathcal{B} = \mathcal{L}(H)$ suffices. (Received February 14, 1975.)
75T-B103	WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Logarithmic and Integration-by-Parts Operators.

U, F,  $p_B$ ,  $p_{AB}$  and the notions of integral and integral function are as in previous abstracts of the author. Suppose  $W \subseteq p_B$ , W is closed under multiplication and each element of W is bounded away from O. L* denotes the set to which T belongs iff T is a function from W into  $p_{AB}$ such that if each of Y and Z is in W, then  $\int_U YT(Z)$  exists and T(YZ) =T(Y) + T(Z). P* denotes the set to which S belongs iff S is a function from W into  $p_{AB}$  such that if each of Y and Z is in W, then  $\int_U YS(Z)$  exists and  $S(YZ) = \int [YS(Z)] + \int [ZS(Y)]$ . Theorem: There is a one-to-one correspondence, Q, from P* onto L*, such that if S is in P* and Z is in W, then Q(S)(Z) =  $\int [(1/Z)S(Z)]$ . (Received February 17, 1975.)

*75T-B104 REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Ravenshaw College, Cuttack 3, Orissa, India. <u>A note on certain reducible</u> cases of the generalized hypergeometric function. Preliminary report.

Generalized hypergeometric functions with positive or negative integral parameter differences between certain numerator and denominator parameters are expressible as finite sums of lowerorder hypergeometric functions. This note presents a number of generalizations of certain reduction formulas of the aforementioned type for the hypergeometric function  $p_{q}^{F}(z)$  and for its basic analogue  $p_{s}^{\Phi}[z]$ , considered recently by Per W. Karlsson [J. Mathematical Phys. 12 (1971), 270-271; Nederl. Akad. Wetensch. Proc. Ser. A 77 = Indag. Math. 36 (1974),

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195-198], H. M. Srivastava [Nederl. Akad. Wetensch. Proc. Ser. A 76 = Indag. Math. 35 (1973), 38-40], and M. Chakrabarty [Nederl. Akad. Wetensch. Proc. Ser. A 77 = Indag. Math. 36 (1974), 199-202]. Several interesting special cases of the main results, contained in Theorems 1 through 3 of this paper, are indicated. (Received February 18, 1975.) (Author introduced by Professor H. M. Srivastava.)

## 75T-B105 ALEXANDER G. RAMM, Institute of Fine Mechanics and Optics, Leningrad 197101, USSR. On integral equations of the first kind with nonnegative kernels.

Consider (1) Af =  $\int_{\mathcal{B}} A(t,y)f(y) dy = g(t)$ , where  $\mathcal{B} \in \mathbb{R}_n$  is a finite domain. Let h(t) > 0, Ah  $\leq \mathbb{C}$  for some h. Denote h/Ah = a(t) > 0,  $f(t) = a(t)\varphi(t)$ . (1) can be written as (2)  $A_1\varphi \equiv \int_{\mathcal{B}} A(t,y)a(y)\varphi(y) dy = g(t)$ . Denote I -  $A_1 = B$ ,  $A_1\emptyset_j = \lambda_j\emptyset_j$ ,  $j = 1, 2, \ldots$ . Suppose that  $\int_{\mathcal{B}} a(t) dt < \infty$ ,  $a(t) \geq m > 0$ ;  $\lambda_j \neq 0$ ,  $|\arg \lambda_j| \leq \pi/3$ ;  $C_1(\Omega) \leq \int_{\Omega} A(t,y)a(y) dy \leq C_2(\Omega)$  for any measurable subset  $\Omega \subset \mathcal{B}$ , mes  $\Omega > 0$ . Denote by  $H_{\pm}$  the spaces  $L_2\mathcal{B}$ ,  $a^{\pm 1}(t)$ ). Theorem. Let (i)  $g \in H_{+}$  and (2) has a solution in  $H_{+}$ ; (ii)  $A_1$  is a compact operator in  $H_{+}$ , its eigenelements form a Riesz basis in  $H_{+}$ . Then the process  $\varphi_{n+1} = B\varphi_n + g$ ,  $\varphi_0 = g$ , converges in  $H_{+}$  to a solution  $\varphi(t)$  of (2); (1) has a solution  $f(t) = a(t)\varphi(t)$  in  $H_{-}$ . The theorem is applicable to the main equation of electrostatics: (3)  $\int_{\mathbf{F}} (4\pi \epsilon_0 r_{ty})^{-1} \sigma(y) dy = V$ , V = const. If  $\mathbf{F}$  is a closed surface we can put h(t) = 1. If  $\mathbf{F}$  has an edge boundary  $\mathscr{L}$  (F is a screen), then h(t) is equal to  $[\rho(t)]^{1/2}$ , where  $\rho(t) = \min_{a' \in \mathcal{L}} |t - a'|$ . Choosing such h we take into consideration the singularity of the solution of (3) on the edge. (Received February 19, 1975.)

*75T-B106 JOHN G. HEYWOOD, University of British Columbia, Vancouver, Canada V6T 1W5 On Function Spaces for Viscous Flow Problems. Preliminary Report

Let  $\Omega$  be an arbitrary open set of  $\mathbb{R}^n$ ,  $n \geq 2$ . Let  $W_2^{-1}(\Omega)$  be the completion of  $C_0^{\infty}(\Omega)$  vector fields in the norm  $||u||_1^2 = ||u||^2 + ||\nabla u||^2$ , where  $||\cdot||$  denotes  $L^2(\Omega)$  norm. Let  $J_1^*(\Omega)$  consist of all solenoidal vector fields which belong to  $W_2^{-1}(\Omega)$ . Let  $J_1(\Omega)$  be the completion in norm  $||u||_1$  of all solenoidal  $C_0^{\infty}(\Omega)$  vector fields. It is shown that there exist domains  $\Omega$  for which  $J_1^*(\Omega)$  is not contained in  $J_1(\Omega)$  if n > 2. The same is true for all  $n \geq 2$  if the completions are taken in norm  $||\nabla u||$  instead of norm  $||u||_1$ . It is shown that the two types of function spaces are identical in the cases of a bounded domain, or an exterior domain, or a half-space, thus reestablishing for these domains the uniqueness theorems of hydrodynamics which depend upon the identification of  $J_1^*(\Omega)$  with  $J_1(\Omega)$ . In some domains such as  $\Omega = \{(x_1, x_2, x_3): x_1 \neq 0 \text{ or } x_2^2 + x_3^2 < 1\}$  for which the two types of function spaces are upon the problems of viscous flow are well posed if appropriate auxiliary flux or pressure conditions are imposed. (Received February 20, 1975.)

*75T-B107 WILLIAM R. MALLORY, Physics & Astronomy, University of Montana, Missoula, Montana 59801, <u>A Special Case of the Kummer Function</u>.

Given the Kummer function,  $M(a,b,z) = 1 + \frac{az}{b} + \frac{a(a+1)z^2}{b(b+1)2!} + \dots$ , the relation  $M(\frac{b}{2} + iv, b, ix) = e^{ix/2} R(x)$  holds, where R is a real function of x, for b, v, and x real. The proof is quite simple: Consider the imaginary part of  $G(x) \equiv e^{-ix/2} M(\frac{b}{2} + iv, b, ix)$ . Let  $G^*$  be the complex conjugate of G. Then  $G-G^* = e^{-ix/2} M(\frac{b}{2} + iv, b, ix) - e^{+ix/2} M(\frac{b}{2} - iv, b, -ix)$ . But by the Kummer

transformation [e.g., M. Abromowitz & I. A. Segun, Handbook of Mathematical Functions, Dover, 1968, p. 505.],  $M(a,b,z) = e^Z M(b-a,b,-z)$ , the two terms are equal and hence Im G=0. Because many other special functions, such as Whittaker's function and the parabolic cylinder functions are related to special cases of the Kummer function, the case discussed herein has implications for these also. (Received February 20, 1975.) (Author introduced by William R. Derrick.)

*75T-B108

R. T. JACOB, JR., University of New Orleans, New Orleans, La. 70122. A class of sequence spaces

Köthe has shown that every perfect nuclear sequence space has the property that each bounded point set in the space is contained in a normal hull of a single point of the space. In this paper, we study sequence spaces having this property. Some simple theorems are proved about matrix transformations involving these spaces and their duals, and most of the known theorems about matrix transformations on the analytic and entire sequence spaces are shown to be corollaries to these general theorems. (Received February 21, 1975.)

*75T-B109 E.B. SAFF, Univ. of South Fla., Tampa, FL 33620 and R.S. VARGA, Kent State Univ., Kent, Ohio 44242, Zeros and Poles of Pade Approximants to e^z.

Letting  $R_{n,\nu}(z)$  denote the Padé approximant to  $e^z$  with numerator degree n, and denominator degree  $\vee$  the following results (among others) are established: <u>Theorem 1</u>. For each fixed  $\nu \ge 0$ , and every  $n \ge 0$ , the Padé approximant  $R_{n,\nu}(z)$  for  $e^z$  has no zeros in the parabolic region  $P_{\nu+1} := \{z=x + iy : y^2 \le 4(\nu+1) \cdot (x+\nu+1), x > -(\nu+1)\}$ . <u>Theorem 2</u>. For every  $n \ge 2$ ,  $\nu \ge 0$ , the Padé approximant  $R_{n,\nu}(z)$  for  $e^z$  has no zeros in the infinite sector  $S_{n,\nu}: = \{z:|\arg z| \le \cos^{-1}(\frac{n-\nu-2}{n+\nu})\}$ . Consequently, for any (fixed)  $\sigma > 0$ , the sequence of Padé approximants  $\{R_{n,f}(\sigma n](z)\}_{n=1}^{\infty}$  is zerofree in the infinite sector  $S_{\sigma} := \{z:|\arg z| \le \cos^{-1}(\frac{1-\sigma}{1+\sigma})\}$ . <u>Theorem 3</u>. If  $n \le \nu+4$ , then  $R_{n,\nu}(z)$  has all its zeros in the open left half-plane. <u>Theorem 4</u>. Given any integer  $\tau$ , there exists an integer  $m=m(\tau)$  such that the approximants  $\{R_{n,n-\tau}(z)\}_{n=m}^{\infty}$  to  $e^z$  have all zeros in the open left half-plane. (Since the poles of  $R_{n,\nu}(z)$  are the same as the negatives of the zeros of  $R_{\nu,n}(z)$ , each of the above theorems on zeros can be restated for the poles). We remark that Theorem 1 improves upon the work of Newman and Rivlin, while Theorem 3 improves upon the work of Ehle, and of Van Rossum. Furthermore the second part of Theorem 2 is sharp at the endpoints  $\sigma = 0^+$  and  $\sigma = + \infty$ , and numerical computations suggest that it is sharp for all  $\sigma$ . (Received February 24, 1975.)

#### *75T-B110 W.LYLE COOK, Eastern Montana College, Billings, Mt, 59101 and T.M.MILLS, Bendigo Institute of Technology, Bendigo, Vic., Australia, 3550. <u>On Berman's phenomenon</u> in interpolation theory.

In 1965,D.L.Berman established an interesting divergence theorem concerning Hermite-Fejer interpolation on the extended Chebyshev nodes.In this paper it is shown that this phenomenon is not an isolated incident. A similar divergence theorem is proved for a higher order interpolation process. The paper closes with a list of several open problems. (Received February 24, 1975.)

*75T-B111 WILLIAM C. TROY, University of Pittsburgh, Pittsburgh, Pa. 15260 Oscillations in a Third Order Differential Equation Modeling a Nuclear Reactor

We consider a third order autonomous non linear ordinary differential equation which models a two temperature feedback nuclear reactor. Using geometric methods and the Brouwer Fixed Point Theorem, we show the existence of periodic solutions over a specific range of parameters. In light of these results, conjectures are given discussing the global behavior of families of periodic solutions bifurcating from the steady state.

(Received February 24, 1975.)

*75T-B112 H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Ravenshaw College, Cuttack-3, India. <u>New generating functions</u> involving several complex variables.II. Preliminary report.

Recently, H. M. Srivastava and R. G. Buschman ["Some polynomials defined by generating relations", *Trans. Amer. Math. Soc.* 206 (1975); see also these *NOTICES* 20 (1973), p. A-633, Abstract 73T-B300] presented a number of generating functions for two classes of polynomials

and for their generalizations in several complex variables. In this sequel to the authors' paper [H. M. Srivastava and Rekha Panda, "New generating functions involving several complex variables", to be published; see also these *NOTICES* 21 (1974), p. A-593, Abstract 74T-B237], certain new classes of generating functions are given for various sequences of functions of several complex variables. The multidimensional generating relations, contained in Theorems 1 through 6 of the present paper, involve either Taylor or Laurent series and provide new extensions of several known results given, among others, by Srivastava and Buschman [*loc.cit.*, Theorems 2, 3 and 4], and David Zeitlin [*Scripta Math.* 29 (1973), 43-48, especially p. 44, Theorem 1]. It is also indicated how the main results of this paper can be appropriately applied to derive the corresponding generating relations associated with the generalized hypergeometric functions in one and more variables. (Received March 3, 1975.)

*75T-B113 WILLIAM CONNETT, University of Missouri-St. Louis, St. Louis, M0 63121 The Dini condition is necessary

It has long been conjectured that The Dini condition is not necessary for the existence of a singular integral operator, K, applied to an  $L^1$  function. It is possible to construct functions  $\Omega(y')$  whose integral modulus of continuity is arbitrarily close to satisfying The Dini condition, but for which K*f(x) does not exist for a continuous f  $\varepsilon L^p$ ,  $1 \le p < \infty$ . In this sense the Dini condition is necessary.

Other results are obtained which show how the smoothness of  $\Omega$  and the smoothness of f interact to produce the existence of K*f(x). The conditions are similiar to those used by Marcinkiewicz to produce the a.e. convergence of the Fourier series of f. (Received March 3, 1975.)

## 75T-B114 LEONARD SARASON, University of Washington, Seattle, Washington 98195. <u>Weak equals</u> strong in corner domains for first order elliptic systems.

Let  $L = \sum A_j(a) D_j$  be a square elliptic system of P.D. Ops. with  $A_j$  smooth, j = 1, ..., n. Let  $\Omega$  be a domain in  $\mathbb{R}^n$  satisfying a cone condition and with  $\partial \Omega = \Gamma$  contained in a finite union of hyperplanes. Theorem. Weak solutions of the problem Lu = f in  $\Omega$ ,  $u = \overline{u}$  on  $\Gamma$  are strong. Proof. The result is local. Let  $\nu$  point out of  $\overline{\Omega}$  from  $x_0 \in \Gamma$ . One shows that the map  $\epsilon \rightarrow U(\cdot - \epsilon \nu)$  is continuous near  $x_0$  from  $\overline{\mathbb{R}}^+$  to  $L_2^{loc}(\Gamma)$ . Hence a mollifier with a shift in the  $\nu$  direction produces the desired approximating sequence. (Received March 3, 1975.)

75T-B115 ROBERTO MORITYON, Universidad Complutense de Madrid, Matemáticas, Madrid - 3 Spain. On some basis of differentiation. Preliminary report.

Corollary 1.3. and Theorem 2 of this paper solve two problems proposed by A. Sygmund. For notation and definitions see M. de Guzmán, <u>Differentiation of integrals in Rⁿ</u>, Universidad de Madrid, Madrid, 1974. Let A be a collection of open bounded sets in Rⁿ. By B(A) we denote the minimal Busemann-Feller basis that is translation invariant and contains A. Let  $f,g:(0,\varepsilon) \rightarrow (0,\infty)$  be such that  $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$  and  $f \leq g$ . Then,  $A_f = \{(0,x)X(0,f(x)) \in R^2: x\in(0,\varepsilon)\}$  and  $A_{f,g} = \{(0,x)X(0,y) \in R^2: x\in(0,\varepsilon), f(x) \le y \le g(x)\}$ . <u>THEOREM 1</u>.- Assume that  $x \rightarrow xf(x)$  and  $x \rightarrow xg(x)$  are continuous and strictly increasing and that  $B(A_{f,g})$  differentiates some space  $\phi(L)$  strictly bigger than  $L(1+\log^+L)$ , then there exists c>0 such that  $B(A_{f,g})$  differentiates  $L^1$ . <u>COROLLARY 1.2</u>.- If moreover  $f(x/2) \ge Kf(x)$ ,  $B(A_{f,g})$ , uifferentiates  $L^1$  if and only if there exists C>0 and  $\delta > 0$  such that  $g(x) \le Cf(x)$  for each  $x\in(0,\delta)$ . <u>COROLLARY 1.3</u>.- If  $\varepsilon=1$ ,  $f(x)=x^2$ , g(x)=x,  $B(A_{f,g})$  differentiates  $L(1+\log^+L)$  and this is the "best" space  $\phi(L)$  it differentiates. THEOREM 2.- Let  $f: (0,\varepsilon) \rightarrow R$  be increasing and such that  $\lim_{x\to 0} f(x)=0$ . Let  $A= \{(0,x)X(0,y)X(0,y)X(0,g)=x^2$ . Then B(A) differentiates  $L(1+\log^+L)$  and this is the "best" space  $\phi(L)$  it differentiates. (Received March 3, 1975.) (Author introduced by Miguel de Guzmán.)

75T-B116 L.A.MEDEIROS and G.PERLA MENZALA, Institute of Mathematics, Federal University of Rio de Janeiro, CP 1835 ZC-00 20.000 Rio de Janeiro,GB Brasil, On a modified Korteweg- de Vries equation. Preliminary report.

#### ABSTRACT

We consider first the problem (1)  $u_t - u_{xxt} + uu_x = 0$  in  $-\infty < x, t < \infty$ with initial data at t = 0 which is 1-periodic and the boundary condition u(x+1,t) = u(x,t) for all x,t; proving the existence and uniqueness of the solutions of such a problem. We use the semi-discrete approach together with the energy method. We also consider the more general equation (2)  $u_t - u_{xxt}$  $+ u_x + \gamma u^m u_x = 0$  in all space-time, where  $\gamma = \text{constant} > 0$  and  $m = \text{odd} \ge 7$  and we apply the nonlinear inverse scattering theory, showing that the scattering operator associated with (2) determines uniquely the constant  $\gamma$ . (Received March 21, 1975.)

#### *75T-B117 P.C. COCHRANE & T.H. MACGREGOR, SUNY AT ALBANY, Fréchet differentiable functionals and support points for families of analytic functions.

Let S*( $\alpha$ ) denote the class of starlike functions of order  $\alpha$  in the open unit disk. Let F be any closed set of functions each of which has the form  $f(z) = z/k_{\pm 1}^{\pi}(1-x_k z)^{t}k$ , where n is a fixed positive integer,  $|x_k| = 1, t_k > 0$  and  $\sum_{k=1}^{n} t_k = 2(1-\alpha)$ . There is a continuous, Fréchet differentiable functional J so that F is the solution set to the extremal problem max Re J(f) over S*( $\alpha$ ). Similar results are proved for families of functions which can be put into one-to-one correspondence with S*( $\alpha$ ).

The support points of  $S^*(\alpha)$  and  $K(\alpha)$ , the functions convex of order  $\alpha$ , are shown to coincide with the extreme points of their respective convex hulls. Also, if a finite collection or support points of  $S^*(\alpha)$  (or  $K(\alpha)$ ) are given then there is a continuous linear functional J which has this set as the solution set to the extremal problem max Re J(f) over  $S^*(\alpha)$  (or  $K(\alpha)$ ). (Received March 5, 1975.)

*75T-B118 DOMINGO A. HERRERO, Universidad Nacional de Río Cuarto, Río Cuarto, Córdoba, ARGENTINA. <u>More about closure of similarity orbits.</u>

Let  $\mathscr{B}(T) = \{WTW^{-1}: W \text{ invertible}\}$  be the similarity orbit of an operator T acting on a complex separable Hilbert space. It is shown that its norm closure  $\mathscr{B}(T)$  always contains: (i) An operator A unitarily equivalent to  $N \oplus B$ , where N is normal, Sp(N)(spectrum) = E(N)(essential spectrum) = F is an arbitrary prescribed nonempty closed subset of E(T),  $E_1(B) = F \cup E_1(T)$ ,  $E_r(B) =$  $F \cup E_r(T)$  (1 = left; r = right), Sp(B) = Sp(T), nullity(B-z) = nullity(T-z) for all  $z \notin E_1(B)$  and null(B-z)* = null(T-z)* for all  $z \notin E_r(B)$ ; (ii) A quasidiagonal operator C such that Sp(C) = Sp(T) and E(C) = w(C) = w(T) (Weyl spectrum); (iii) A normal operator L such that Sp(L) = Sp(T) and E(L) = Sp(T)-(normal eigenvalues). Closed similarity orbits, as well as orbits with minimal(with respect to inclusion) closure are completely characterized. As a corollary: A compact set F has the property that  $\mathscr{B}(T)^- = \mathscr{B}(A)^-$  whenever Sp(T) = Sp(A) = F if and only if it is perfect and nowhere dense. (Received March 7, 1975.) *75T-BL19 M. J. FRANK, University of Wisconsin, Milwaukee, Wisconsin 53201 and B. SCHWEIZER, University of Massachusetts, Amherst, Massachusetts 01002. On the duality of generalized infimal and supremal convolutions.

Fix two real intervals I and J, and denote by M (resp., N) the set of non-decreasing functions  $F: I \rightarrow J$  (respectively,  $f: J \rightarrow I$ ). Given two continuous and order-preserving binary operations 0 and  $\Box$  on I and J, resp., define the operations  $\Delta$  on M and  $\nabla$  on N by:

 $(F\Delta G)(x) = \sup_{y \circ z} = x^{F(y) \square G(z)}$ , F,G  $\in M$ , x  $\in$  I; and  $(f \nabla g)(t) = \inf_{u \square V} = t^{f(u) \circ g(v)}$ , f,g  $\in N$ , t  $\in$  J. Let  $\phi^*$  denote any quasi-inverse of a non-decreasing function  $\phi$ . (Connecting the graph of  $\phi$  at any points of discontinuity and reflecting the connected graph in the identity yields the connected graph of  $\phi^*$ .) Note that M and N are dual under the mapping  $\phi \rightarrow \phi^*$ . THEOREM:  $(F \Delta G)^*(t) = (F^* \nabla G^*)(t)$  and  $(f \nabla g)^*(x) = (f^* \Delta g^*)(x)$  at every point of continuity. Full equality holds under some mild assumptions. This result has interesting applications when either M or N is taken to be the space of probability distribution functions. (Received March 10, 1975.)

75T-B120 WILLIAM L. WHITE, University of Mississippi, University, MS. 38677 <u>A Subclass of Bazilevič Functions</u>. Preliminary report.

Let S be the class of functions analytic and univalent in the open unit disc normalized by f(0) = 0, f'(0) = 1. Also let K, S^{*} and B( $\alpha$ ,  $\beta$ , g, P) be its subclasses of convex, starlike and Bazilevič functions, respectively (see R. Singh, Proc. Amer. Math. Soc. 38(1973), 261-271). Define

(1) 
$$F(z)^{\alpha} = F(z, \delta, \gamma, c)^{\alpha} = cZ^{\alpha-c} \int_{0}^{z} t^{c-1} \left(\frac{f(t)}{t}\right)^{\delta} \left(\frac{g(t)}{t}\right)^{\gamma} dt$$

<u>Theorem 1.</u> If  $f \in S^*$ ,  $g \in K$ ,  $0 < \alpha \le c \le 1$ ,  $2\delta + \gamma \le 2\alpha$  and  $2\delta + \gamma \le 2c$ , then  $F \in S^*$ . <u>Theorem 2</u>. If  $F \in S^*$ ,  $g \in K$ ,  $0 < \alpha \le c \le 1$ ,  $\delta \ge 0$ ,  $\gamma \ge 0$  and  $\alpha - \delta - \frac{\gamma}{2} = 0$ , then f is starlike in  $|z| < c/((\alpha + 1) + \sqrt{(\alpha + 1)^2 + c^2 - 2\alpha c})$  and the result is sharp. <u>Theorem 3</u>. If  $f \in B(\alpha) = B(\alpha, 0, g, P)$ ,  $0 < \alpha \le c \le 1$ , and F defined by (1) with  $\alpha = \delta$ ,  $\gamma = 0$ , then  $F \in B(\alpha)$ . Other results are obtained. (Received March 10, 1975.)

*75T-B121 SADANAND SRIVASTAVA, Bowie State College, Bowie, Md. 20715. Fixed Point Theorems for Quasi-Nonexpansive Mappings, Preliminary report.

<u>Theorem 1.</u> Let C be a weakly compact convex subset of a normed linear space X. Let T: C  $\longrightarrow$  C be a quasi non-expansive map i.e.  $||Tx-Ty|| \leq 1/3 \{||x-Tx|| + || x-y|| + || y-Ty|| \}$ then the set  $\{ || x-Tx|| | x \in C \}$  has a smallest number. <u>Theorem 2</u>. Let X be a normed linear space. Let C be weakly compact subset of X, and T: C  $\rightarrow$  C be a quasi non-expansive. Suppose T does not have a fixed point, then there exists a T-invariant closed convex subset of X such that  $\delta$  (H) > O and  $|| Tx-x || = \delta$  (H) for all x in H. Moreover if for any closed convex T-invariant subset H of X with  $\delta$  (H) > O there exists an x in H such that

 x-y 
 < δ (H), y ε H. Then T has a unique fixed point. (Received March 12, 1975.)</li>
 *75T-B122 BEATRIZ P. NEVES, Federal University of Rio de Janeiro, R.J., Brasil. Regular solutions of the initial-periodic boundary value problem for a nonlinear partial differential equation.

We prove existence and uniqueness of global solutions for the following initial-periodic value problem in the half plane  $t \ge 0 -\infty < x < \infty$  $u_t - (f(u))_x - \delta u_{xxt} = 0$ ,  $\delta > 0 u(x,0) = u_0(x)$  and u(x+1,t) = u(x,t) where u = u(x,t) and the subscripts indicate partial derivatives. We have the following result Theorem: Suppose that  $f(s) \in C^{\infty}(\mathbb{R})$  and  $\frac{df}{ds}(s) \ge 0$ , then, for each  $\delta > 0$ and for every initial data  $u_0 \in H^m(0,1)$ ,  $\frac{d^{j}u_0}{dx^{j}}(0) = \frac{d^{j}u_0}{dx^{j}}(1)$ ,  $j=0,1,\ldots,m-1$ where m is an integer  $\ge 2$ , there exists a unique solution u(x,t) with the same regularity as the data. The proof is done by the semi-discrete approximation method similar to that presented by Sjöberg (in Journal of Math. Analy. and Applications 29, 569-579 (1970)). (Received March 12, 1975.) (Introduced by G. Perla Menzala.) <u>Theorem 1</u>. If x is a null sequence not in  $\ell$ , and A is a matrix transformation such that Ay is in  $\ell$  for every subsequence y of x, then (1)  $\sum_{p=1}^{\infty} |a_{pq}| < \infty$  for q = 1, 2, 3, ..., and (2) if  $\lim_{q} \sum_{p=1}^{\infty} a_{pq} = L$ , then L = 0. This theorem may be used to answer in the affirmative the following question proposed by J. A. Fridy. Is a null sequence x necessarily in  $\ell$  in case there is a sum-preserving  $\ell - \ell$  matrix A such that Ay is in  $\ell$  for every subsequence y of x? <u>Theorem 2</u>. If x is a null sequence not in  $\ell$ , and A is a matrix method such that Ay is in  $\ell$  for every rearrangement y of x, then  $\lim_{q} \sum_{p=1}^{\infty} |a_{pq}| = 0$ . (Received March 13, 1975.)

#### 75T-B124 H. SILVERMAN, D.N. TELAGE, University of Delaware, Newark, Delaware, 19711, <u>Spiral-Like Functions and Related Classes With Fixed Second</u> <u>Coefficient</u>,

Denote by  $S_p(\lambda, \alpha)$   $(|\lambda| < \frac{\pi}{2}, 0 \le \alpha \le \cos \lambda, 0 \le p \le \cos \lambda - \alpha)$  the class of functions  $g(z) = 1 + 2a_2z^2 + \ldots$  analytic in the unit disk |z| < 1for which  $Re\{e^{i\lambda}g(z)\} > \alpha$  with  $|a_2| = p$ . We determine the largest disk  $|z| < r = r(\lambda, \alpha, \gamma, \beta, p)$  in which functions in  $S_p(\lambda, \alpha)$  are contained in  $S_p(\gamma, \beta)$ . By specializing our parameters and our function g(z), we obtain results relating subclasses of spiral-like functions to subclasses of starlike functions, and subclasses of functions for which zg'(z) is spiral-like to subclasses of convex functions. When p = 0, results concerning odd functions are found. (Received March 13, 1975.)

#### *75T-B125 ARTHUR LIEBERMAN, The Cleveland State University, Cleveland, Ohio 44114. Entropy of States of a Gage Space.

Let (H, A, m) be a regular gage space. Let  $\rho$ ,  $\sigma$ , and  $\psi = \lambda \rho + (1-\lambda)\sigma$ ,  $0 < \lambda < 1$ , be regular states. The density operator  $D_{\rho}$  of a regular state is a non-negative (possibly unbounded) self-adjoint measurable operator. Let F be a continuous convex function on  $[0, \infty)$  and define the entropy of  $\rho$  by  $e(\rho) = m(F(D_{\rho}))$ . Conditions are obtained, in terms of  $e(\rho)$  and  $e(\sigma)$ , for  $e(\psi)$  to be  $-\infty$ , finite,  $\infty$ , or undefined. If both  $\rho$  and  $\sigma$  have finite entropy, then  $\psi$  has finite entropy and  $e(\psi) \ge \lambda e(\rho) + (1-\lambda)e(\sigma)$ ; if A = B(H), F is strictly convex, and  $\rho \neq \sigma$ , then strict inequality is obtained. If (H, B, m|B) is a sub-gage space of (H, A, m), then  $e(\rho|B) \ge e(\rho)$ . These results are restated as inequalities concerning the trace of a convex function of an operator. (Received March 14, 1975.)

*75T-B126 F. JAVIER THAYER, Tulane University, New Orleans, Louisiana 70118. Almost Diagonal C-* Algebras.

Call a separable C-* algebra A almost diagonal iff it satisfies one of the following equivalent conditions:

There is a sequence of mutually orthogonal projections  $\{p_i\}$  with sum 1 such that  $A \in \{p_i\}' + K(H)$ ; there is a finite type 1 algebra  $\mathcal{A}$  such that  $A \in (\mathcal{L} + K(H))$ ; for every finite sequence  $x_1, \ldots, x_n \in A$ , lim inf  $\{Max || [x_i, p] || : p$  a finite rank projection  $\} = 0$ . Theorem: Let A be a separable C-* algebra,  $\{\phi(\lambda)\}_{\lambda \in X}$  a measurable family of representations of A such that 1) for almost all  $\lambda \in X$ ,  $\phi(\lambda)(A)$  is almost diagonal; 2)  $\phi$  is almost norm separately valued. Then if  $\mu$  is a Radon measure,  $\int^{th} \varphi(\lambda) d\mu(\lambda)$  has almost diagonal image. Corollary. All liminal algebras are almost diagonal.

(Received March 14, 1975.) (Author introduced by J. Thomas Beale.)

75T-B127 MARVIN W. GROSSMAN, Temple University, Philadelphia, Pa. 19121. Korovkin type theorems with respect to a Markov operator. Preliminary report.

It is shown that H. Bauer's approach to generalized Korovkin theorems (cf. Ann. Inst. Fourier 23 (1973), 245-260, and compare H. Berens & G.G. Lorentz, Approx. Theory, Academic Press 1973) can be adapted to the case of a Markov operator T in place of the identity operator. For convenience we assume X is compact and H is a linear subspace of C(X) with 1  $\epsilon$  H. We

introduce and study the Choquet boundary of X with respect to H and T denoted  $\vartheta_H^1 X$  (which can be empty even if H and TH separate points).

THEOREM. Let  $T:C(X) \rightarrow C(X)$  be a Markov operator (T is positive, linear and T(1) = 1). Then the following statements are equivalent:

(1)  $\hat{f}_{T} = Tf$  for all  $f \in C(X)$  where  $\hat{f}_{T} = \inf \{Th | h \in H, h > f\};$ 

(2)  $\partial_H^T X = X$  where  $\partial_H^T X$  is the set of x in X such that  $\varepsilon_X^{oT}$  is the unique probability measure  $\mu_X$  on X satisfying  $\int h \ d\mu_X = Th(x)$  for all  $h \in H$ ;

(3) for every net  $\{L_i\}$  of monotone maps  $L_i:C(X) \rightarrow C(X)$  which converges pointwise (uniformly) on X to T on H, necessarily  $\{L_i\}$  converges pointwise (uniformly) on X to T on C(X).

If T is a lattice homomorphism, then the above are equivalent to: (4) for every x in X,  $\varepsilon_{0}^{\circ T}$  is  $\mathcal{E}_{H}^{-maximal}$  in the Choquet ordering ( $\mathcal{E}_{H}^{-}$ the smallest

inf-stable closed subset of C(X) containing H). The equivalence of (1) and (3) holds in the more general form: {f  $\in C(X) | \hat{f}_T = \hat{f}_T$ } coincides with the Korovkin closure of H with respect to T. (Received March 14, 1975.)

*75T-B128 RHONDA J. HUGHES, University of Illinois at Chicago Circle, Box 4348, Chicago, Illinois 60680, <u>Semi-groups of unbounded linear</u> operators in <u>Banach</u> space. I.

Let X be a Banach space, and  $\{T_t\}_{t>0}$  a one-parameter family of unbounded linear operators in X for which the semi-group property  $T_sT_tx = T_{s+t}x$ , s,t > 0, is satisfied on a suitable subspace D of X; for each  $x \in D$ ,  $\|T_tx-x\| \longrightarrow 0$  as  $t \to 0^+$ , and  $T_tx$  is a continuous function of t > 0. Then the notion of infinitesimal generator is generalized to this unbounded setting; the operator defined is essentially the classical generator with a more restricted domain. For each  $w \in R$ , define  $N_w(x) = \sup_{t>0} e^{-wt} \|T_tx\|$ , where

x  $\in$  D, and let  $\Sigma_{w} = \{x \in D: N_{w}(x) < \infty\}$ . Under certain conditions (which include the case where the operators  $T_{t}$  are closed), the spaces  $\Sigma_{w}$  are Banach spaces with norm N. They are used to prove the following: <u>Theorem</u>. Let  $w \in R$ ,  $x \in \Sigma_{w}$ . Then the resolvent equation  $(\lambda I - A)y = x$ ,  $Re \ \lambda > w$ , has a unique solution (call it  $J_{\lambda}^{W}x$ ) belonging to  $\Sigma_{w}$ , and  $J_{\lambda}^{W}x = \int_{0}^{\infty} e^{-\lambda t}T_{t}xdt$ . Moreover,  $N_{w}((Re \ \lambda - w)J_{\lambda}^{W}x) \leq Nw(x)$ . In addition, a Hille-Yosida type theorem is proved; applications include the classical Weyl fractional integral in  $L^{P}(0,\infty)$ , and fractional powers of closed operators. (Received March 17, 1975.) (Author introduced by Shmuel Kantorovitz.)

75T-B129 JOSE G. LLAVONA. F. of Mathematics. University of Madrid. Madrid-3-. Spain. <u>Bornologies of Approximation</u>. Preliminary report.

Let E and F be two non-zero Banach spaces over R, and  $\beta$  a convex bornology on E, finer than  $\beta_c$  (the compact bornology). Let  $C^{\circ}_{\beta}(E,F)$  be the space  $C^{\circ}(E,F)$  with the topology defined by the family of semi-norms:

$$\begin{split} \|\beta\|_{k,n} &= \underset{o\leqslant j \neq n}{\text{Max}} \left[ \sup_{i \neq j} \left\{ \|D_{f_i(X)(Y)}\| : X \in K, Y \in K \right\} \right] & \text{Where } K \text{ is a compact and runs} \\ \text{ over a basis of } \beta. \\ \text{THEOREM 1. If } n \geqslant 1, \text{ the following conditions are equivalent: 1)} C^{\circ}(E) \cong F \text{ is dense} \\ \text{in } C_{\beta}^{\circ}(E,F) \text{ for every Banach space } F. 2) \text{ There exists a family } F_i \text{ in } C^{\circ}(E) \boxtimes F \\ \text{such that the identity map on } E \text{ can be approximated in } C_{\beta}^{\circ}(E,E) \text{ by maps belonging to } F_i. 3) \text{ There exists a family } F_2 \text{ in } E' \otimes E \text{ such that the identity map on } E \\ \text{can be approximated in } C_{\beta}^{\circ}(E,E) \text{ by maps belonging to } F_2. 4) \\ \beta \text{ is a bornology of approximation on } E. \\ \text{TEHOREM 2. Let } E \text{ and } F \text{ be two non-zero Banach spaces over } R, \text{ and } f \text{ a bornology of approximation on } E. \\ \text{If } A \text{ is a pseudo-algebra of } C(E,F), \text{ contained in } \\ \end{array}$$

 $C^{n}(E,F)$ , satisfyiging the Nachbin's conditions and such that Aoff CA, where f is a family which satisfies (2) (Theorem 1), then A is dense in  $C^{n}_{P}(E,F)$ . (Received March 17, 1975.) (Author introduced by Mr. Alfonso Casal.)

*75T-B130 HENRY B. COHEN, University of Pittsburgh, Pittsburgh Pa. 15260. Integration in locally convex spaces, Preliminary report.

Let X be a locally convex space, S a set, and u a non-negative finitely additive set function defined on an algebra of subsets of S. We define the notion of integrability and the more special notion of Bochner-integrability of a function from S into X that is the limit in measure of a net of simple integrable functions. Thus an integrable function need not be u-separably valued or measureable in a strong sense. Both integration processes define an indefinite integral and therefore provide a good supply of potential derivatives for vector-valued measures. (Received March 20, 1975.)

*75T-B131 W.M. BOGDANOWICZ & J.P. MCCLOSKEY, Catholic University of America, Washington, D.C. 20064. A Characterization Of Measurability With Respect To The Smallest Sigma Algebra Extending A Sigma Ring V By Means Of Measurability With Respect To A Sigma Algebra In Which V Forms An Ideal.

Let V be a sigma ring of subsets of a space X such that X is not an element of V. We make the following definition:

<u>DEFINITION</u>. Let  $V_1$  and  $V_2$  be two nonempty collections of subsets of a space X. Then  $V_1$  is an ideal in  $V_2$  if and only if the following two conditions hold:

1. The collection  $V_1$  is contained in the collection  $V_2$ .

2. Let the set N be in  $V_1$  and the set M in  $V_2$ . If M is a subset of N then M is in  $V_1$ .

Denote by M(V,R) the space of all V-measurable functions f:  $X \rightarrow R$ , where  $R = (-\infty, \infty)$ . Now let B represent the collection of all sets that are complements of sets in V. Notice that B is a filter base.

<u>THEOREM</u>. Let  $V^a$  be the smallest sigma algebra extending V. Now let W be any sigma algebra extending V such that V is an ideal in W. Then a function f in M(W,R) belongs to M( $V^a$ ,R) if and only if the limit of f with respect to the filter base B exists and is finite. (Received March 20, 1975.)

75T-B132 WAYNE C. BELL, North Texas State University, Denton, Texas 76203. <u>An absolutely</u> <u>continuity condition for finitely additive set functions</u>.

Setting and notions are as in previous abstracts of W. D. L. Appling. Theorem 1: If  $\xi$  and  $\mu$  are in  $P_a^+$  and  $\mu(v) = 0$  implies  $\xi(v) = 0$  then  $\xi \in \varphi_{\mu}$  iff  $S_{\mu}^{+} \subseteq M_{\xi}$ . The following theorem of Appling's (two inclusion theorems for real variable summable set functions. Portugaliae Math., Vol. 29, 1970, pp. 101-111) is proved without the uniform boundedness principle: Theorem 2: If  $\xi$  and  $\mu$  are in  $P_A^+$  and  $\xi \in \varphi_{\mu}$  then  $\xi \in Lip(\mu)$  iff  $S_{\mu}^{+} \in S_{\xi}$ . (Received March 24, 1975.)

*75T-B133 V.K. KRISHNAN, St. Thomas College, Trichur, Kerala, India. Gap Tauberian theorems for logarithmic summability (L).

A series  $\sum_{n=0}^{\infty} a_n$  or, equivalently, the sequence  $A_n = a_0 + \ldots + a_n$ ,  $n = 0 \ 1, 2, \ldots$ , is said to be summable (L) to A if  $L(x) \equiv -\frac{1}{\log(1-x)} \sum_{n=0}^{\infty} \frac{A_n}{n+1} x^{n+1} + A$  as  $x \neq 1 \neq 0$ , it being assumed that the series defining L(x) converges for  $0 \le x \le 1$ . It is shown that if the series  $\Sigma a_n$  is summable (L) to A and if  $a_n = 0$  for  $n \neq n_k$  where  $\{n_k\}$  is a sequence of positive integers greater than 2 with log log  $n_{k+1}$  - log log  $n_k \ge \delta > 0$ , k =

1,2, ..., then  $\Sigma$  a converges to A. (Received March 24, 1975.) (Author introduced by

M. S. Ramanujan.)

75T-B134 HUGO JUNGHENN, The George Washington University, Wash.,D.C. 20006. <u>Some general</u> results on fixed points and invariant means, Preliminary report.

Let S be a semitopological semigroup, C(S) the space of all bounded continuous  $f: S \rightarrow R$ , and  $r_s$ ,  $l_s$  the right and left translation operators on C(S). Let  $\tau$  be a locally convex topology on C(S) between the pointwise and norm topologies, and let  $F = \{f \in C(S): the convex hull of r_s f \in C(S) : the convex hull of r_s f \in C(S) : the convex hull of r_s f = f \in C(S) : the convex hull of r_s f = f \in C(S)$ is  $\tau$ -relatively compact} . If S x XoX is an affine action of S on a convex compactum X ( i.e. a convex compact subset of a locally convex topological vector space), define  $T_x: C(X) \rightarrow C(S)$ by  $T_xh(s) = h(sx)$  (x  $\epsilon X$ , s  $\epsilon S$ ). Call the action  $\tau$ -affine if x  $\rightarrow T_xh$  is  $\tau$ -continuous for each affine h  $\varepsilon$  C(X). THEOREM. F is a right translation invariant left introverted norm closed linear subspace of C(S) containing the constant functions. Moreover if F is left translation invariant then F has a left invariant mean (LIM) iff every au -affine action of S on a convex compactum has a fixed point. From this theorem may be deduced many of the known theorems, and several new ones, relating fixed points of actions to invariant means on function spaces. As an example we have: COROLLARY. WAP(S) has a LIM iff every separately continuous quasiequicontinuous affine action of S on a convex compactum has a fixed point. (An action  $SxX \rightarrow X$ of S on a uniform space X is <u>quasi-equicontinuous</u> if the family of mappings  $x \rightarrow sx$  (s  $\epsilon$  S) of X into itself is quasi-equicontinuous in the sense of Arzela ( see Dunford and Schwarz, Linear Operators I, Interscience, N.Y., 1958, p. 269)). Similar results are obtained for the case of multiplicative invariant means. (Received March 28, 1975.)

75T-B135 S.M. SHAH, University of Kentucky , Lexington, Kentucky 40506 Polynomial Approximation and Entire Functions II

This abstract is in continuation of one with the same title in Notices, April 75, p.A-387. The functions  $\mathfrak{G}(\mathbf{x})$ ,  $\beta(\mathbf{x})$ ,  $f(\mathbf{x})$ ,  $E(\mathbf{f})$  and  $F(\mathbf{x},\mathbf{c})$  satisfy the conditions there. Theorem 1. Let f(z) be entire and  $\mu(\mathbf{r})$  the maximum term and  $\nu(\mathbf{r})$  the rank. Then

(*) 
$$\lim_{r \to \infty} \inf \frac{\alpha(\log M(r))}{\beta(\log r)} = \lim_{r \to \infty} \inf \frac{\alpha(\log \mu(r))}{\beta(\log r)} = \lim_{r \to \infty} \inf \frac{\alpha(\nu(r))}{\beta(\log r)};$$

and similar equalities hold when  $\lim_{n\to\infty} \inf \alpha(n)/\beta(\frac{1}{n} \log |\frac{1}{a_n|} \leq \lambda (\alpha,\beta,f)$  where  $\lambda(\alpha,\beta,f)$  is the generalized

order of f defined by (*). (ii) Assume that  $|a_n/a_{n+1}|$  is ultimately a non-decreasing function of n. Then there is equality sign in (i). Theorem 3. Let  $f(x) \in \mathbb{C}[-1, 1]$  and suppose that f(x)is not a polynomial and  $(\mathbb{E}_n(f))^{1/n} \to 0$  as  $n \to \infty$ . Then f(x) is the restriction of an entire function f(z) and

$$\begin{split} \lambda(\alpha,\beta,f) &= \sup_{\substack{n \\ n_k \\$$

where supremum is taken over all sequences  $\{n_k\}$  of all strictly increasing positive integers. (Received April 3, 1975.)

#### 75T-B136 RICHARD F. BASENER, Lehigh University, Bethlehem, Pennsylvania 18015 Nonlinear Cauchy-Riemann equations, Preliminary Report.

<u>Definition</u>. Let  $\Omega$  be an open subset of  $\boldsymbol{\xi}^n$ , and let  $f \in C^{\infty}(\Omega)$ . We say that f is <u>q-holomorphic</u> on  $\Omega$  if for each  $\mathbf{x} \in \Omega$  the rank of the matrix

$$\begin{pmatrix} f_{\overline{z}_1}(\mathbf{x}) & f_{\overline{z}_1 z_1}(\mathbf{x}) & \cdot & \cdot & f_{\overline{z}_1 z_n}(\mathbf{x}) \\ \cdot & & \cdot & \cdot & f_{\overline{z}_n z_1}(\mathbf{x}) & \cdot & \cdot & f_{\overline{z}_n z_n}(\mathbf{x}) \end{pmatrix}$$

is at most q. We say that f is <u>holomorphic in codimension q</u> if locally there exist new holomorphic coordinates  $w_1, \ldots, w_n$  so that f is holomorphic in  $w_1, \ldots, w_{n-q}$ . Holomorphic in codimension q implies q-holomorphic; partial converses can be obtained. <u>Theorem</u>. Let  $\Omega$  be a smoothly bounded, bounded open subset of  $\xi^n$ .  $\Omega$  "convex wrt the family of functions holomorphic in codimension q" implies  $\partial \Omega$  is q-pseudoconvex. A "local converse" is true under the assumption of strict q-pseudoconvexity of  $\partial \Omega$ . (Received April 4, 1975.) *75T-B137

Theorem 1 Let X be a Banach space. Let D be open bounded subset of X. Let  $F_1, F_2$ :  $\overline{D} \to X$  be two densifying mappings such that  $||F_1(x)-F_2(x)|| \leq ||F_1(x)-x|| \quad \forall x \in \partial D$ . Then if deg  $(I-F_2, D, 0)$  is defined so is deg $(I-F_1, D, 0)$  and deg $(I-F_2, D, 0) = deg(I-F_1, D, 0)$ . Furthermore if deg $(I-F_2, D, 0) \neq 0$ , then  $F_1$  has a fixed point. Theorem 2 Let X be a Banach space. Let D be bounded open subset of X. Let  $F_1, F_2 : \overline{D} \to X$  be densifying mappings satisfying  $||F_1(x) - x|| \leq F_2 ||x||$ . Furthermore suppose that deg $(I-F_1, D, 0) \neq 0$ . Then there exists an  $x \in \overline{D}$  such that  $F_1(x) + F_2(x) = x$ . (Received April 4, 1975.) (Author introduced by Dr. S. P. Singh.)

*75T-B138 Lawrence Fialkow, Western Michigan University, Kalamazoo, Mich. A note on direct sums of quasinilpotent operators.

*75T-B139 Andre de Korvin, Indiana State University, Terre Haute, Indiana 47809 and Mr. Vo Van Tho, Indiana State University, Terre Haute, Indiana 47809 Strong and Weak Non Linear Integration of Totally Measurable Functions.

For notations and definitions see [Abstract 75T-B90, these Notices 22(1975) A-387]. Let  $M_E$ denote totally measurable functions. A measurable function f with values in E is said to be strongly m integrable if there exists a sequence  $f_n \in M_E$  such that  $\sup ||f_n|| < \infty$  and  $f_n \to f m$  a.e. We define (s)  $/fdm = \lim (s) /f_n dm$  Theorem Assume m is v.s.r. and  $sv(m_{\alpha}, )$  and  $sv[m_{y^{*}\alpha^{*}}]$  are countably subadditive for any  $\alpha > 0, y^{*} \in F^{*}$ . Every bounded measurable function has a strong integral. If f is in  $M_E$ , f is called weakly integrable if there is  $y \in F$  such that  $\langle y, y^{*} \rangle = (s) / fdm_{y^{*}}$ . We write y = (w) / fdm. Theorem If  $f_n$  converges to f in measure, under some conditions (w) / f dm + (w) / fdm. (Received April 7, 1975.) 75T-B140 TSAI-SHENG LIU, University of Oklahoma, Norman, Oklahoma 73069. Oscillation of

bounded solutions of even order differential equations with retarded arguments. Preliminary report. We are concerned with the equation

(1)  $y^{(n)}(t) - \int_{j=1}^{n} p_j(t)y(g_j(t)) = 0$ ,  $(n \ge 2$  an even integer), where (i)  $g_j(t) \le t$ ,  $g'_j(t) > 0$ ,  $g_j(t) \rightarrow \infty$  as  $t \rightarrow \infty$  and  $g_k(t) < t$  for some  $1 \le k \le m$ ,  $t \ge t_0$ ; (ii)  $p_j(t) \ge 0$ ,  $p'_j(t) \le 0$  and  $p_k(t) > 0$  for the same k as in (i),  $t \ge t_0$ . An extensible solution of (1) is called <u>oscillatory</u> if it has no last zero, otherwise it is called <u>non-oscillatory</u>. Lemma 2 in the paper [Kiguradze, "The problem of oscillation of solutions of non-linear differential equations," Differential Equations, (8)1(1965), 773-782] is employed to extend to higher order equations for Theorems 2.1-2.4 due to Ladas and Lakshmikantham ["Oscillations caused by retarded actions," Applicable Anal. 4(1974), 9-15]. <u>Theorem</u>. If  $(t - g_k(t))^n p_k(t) \ge n!$  for all sufficiently large t, then every extensible bounded solution of (1) is oscillatory. Some examples are given. (Received April 7, 1975.) A-466 75T-B141 JAMES V. PETERS, Stevens Institute of Technology, Hoboken, New Jersey 07030. Phrägmen-Lindelöf indicators for Radon transforms. Preliminary report.

A result analogous to the classical Paley Wiener theorem is obtained for the Radon transform on  $\mathbb{C}^n$  in terms of analytic multipliers. Specifically, for each point  $z_0 \in \mathbb{C}^n$ , a function  $F_{z_0}$  is associated with the Radon transform f of a continuous, integrable function f. The support of  $F_{z_0}$  is shown to be a cone and the entire functions bounded on this cone are termed bounded analytic multipliers. Theorem 1. A point  $z_0$  is interior to the convex hull of the support of f iff the only bounded analytic multipliers of  $f_{z_0}$  are constants. Theorem 2. If  $f \in \mathbb{C}^{m+1}$  locally then  $f \in \mathbb{C}^m$  locally. This result is sharp. Using this theorem, the singular support of f is determined up to its convex hull. (Received April 10, 1975.)

75T-B142 S.M. SHAH, University of Kentucky, Lexington, Kentucky 40506 and HERB SILVERMAN, University of Delaware, Newark, Delaware 19711 Entire Functions with Maximum Deficiency Sum and their Means.

Let f(z) be an entire function of finite order and write  $M_0(r) = \exp(T(r))$ ,  $M_s(r) = \{\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^s d\theta\}^{1/s}$  (0 < s <  $\infty$ ). Theorem 1. If f has the maximum deficiency sum  $\sum_{a\neq\infty} \delta(a,f)=1$ , then (*)  $\lim_{r\to\infty} (M_0(r)^{(\pi-\varepsilon)}/M_s(r)=0$  (0 < s <  $\infty$ ). The result is best possible in that  $\pi-\varepsilon$  cannot be replaced by  $\pi$ . Theorem 2. Let  $\rho$  be a positive integer and  $f_i(z)=z^{k_i} \exp(Q_i(z))P_i(z)$  where  $k_i$  is a non-negative integer,  $Q_i(z)=a_iz^{\rho}+b_iz^{\rho-1}+\ldots$ ,  $P_i(z)$  the canonical product such that log  $M(r,P_i)=o(r^{\rho})(i=1,2)$ . Let  $f=f_1+f_2$  and suppose that  $|a_2| < |a_1|$  and that either  $a_2=0$  and  $f_2$  does not reduce to a constant or arg  $a_2=arg a_1$ . Then  $\sum_{a\neq\infty} \delta(a,f)=1-\frac{|a_1-a_2|}{|a_1|} < 1$  and (*) holds. Theorem 3 gives functions for

which lim sup  $M_{o}(r)/M_{s}(r)=1$ . (Received April 10, 1975.)

*75T-B143 Andre de Korvin, Indiana State University, Terre Haute, Indiana 47809 Integration of Measures. Preliminary Report.

#### *75T-B145 J.P. PENOT Université de Pau, B.P. 290, 64016 PAU, France. A vectorial form of Gronwall's lemma

The following generalization of Gronwall-Bellman's lemma is related to the study of ordinary differential equations on closed convex subsets of Banach spaces. Theorem : Let (E,<) be an ordered Banach space,  $T = [0, \theta]$  be an interval of R, a :  $T \rightarrow L(E,E)$ , b :  $T \rightarrow E$  be integrable functions. Suppose that a(t).  $p \ge 0$  for almost all  $t \in T$  and every  $p \ge 0$ . Then every continuous map  $x : T \rightarrow E$  such that  $x(t) \le \int_0^t a(s).x(s)ds + b(t)$  satisfies the estimate  $x(t) \le y(t) + b(t)$ , where  $y : T \rightarrow E$  is the solution of the differential equation  $\dot{y}(t) = a(t).y(t) + a(t).b(t)$ , y(0) = 0. In particular, if  $b(t) = \int_0^t c(s)ds + b_o$ , with  $c : T \rightarrow E$  integrable, x satisfies the estimate  $x(t) \le z(t)$ , where z is the solution of  $\dot{z}(t) = a(t).z(t) + c(t)$ ,  $z(0) = b_o$ . When a(t)oa(s) = a(s)oa(t) for every  $t, s \in T$  one has the analogue of the usual estimate :  $x(t) \le exp(A(t)).b_o + \int_0^t exp(A(t)-A(s)).c(s)ds$  with  $A(t) = \int_0^t a(s)ds$ . Some comparison results can also be given for nonlinear differential equations in ordered Banach spaces. (Received April 15, 1975.)

*75T-B146 H. GUGGERHEILER, Polytechnic Institute of New York, Brooklyn NY 112Cl. Differentiability of the conjugate point.

Let  $L_n [x] = 0$  be a linear DE for which the conjugate point on (a, b)if  $\eta(t) = z_{j,n-j}(t)$  in Barrett's notation. Let X be a fundamental vector of volutions of  $L_n = 0$ , and put  $i = \min(j, n-j)$ . Theorem:  $\eta(t)$  is  $\sigma^{i+1}$  on (a,b) and  $-\eta'(t) =$ det  $(X(t), \dots, X^{(j-2)}(t), X^{(j)}(t), X(\eta(t)), \dots X^{(n-j-1)}(\eta(t))$ det  $(X(t), \dots, X^{(j-1)}(t), X(\eta(t)), \dots, X^{(n-j-2)}(\eta(t)), X^{(n-j)}(\eta(t))$ for n = 2 where  $\eta = z_{11}$ , a result of Borûvka asserts that  $\eta$  is  $\sigma^3$ if L[x] = x'' + p(t)x. If  $L[x] = x^{(n)} + p(t)x$  then  $\eta$  is  $\sigma^n$ 

excert at points t at which  $z_{j,n-j} = z_{s,n-s}$  for  $j \neq s$ .

(Received April 17, 1975.)

### *75T-B147 JOHN TRIPP, Claremont Graduate School, Claremont, California 91711. Constructing Multiplications on Banach Spaces.

This note is concerned with the problem of constructing nontrivial Banach algebras by defining multiplications on a Banach space. The basic technique is to define an injective linear map from the Banach space onto a subalgebra of a topological algebra, and to let the Banach space take on the multiplication of the topological algebra. <u>Theorem 1</u>. Let B be an infinite dimensional separable Banach space. Then B can be given a multiplication so that it becomes a radical Banach algebra of power series of any finite or countably infinite number of indeterminates, commutative or not; a radical Banach algebra of Dirichlet series; or a radical Banach algebra of strictly lower triangular matrices, containing all strictly lower triangular matrices with a finite number of nonzero entries. <u>Theorem 2</u>. Let B be an infinite dimensional Panach space with an unconditional Schauder basis. Then B can be given a multiplication so that it becomes a Banach algebra of lower triangular ratrices, containing all lower triangular matrices with a finite number of nonzero entries. <u>Theorem 2</u>. Let B be an infinite dimensional Panach space with an unconditional Schauder basis. Then B can be given a multiplication so that it becomes a Banach algebra of lower triangular ratrices, containing all lower triangular matrices with a finite number of nonzero entries. (Received April 18, 1975.)

#### *75T-B148 IRA N. ROSENHOLTZ, University of Wyoming, Laramie, WY 82071 On A Fixed Point Problem of D.R. Smart

In his book, Fixed Point Theorems, D.R. Smart poses the following problem which he says appears to be open: "Does every shrinking (i.e. contractive) mapping of the closed unit ball in a Banach space have a fixed point?" We answer this question in the negative by exhibiting a contractive mapping from the closed unit ball in a Banach space to itself which has no fixed point. Furthermore, our mapping has the additional properties that it is affine, a homeomorphism onto its image, and its inverse is Lipschitz. (Received April 18, 1975.)

SCOTT WOLPERT, Stanford University, Stanford, California 94305. 75T-B149 Non Completeness of Weil-Petersson Metric for Teichmüller Space . Preliminary report.

Let T(p,n) be the Teichmüller space of a compact Riemann surface of genus p with n punctures, where 2p-2+n > 0 (the signature  $(p,n; \gamma_1, \ldots, \gamma_n)$  need only satisfy  $2p-2+\sum_{i}(1-r_{i}^{-1})>0$ ). It is shown that the Weil-Petersson metric is not complete. A path of finite length tending to the boundary is constructed by "pinching" a given surface of type (p,n). A theorem of Jenkins allows one to exhibit the quadratic differentials which are dual to the tangents of this curve, thus enabling estimates on the metric. A partial consequence is that the Weil-Petersson metric is not equivalent to either the Caratheodory or Kobayashi-Teichmüller metrics which are complete. Details will appear elsewhere. It has been communicated to the author that Mr. T.C. Chu of Columbia University has found similar results. (Received April 18, 1975.)

#### Frank N. Huggins, University of Texas, Arlington, Texas 76019 75T-B150 Bounded Slope Variation and Generalized Convexity

The relationship between the class of functions which have bounded slope variation with respect to an increasing function m over [a,b] and the class of functions which are convex with respect to m on [a,b] is investigated. Some representative results are: LEMMA. If F is the Lane integral of a nondecreasing function f with respect to an increasing function m on [a,b], then F is convex with respect to m on [a,b]. THEOREM. If m is a continuous increasing function on [a,b], then the function F has bounded slope variation with respect to m over [a,b] if and only if F = G - H on [a,b] where each of G and H is convex with respect to m on [a,b] and has bounded slope variation with respect to m over [a,b]. (Received April 21, 1975.)

G.FREUD, Ohio State University, Columbus, Ohio 43210 and 75T-B151 A.R. REDDY, Michigan State University, East Lansing, Michigan 48824 Rational Approximation

We announce here the following: THEOREM 1: There is a polynomial  $P_n^*(x)$  of degree at most n for which for all large n

$$\left\|e^{-\left|x\right|} - \frac{1}{P_{n}^{\star}(x)}\right\|_{L_{\infty}(-\infty,\infty)} \leq (C_{1}\log n)n^{-1}$$

THEOREM 2: For every polynomial  $P_n(x)$  of degree at most n, and all large n

$$\left|e^{-\left|x\right|} - \frac{1}{P_{n}(x)}\right|_{L_{\infty}(-\infty,\infty)} \geq C_{2}n^{-1}$$

THEOREM 3: There is a rational function  $r_n^{*}(x) = \frac{P_n(x)}{Q_n(x)}$  of degree at most n, for which for all large n

$$\left\|e^{-\left|x\right|} - r_{n}^{\star}(x)\right\|_{L_{\infty}(-\infty,\infty)} \leq e^{-C_{3}\sqrt{n}}$$

THEOREM 4: For every rational function  $r_n(x)$  of degree at most n and all large n  $-C_{A}\sqrt{n}$ le

$$|\mathbf{x}| - \mathbf{r}_{n}(\mathbf{x}) \|_{\mathbf{L}_{\infty}(-\infty,\infty)} \ge e$$

 $C_1, C_2, C_3$ , and  $C_4$  are suitable positive constants. (Received April 21, 1975.)

*75T-B152 R. S. DAHIYA, Iowa State University of Science and Technology, Ames, Computation of two-dimensional Laplace transforms. Iowa 50010 Theorem. Let (i)  $F(p_1, p_2) \stackrel{:}{=} f[x_1+1)^{1/n_1}$ ,  $(x_2+1)^{1/n_2}]$ , valid in a pair of associated half planes  $H_{p_1}$  and  $H_{p_2}$  'say' and  $R(n_1, n_2) > 0$ . (ii)  $P_i^{k} \exp(-\lambda_i \psi_i(p_i)) \{\psi_i(p_i)\}^{\ell} \stackrel{:}{=} h_i(\lambda_i, x_i)$ , where  $h_i(\lambda_i, x_i)$  is a continuous function of  $x_i(i=1,2)$ , in a  $(0 \le x_i \le x_i)$  or in  $(0,\infty)$ . (iii)  $\frac{F(x_1, x_2)}{x_1 x_2}$  is integrable L in  $(0,\infty)$  or in  $(0 \le x_i \le x_i)$ . Then (1)  $\frac{p_1 p_2}{n_1 n_2} \int_{0}^{\infty} \int_{0}^{\infty} h_1(p_1, x_2) h_2(p_2, x_2) \frac{F(x_1, x_2)}{x_1 x_2} dx_1 dx_2$  $\stackrel{:}{=} \frac{(x_1 x_2)^{\ell} \exp(n_1 \phi_1(x_1) + n_2 \phi_2(x_2)) f[\exp(\phi_1(x_1)), \exp(\phi_2(x_2))] \phi_1'(x_1) \phi_2'(x_2)}{\{(\exp(n_1 \phi_1(x_1) - 1)(\exp(n_2 \phi_2(x_2) - 1)\}^{1-k}\}$ 

Provided  $\phi_1(x_1)$  and  $\phi_2(x_2)$  are two strictly monotonic and continuous functions of  $x_1$  and  $x_2$ , tending to zero and infinity with  $x_1$ . The above theorem is used to obtain several Laplace transforms and integrals. (Received April 21, 1975.)

### **Applied Mathematics**

75T-C30 Roberto Triggiani, State University of New York at Albany, <u>On the implication: controllability ⇒ stabilizability in Banach space</u>

The model is a linear system defined on Banach (state and control) spaces, with the operator acting on the state, only the infinitesimal generator of a C_o semigroup. The stabilizability problem of expressing the control through a bounded operator acting on the state as to make the resulting feedback system globally #ymptotically stable is considered. M. Slemrod has recently shown (SIAM J.Cont. Aug 74), for the wave equation case, that dense controllability implies stabilizability, an extension of the classical result for finite dimensional systems. In contrast with the finite dimensional theory we give a few counter-examples of systems which are densely controllably in the space and yet are not stabilizable, even if some further 'nice properties' hold. Use is made of the notion of essential spectrum and its stability under relatively compact perturbations. On the positive side it is shown however that for classical selfadjoint boundary value problems, retarded systems etc. controllability on a suitable <u>finite</u> dimensional subspace still yields stabilizability on the whole space. (Received January 30, 1975.)

### *75T-C31 VIJAY B. AGGARWAL. University of Vermont, Burlington, Vermont 05401 Hierarchy of Group-free automata under series-parallel decomposition.

One can construct any group-free finite automaton as a series-parallel interconnection of the two-state identity-reset automata. The decomposition of group-free automata in which capabilities are preserved is discussed. An infinite hierarchy of properly-included classes

 $\mathfrak{n}_0 \subset \mathfrak{n}_1 \subset \mathfrak{n}_2 \subset \ldots \subset \mathfrak{n}_n \subset \ldots$ 

of group-free semiautomata is obtained such that no member of  $\P_{i+1}^{-\P_i}$  can be constructed as series-parallel connections of members of  $\P_i$  alone. In each class  $\P_{i+1}^{-\P_i}$  a semiautomaton  $\overline{U}_{i+1}$  is obtained such that every member of the class has the capability of  $\overline{U}_{i+1}$ . Applications of these results to the Krohn-Rhodes theory are discussed. (Received February 24, 1975.) (Author introduced by Robert K. Wright.)

*75T-C32 CHARLES E. BLAIR, Mathematics Dept., Carnegie-Mellon University, Pittsburgh, Pa. 15213. <u>Minimal Inequalities for Mixed-Integer</u> <u>Problems</u>

Let Q = the rationals,  $q_1, \ldots q_n \in Q^k$ . If  $w \in R^k$ 

let  $\mathbf{S}_{\mathbf{w}} = \{ (\mathbf{t}_1, \dots, \mathbf{t}_n) \mid \sum_{i=1}^{n} \mathbf{t}_i \mathbf{q}_i = \mathbf{w} \quad \mathbf{t}_i \geq 0 \quad 1 \leq i \leq n \quad \mathbf{t}_i \text{ integer } 1 \leq i \leq m \leq n \}$ Suppose every member of  $S_v$  satisfies (I)  $\sum_{i=1}^{n} a_i t_i \ge K$  and that (I) is minimal. (i. e., if we replace  $a_i$  by  $a_i' < a_i$  or replace K by K' > K (I) is no longer true for every member of S.,.)  $F(w) = \min \sum_{i=1}^{n} a_i t_i$ , subject to  $(t_1, \dots, t_n) \in S_w$ . Then Theorem: Let (1) For  $1 \le i \le m$ ,  $F(v-q_i) + F(q_i) = F(v)$ (2) For m+1  $\leq i \leq n$ , there is an  $\alpha > 0$  F(v- $\alpha q_i$ ) + F( $\alpha q_i$ ) = F(v). (Received March 12, 1975.) R.G.JEROSLOW, Carnegie-Mellon University, Pittsburgh, Pa., 15213. *75T-C33 Minimal Inequalities. Let the constraint set for a consistent mixed integer program  $\sum_{a \in A} mt + \sum_{b \in B} p_{a r} r = d$ (P)  $t_a, r_b \ge 0 \text{ and } t_a \text{ integer } (a_eA, b_eB)$ be given, with the  $m_a, p_b, d_eR^m$  (possibly  $A \cup B$  is infinite). Let a valid (P) inequality  $\begin{array}{ccc} (\text{CUT}) & & & & \sum \pi_a t_a & + \sum \sigma_b r_b \geq \pi_0 & ( \ \pi_a, \sigma_b & e \ \text{R} \cup \left\{ \ - \ \infty \right\}^2, \ \pi_0 e \text{R} ) \\ \text{be given. We say that (CUT) is <u>minimal</u>, if no <math>\pi_a \text{ or } \sigma_b$  can be decreased, nor can  $\pi_0$  be increased, still retaining validity of the inequality. Let G(d) be the infimum of the left-hand-side of (CUT) as a function of d, for d with (P) consistent. Lemma: G is subadditive. Theorem: Suppose that there are bounds  $T_a$  on  $t_a$  and  $R_b$  on  $r_b$ ,  $a_{{\mathfrak g}}A,\ b_{{\mathfrak g}}B,$  for any solution to (P). Then for (CUT) to be minimal, it is necessary and sufficient that: (1) If  $\pi_a > -\infty$ , then  $G(d - m_a)$  is defined and  $G(m_a) + G(d - m_a) \neq G(d)$ .

(2) If  $\sigma_{\overline{b}} > -\infty$ , then for all sufficiently small  $\delta > 0$ ,  $G(d - \delta p_b)$  is defined and  $\overline{G}(p_b) - \lim \left\{ G(d) - G(d - \delta p_b) \right) / \delta \mid \delta \rightarrow 0 + \left\} \cdot$  (3)  $G(d) = \pi_0, G(m_a) = \pi_a, \overline{G}(p_b) = \sigma_b, a_{\delta}A, b_{\delta}B.$ <u>Remark:</u> If  $A \cup B$  is finite, then the equation in (2) becomes

 $G(\delta p_b) + G(d - \delta p_b) = G(d)$ . (Received March 17, 1975.)

75T-C34 EDUARDO D. SONTAG, Ctr.for Math.Syst.Theory, Dept.Math., U.of Florida, Gainesville Fl 32611. <u>Matrix-Fatou rings and projective modules</u>. Preliminary report.

Let R be an integral domain with quotient field Q. DEFINITION. R is <u>matrix-Fatou</u> if for every  $F \in Q^{m \times n}$ ,  $G \in Q^{m \times n}$ , and  $H \in Q^{n \times p}$  for which  $GF^{i}H \in R^{m \times p}$ ,  $i = 0, 1, \ldots$ , there exist  $A \in R^{n \times n}$ ,  $B \in R^{m \times n}$ , and  $C \in R^{n \times p}$  such that  $GF^{i}H = BA^{i}C$  for all i. This is a strong form of the descent problem for linear systems (c.f. Rouchaleau and Wyman [J.Comp.Syst.Sc. 2(1974):129-142]). THEOREM 1. Assume that R is matrix-Fatou and P is a f.g. projective R-module; then P is free. THEOREM 2. Assume that R = A[X], where A is a principal ideal domain; then R is matrix-Fatou. The second result gives the first examples of matrix-Fatou rings which are not principal ideal domains. (Received March 28, 1975.)

*75T-C35 FRANCO P. PREPARATA and DAVID E. MULLER, University of Illinois, Urbana, Illinois

#### 61801. Parallel evaluation of division-free expressions.

The problem of the parallel evaluation of division-free arithmetic expressions is investigated, under the assumption that a sufficiently large number of processors is available. A given arithmetic expression involving only addition, multiplication and |E| distinct variables (a primitive expression) is constructively restructured so that the depth of the resulting computation tree is no greater than  $\log |E|/\log \beta$  where  $\beta$  is the positive real root of the equation  $z^4 = 2z + 1$ , giving  $1/\log_2\beta = 2.0806...$  This shows that if the operations of addition and multiplication take unit time, E can be evaluated in at most  $2.0806 \log_2|E|$  steps. We also consider a family  $\{E_j\}$  of primitive expressions, where the computation tree T_j of E_j is recursively defined by T_j = T_{j-3}(T_{j-3}T_{j-4} + T₀) + T₀; E_j can be evaluated in j steps by our algorithm and  $|E_j|$  grows as  $c\beta^j$ , for some constant c. We formulate the conjecture that the evaluation of E_j cannot be further sped-up by algebraic manipulations; this conjecture suggests that a lower-bound to the evaluation time of certain division-free expressions is 2.0806  $\log_2 |\mathbf{E}|$  - c', where c' is a constant. (Received March 31, 1975.) (Authors introduced by Professor James Armstrong.)

### *75T-C36 T. ERBER, Illinois Institute of Technology, Chicago, Illinois 60616. <u>Čerenkov-Magnetobremsstrahlung</u>

High energy electrons traversing intense magnetic fields in the presence of a (tenuous) medium with index of refraction n = 1 +  $\Delta$ n,  $|\Delta$ n| << 1 radiate by means of a synergic Cerenkov-brems-strahlung process. The spectral intensities are

$$\frac{\alpha\sqrt{3}}{2\pi}\frac{c}{\lambda_{c}}\frac{H}{H_{cr}}\left[1-2\Delta n\left[E/mc^{2}\right]^{2}\right]^{-1/2}\kappa(z) \quad ; \quad z = \frac{2\hbar\omega}{3TE}\left[1-2\Delta n\left[E/mc^{2}\right]^{2}\right]^{3/2}$$

$$\frac{\alpha c}{\lambda_{c}} \frac{\left(\frac{\pi \omega}{E} m c^{2}\right)^{1/2}}{E} \frac{H}{H_{cr}} \left[ -2 \operatorname{Ai}^{\prime}(-\hat{z}) + \hat{z} \left(\frac{1}{3} + A\hat{i}(\hat{z})\right) \right] \quad ; \quad \hat{z} = \left(\frac{\pi \omega}{E}\right)^{2/3} \left[ 2\Delta n \left[ E/mc^{2} \right]^{2} - 1 \right]$$

in appropriate ranges on either side of the threshold. For notation see Rev. Mod. Phys. <u>38</u>, 626-529 (1966). (Received April 8, 1975.)

#### *75T-C37 GAIL A. CARPENTER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 Periodic Solutions of the Hodgkin-Huxley Equations of Nerve Impulse Transmission

A nonlinear diffusion equation coupled with k "fast" and  $\ell$  "slow" variables models excitable membranes such as nerve and muscle fibers. Isolating block techniques give sufficient conditions for the existence of periodic traveling wave solutions of such a system provided that the time constants of the fast variables are sufficiently large and the time constants of the slow variables are sufficiently small. The Hodgkin-Huxley equations (k = 1,  $\ell$  = 2) satisfy these conditions. (Previous results provide periodic solutions only if the ratio of the two slow variables is large or small.) (Received April 10, 1975.)

#### *75T-C38 STEPHEN GROSSBERG, M.I.T., Cambridge, Massachusetts 02139 Adaptive Pattern Classification and Universal Recoding, Preliminary Report.

Theorems are proved concerning how training with spatial patterns causes adaptive classification of these patterns into convex regions. Moreover, given any fixed k  $\geq 2$ , m  $\geq 2$ , and n  $\geq 1$ , it is shown how any n patterns in R^k can be recoded into any n patterns in R^m in one learning trial. An expectation mechanism is needed to facilitate search for uncommitted classifying vectors. The simplest learning mechanism (excluding expectation) is  $z_{ij} = (-z_{ij} + \theta_i)e_j$ , where  $\theta_i = I_i(\sum_{r=1}^k I_r)^{-1}$  and  $e_j = 1$  if  $\sum_{r=1}^k \theta_r z_{rj} > \max_{p \neq j} \sum_{r=1}^k \theta_r z_{rp}$ , 0 otherwise. This system simplifies the concept

of an adapting retina sending signals via cross-correlational synapses to an adapting, contrast enhancing cortex capable of STM. (Received April 9, 1975.)

### Geometry

*75T-D4 KINETSU ABE and JOSEPH ERBACHER, University of Connecticut, Storrs, Connecticut 06268. <u>Contact structures on the generalized Brieskorn manifold</u>.

We announce the following:

<u>Theorem 1</u>. All the generalized Brieskorn manifolds admit normal contact structures. Furthermore, all of them have closed curves as the leaves of their associated foliations, and most of them are non-regular.

<u>Theorem 2</u>. All the odd dimensional standard spheres and all the odd dimensional exotic spheres bounding parallelizable manifolds admit infinitely many seemingly different contact structures such as described in Theorem 1. In particular, all the odd dimensional standard spheres admit infinitely many mutually distinct contact structures. Here two contact structures are the same if there exists a contact transformation between them.

<u>Theorem 3</u>. Let  $P_1(Z_0, Z_1, Z_2) = Z_0 + Z_1 + Z_2^2$  and  $P_2(Z_0, Z_1, Z_2) = Z_0 + Z_1 + Z_2^2$  be two Brieskorn polynomials. Then  $P_1$  and  $P_2$  induce contact structures on  $S^3$  such as in Theorem 1. These structures are distinct if  $(\frac{a_1}{(a_1, a_2)}, \frac{a_2}{(a_1, a_2)}) \neq (\frac{b_1}{(b_1, b_2)}, \frac{b_2}{(b_1, b_2)})$ , where (i,j)

denotes the greatest common divisor of i and j . (Received March 13, 1975.)

*75T-D5 ROBERT E. JAMISON, Louisiana State University, Baton Rouge, Louisiana 70803. A General Theory of Convexity

By means of the notion of an alignment on a space -- a family of distinguished subsets of the space satisfying two set theoretic properties of convex sets: closure under (1) arbitrary intersection and (2) nested union -- the foundations are laid for a general theory of convexity broad enough to encompass formulations of convexity in linear spaces, metric spaces, semigroups, modules over rings, and so forth. Although some combinatorial results on ceparation of convex sets are proved, the main emphasis is placed on the continuity of the hull operator induced by an alignment on the subsets of a compact  ${\mathbb T}_{\phi}$  space. The paper gives a range of examples of continuous alignments and provides some criteria for the continuity of alignments induced by continuous functions or by restriction to subspaces. The interplay between the possible topological and alignment properties of the space are also investigated. Sample results include: a) the only continuous alignment on the circle is the one in which all subsets are convex; b) a finite dimensional continuum must be an AR if it possesses a continuous alignment satisfying a Tchebyschev approximation property. (Received March 24, 1975.)

### Logic and Foundations

75T-E36 STEVEN GARAVAGLIA, Yale University, New Haven, Connecticut 06520. <u>Completeness</u> for topological languages. Preliminary report.

In many-sorted logic there is a recursive set  $\Sigma$  of axioms for the class of structures  $(X, T, \epsilon)$ where X is a first order structure, T is a base for a topology on the domain of X, and  $\epsilon$  is the membership relation between elements of the domain of X and elements of T. A sentence  $\alpha$  of many-sorted logic is called invariant iff for all  $(X, T, \epsilon)$  satisfying  $\Sigma$ , if T* is the topology generated by T then  $(X, T, \epsilon)$  $\models \alpha$  iff  $(X, T^*, \epsilon) \models \alpha$ . (1) Given any sentence  $\alpha$  in Sgro's topological language L(Q) [these *cNotices*) 19(1972), A-765] there is an invariant sentence  $\overline{\alpha}$  of many-sorted logic such that for any topological structure (X, T) we have  $(X, T) \models \alpha$  iff  $(X, T, \epsilon) \models \overline{\alpha}$ . Consequently, the completeness, compactness, and LST theorems for L(Q) follow from those for many-sorted logic. (2) The set of invariant sentences is r.e. but not recursive. (Received January 27, 1975.) Let  $\Sigma$  be a universal class with the amalgamation property. A structure  $A \in \Sigma$ is atomic homogeneous if for all  $B_1$ ,  $B_2 \leq A$  if  $B_1$  has a set of generators of power  $\langle |A|$  and if f is an isomorphism of  $B_1$  and  $B_2$  then f lifts to an automorphism g of A. <u>Theorem 1</u> Let A be a countable structure in  $\Sigma$ . Then A has a countable atomic homogeneous extension  $B \in \Sigma$ . <u>Corollary</u> Every countable graph has a countable ultrahomogeneous extension. <u>Theorem 2</u> Suppose that  $\Sigma$  has the property that for all infinite structures  $A \in \Sigma$  the set of isomorphism types of finitely generated extensions of A has power  $\leq |A|$ . Then  $\forall \lambda \geq |A|$  there is an atomic homogeneous extension B of A in  $\Sigma$  of power  $\lambda$ . <u>Definition</u>  $A \in \Sigma$ is maximal if A has no proper extension in  $\Sigma$ . <u>Theorem 3</u> If  $A \in \Sigma$  is maximal then A is atomic homogeneous. (Received February 12, 1975.)

### 75T-E38 S. Shelah, Hebrew University, Jerusalem, Israel, <u>Various results in mathematical</u> logic, Preliminary report

(1) It is consistent with ZFC e.g. that every  $2^{N_0}$ -free abelian group is free(assuming large cardinals). (2) Some of the problems of Friedman of Beth theorem are solved e.g. not weak Beth for  $(L_{\mu+\omega}, L_{\omega,\lambda+})$  when  $\mu = 2^{\lambda^+}$ ; (3) Stability theory is generalized e.g. to the class of generic models of a theory.E.g. we characterize the stability spectrum the order property, and characterize the class of cardinals in which there is a universal-homogeneous model.(4) e.g.  $(\aleph_{\omega}, \aleph_{0}) \rightarrow (2^{\aleph_{0}}, \aleph_{0})$ . (5) e.g. if  $\varphi \in L_{\omega_{1},\omega}$  (Q) is categorical in each  $\aleph_{n}$ , it is categorical in each  $\aleph$ . This is generalized to abstract classes. (6) Stability and categoricity theory is generalized to "over a fixed predicate P". (7) E.g. the functions of the number of non-isomorphic models of a totally transcendental theory are classified. (Received February 20, 1975.)

### *75T-E39 WILLIAM BOOS, 824 East College Street, Iowa City, Iowa 52240. <u>Compactness and</u> <u>indescribability below the continuum</u>. Preliminary report.

$$\begin{split} & \mathsf{c}(\aleph,\lambda) \ \text{for } \lambda \leq \aleph \ \text{means whenever } \Sigma \subseteq \mathsf{L}_{\aleph\lambda} \ \text{has power } \aleph \ \text{and has no model, some } \Gamma \subseteq \Sigma \ \text{of power} \\ & \mathsf{c}(\aleph,\lambda) \ \text{for } \lambda \leq \aleph \ \text{means whenever } \Sigma \subseteq \mathsf{L}_{\aleph\lambda} \ \text{has power } \aleph \ \text{and has no model, some } \Gamma \subseteq \Sigma \ \text{of power} \\ & \mathsf{c}(\aleph, \aleph) \ \text{no model; } [\sigma] < \tau \ \text{ is the set of subsets of } \sigma \ \text{ of order type } < \tau, \ \text{and } \mathbb{B}_{\mu} \ \text{ is the regular open} \\ & \text{algebra of } 2^{\omega \times \mu} \ \underline{Proposition} \ c(\aleph, \aleph) \ \text{holds whenever } \varkappa \ \text{ is ordinal } \Pi_1^2 \ \text{-indescribable (whatever the size of } 2^{\mathfrak{C}}). \ \underline{Proposition} \ \mathbb{F} \text{ or each } \lambda, \varkappa \ \text{with } \lambda \leq \varkappa \ (\lambda = \varkappa), \ \mathsf{c}(\aleph, \lambda) \ \text{holds iff } "[\aleph] \ \overset{<}{\sim} \lambda \ \text{ is } \Pi_1^1 \ \text{-indescribable} \\ & \mathsf{able}", \ \text{i. e., whenever } \varphi \ \text{ is } \Pi_1^1, \ A \subseteq [\varkappa]^{<\lambda}, \ (A)^{=} = \varkappa \ \text{ and } \langle [\varkappa]^{<\lambda}, \ \in, \ A \rangle \models \varphi, \ \text{ there is a } \mu < \varkappa \ \text{ such that} \\ & \langle [\mu]^{<\lambda}, \in, A \cap [\mu]^{<\lambda} \rangle \models \varphi \ (\langle [\mu]^{<\mu}, \in, A \cap [\mu]^{<\mu} \rangle \models \varphi). \ \underline{Proposition}. \ \text{Whenever } n, \varkappa \ \text{ are such that } n \geq 1, \\ & \mathsf{a}(\aleph) = \aleph_{\varkappa + n} = \mu, \ \text{ and } \varkappa \ \text{ is } \Pi_1^{n+1} \ \text{-indescribable, } [\mathbb{C}(\aleph, \aleph)] \ \mathbb{B}_{\mu} = \mathbb{1}. \ \underline{Proposition}. \ \text{Whenever } \lambda < \varkappa < 2^{\lambda} \ \text{ and} \\ & \varkappa \ \text{ ordinal } \Sigma_1^2 \ \text{-indescribable, } (\varkappa \ \text{ is } \alpha \ \text{-indescribable for all } \alpha < \varkappa)^{\overline{\mathbf{L}}}. \ \underline{Proposition}. \ \text{If } n, \varkappa \ \text{ are such that} \\ & \mathsf{tat } n \geq 1, \ \neg_n(\aleph) = \aleph_{\varkappa + n} = \mu^+ \ \text{ and } \varkappa \ \text{ is } \mu \ \text{-supercompact, } [\aleph \ \text{ is ordinal } \Pi_1^{n+1} \ \text{-indescribable}] \ \mathbb{B}_{\mu}^{-1} = \mathbb{1}. \end{aligned}$$

*75T-E40 J.N. CROSSLEY, Monash University, Clayton 3168, Australia and ANIL NERODE, Cornell University, Ithaca 48540. <u>Effective dimension.</u>

We prove effective generalizations of the classical results that there exist unique vector spaces of given dimension over a fixed field K and that there exist unique algebraically closed fields of given transcendence degree and specified characteristic. We work inside a universal recursively presented model  $\mathcal{W}$ , which has x = x as a minimal formula. A substructure  $\mathcal{O}$  is said to be <u>soundly based</u> with basis A if there is a recursively enumerable independent set  $A^+$  of elements (satisfying the minimal formula) and  $A = A^+$  $\cap \mathcal{O}$  is a basis (i.e. independent generating set) for  $\mathcal{O}$ . Theorem 1. If  $\mathcal{O}$ .  $\mathcal{B}$  are soundly based algebraically closed substructures of  $\mathcal{H}$  and A, B are bases for  $\mathcal{H}, \mathcal{B}$  respectively then A is recursively equivalent to B (as sets) if, and only if,  $\mathcal{H}$  is recursively equivalent to  $\mathcal{F}$  as  $\mathcal{H}$ -substructures. Theorem 2. If A, B are subsets of a recursive (or recursively enumerable) basis for  $\mathcal{H}$  and A is recursively equivalent to B (as sets) then the algebraic closures of A, B are recursively equivalent as substructures of  $\mathcal{H}$ . (Received March 4, 1975.)

#### 75T-E41 DAVID W. KUEKER, University of Maryland, College Park, Maryland 20742. <u>Game</u> sentences and models with many automorphisms, Preliminary report.

Let  $\kappa$  be a regular cardinal. Let  $L(\kappa)$  be the logic containing all atomic formulas and closed under negation, arbitrary infinite conjunction and disjunction, and quantification by well-ordered quantifiers of length  $\leq \kappa$  provided the result has  $< \kappa$  free variables (cf. Keisler, Formulas with linearly ordered quantifiers, in Springer Lecture Notes, vol. 72).

THEOREM 1. Assume  $|A| = \kappa$ . Then there is a prenex sentence  $\sigma$  of  $L(\kappa)$  such that for every **3**, **3**  $= \sigma$  iff  $\Im \equiv \mathfrak{B} L(\kappa)$ .

THEOREM 2. Assume  $|A| = \kappa$  and  $(\sharp \equiv \mathfrak{DL}(\kappa)$  for some  $\mathfrak{D} \not\cong \mathfrak{A}$ . Then  $\mathfrak{T}$  has  $2^{\mathsf{K}}$  automorphisms.

A model  $\mathfrak{A}$  is  $(\kappa, L_{\alpha_{\omega}})$ -homogeneous if for any  $\alpha < \kappa$  and sequences  $\{a_{\xi}\}_{\xi < \alpha}$ , and  $\{b_{\xi}\}_{\xi < \alpha}$  from A,  $(\mathfrak{A}, a_{\xi})_{\xi < \alpha} \equiv_{\alpha_{\omega}} (\mathfrak{A}, b_{\xi})_{\xi < \alpha}$  provided  $(\mathfrak{A}, a_{\xi})_{\xi \in I} \equiv_{\alpha_{\omega}} (\mathfrak{A}, b_{\xi})_{\xi \in I}$  for all finite  $I \leq \alpha$ . If  $\mathfrak{A}$  and  $\mathfrak{B}$  are both  $(\kappa, L_{\alpha_{\omega}})$ -homogeneous and  $\Im \equiv_{\alpha_{\omega}} \mathfrak{D}$  then  $\Im \equiv \mathfrak{I}(\kappa)$ .

COROLLARY. Assume  $\kappa^{\kappa} = \kappa$ . Let  $\sigma$  be a complete (w.r.t.  $L_{\infty\omega}$ ) sentence of  $L_{\kappa}^{+}\omega$  which has a  $(\kappa, L_{\infty\omega})$ -homogeneous model of power >  $\kappa$ . Then  $\sigma$  has a model of power  $\kappa$  with  $2^{\kappa}$  automorphisms. (Received March 20, 1975.)

### *75T-E42 STANLEY BURRIS, University of Waterloo, Waterloo, Ontario N2L 3G1 and HEINRICH WERNER, FB4 AG1 Technische Hochschule, D-61 Darmstadt, Germany. <u>Decidable Theories</u>

For algebras which have a certain type of representation by continuous function from a Boolean space into a finite discrete algebra one can manufacture a translation of each sentence in the language of these algebras with quantification over congruences into a sentence in the language of Boolean algebras with quantification over filters.

<u>THEOREM</u>: Every variety generated by a weakly independent set of quasi-primal algebras has a decidable theory and its countable members have a decidable theory with quantification over congruences. Examples for such varieties are Boolean lattices, relatively complemented distributive lattices and all residually finite varieties of Post-algebras, double p-algebras, monadic algebras, cylindric algebras, relation algebras, and arithmetical rings. (Received March 26, 1975.)

### 75T-E43 JAN MYCIELSKI, Math. Dept., University of Colorado, Boulder, Colorado 80302. Measure and category of some sets of models. Preliminary report.

Let  $\mathfrak{A} = \langle A, R \rangle$ , where  $R \subseteq A \times A$  and  $|A| = \aleph_0$ . Consider two conditions: (1) For every finite set  $X \subset A$  there exists an automorphism  $\alpha$  of  $\mathfrak{A}$  such that  $\alpha(X) \neq X$ . (2) Every isomorphism between two finite substructures of  $\mathfrak{A}$  is extendible to an automorphism of  $\mathfrak{A}$ . For any  $S \subseteq A \times A$  we put  $\mathfrak{A}(S) = \langle A, R, S \rangle$ . We make the standard topology and measure in  $2^{A \times A}$ .  $\sigma$  runs over all sentences of a first order language of type  $\langle 2, 2 \rangle$ . <u>Theorem 1</u>. If (1) then both sets  $\{\sigma: \{S: \mathfrak{A}(S) \models \sigma\}$  is comeager} and  $\{\sigma: \{S: \mathfrak{A}(S) \models \sigma\}$  is of measure 1} are complete theories. Problems. Do these theories coincide? Are they decidable mod Th( $\mathfrak{Y}$ )? By the next theorem, if (1) and (2), then both answers are yes. <u>Theorem 2</u>. If (1) and (2) then there exists  $S_0 \subseteq A \times A$  such that  $\{S: \mathfrak{Y}(S) \cong \mathfrak{Y}(S_0)\}$  is comeager and of measure 1 and Th( $\mathfrak{Y}(S_0)$ ) is decidable mod Th( $\mathfrak{Y}$ ). Both theorems generalize to arbitrary finite relational similarity types. (Received April 7, 1975.)

*75T-E44 PETR ŠTĚPÁNEK and BOHUSLAV BALCAR, Charles University, Prague, Sokolovská 83, 186 00 Praha 8, Czechoslovakia. <u>Embedding Boolean</u> <u>algebras into rigid complete Boolean algebras</u>, Preliminary report.

<u>Theorem</u>. Any Boolean algebra B can be completely embedded into a complete Boolean algebra C with no non-trivial  $\sigma^*$ -complete one-one endomorphism. Moreover, if B satisfies the K-chain condition for an uncountable cardinal K, the same holds true for C.

The Theorem gives stronger conditions on C than the earlier results of K. McAloon. Its proof follows from recent work of S. Shelah and the authors. (Received March 10, 1975.)

*75T-E45 HARVEY FRIEDMAN, State University of New York at Buffalo, 4246 Ridge Lea Road, Amherst, New York 14226. <u>Programs and results in logic I</u>.

There is an explicitly defined countably additive translation invariant extension of Lebesgue measure on the circle, not obtainable by adding new sets of measure 0. Every countably additive translation invariant measure on the circle can be extended to another not by just adding sets of measure 0. If transfinite induction can be proved for all formulae in HA on a primitive recursively presented linear ordering, then the ordering is well ordered of length  $< \epsilon_{0}$ . Various forms of ordinal categoricity are considered in

set theory, some of which are shown to imply V = L, as in my March 6 lecture at M.I.T. We analyze which sentences of predicate calculus with =, are true in all structures whose domain is the whole universe, and give an axiomatization. We consider formal theories ranging from PA to ZFC in which self-referential truth predicates are added, in letters of 3/13, 3/21, 3/28 to Saul Kripke. Kripke has priority on a number of the results. We are giving an answer to Hilbert's second problem that Hilbert may or may not have liked, in a series of reports, all entitled "The analysis of mathematical texts, and their calibration in terms of intrinsic strength". A purely geometric characterization of the unit interval is afforded by: [0,1] is the only complete, dense linear ordering with endpoints 0, 1, such that there is a continuous  $F: [0,1] \times [0,1] \rightarrow [0,1]$  obeying a) F(0,x) = x b) F(1,x) = 1 c) F(t,1) = 1 d)  $s < t \rightarrow F(s,x) < F(t,x)$ e)  $x < y \rightarrow F(t,x) < F(t,y)$ . (Obtained in 1971.) (Received April 14, 1975.)

### Statistics and Probability

*75T-F7 F. ALBERTO GRUNBAUM, University of California, Berkeley, California 94720 The Spectrum of a Random Matrix

Let  $V \equiv Hom (\mathbb{R}^3)$  denote the set of 3×3 real matrices. Let  $X = \{x_{ij}: 1 \leq i, j \leq 3\}$  be a set of independent N(0,1) random variables. There exists a map  $\Phi: V \rightarrow V \times V \times V$  satisfying  $\Pi_1 \Phi = I$  and such that the following holds:

THEOREM: If the joint distribution of the random variables  $tr(A+X)^k$  and  $tr(B+X)^k$  (k=1,2,3) coincide, then A is orthogonally equivalent to one of the 3 components in  $\Phi(B)$ . (Received March 6, 1975.) (Author introduced by Professor Jacob Feldman.)

75T-F8 Mark Blondeau Hedrick, 222 Kalmer, Paladena, TX 775D2, <u>Permanental minors and</u> <u>critical points of the permanent relative to the set of doubly stochastic matrices</u>

The author has recently announced in the Bulletin of the AMS a necessary condition on a matrix A in order that it be a critical point of the permanent relative to the set  $D_n$  of n x n doubly stochastic matrices. In this paper, he announces a structural theorem for matrices in  $D_n$ . He then uses this theorem to show that the condition is also sufficient. Specifically, he defines an algorithm on a fixed ( $D_1$ )-pattern in  $D_n$  write reduces the number of variables until the remaining ones are independent. He then uses this theorem to show that if perA equals perA( $i \mid j$ ) for every positive  $a_{ij}$ , then A is a critical point of the permanent relative to  $D_n$ .

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The author also proves that if A is a matrix at which the permanent achieves a local maximum relative to  $D_n$ , then A is on the boundary of  $D_n$ . (Received March 28, 1975.) (Author introduced by Professor Richard Sinkhorn.)

75T-F9 Robert M. Cranwell and Neil A. Weiss, Arizona State University, Tempe, AZ 85281. Limit theorems involving stationary point processes. Preliminary report.

Suppose that  $A_o(x)$  particles are initially placed at  $x \in Z^d$  and the particles move independently according to the transition function of a random walk. For any finite nonempty subset B of  $Z^d$ , let  $S_n(B)$  denote the total occupation time of B by time n. Under the assumption that  $A_0$  is covariance stationary, the asymptotic behavior of the variance of  $S_n(B)$  is established. Furthermore, under the additional assumption that  $A_0$  is ergodic, strong laws of large numbers and central limit theorems are proved for  $S_n(B)$ . (Received April 7, 1975.)

75T-F10 PHILIP E.PROTTER, University of California, San Diego, La Jolla, Ca. 92037. Existence, Uniqueness, and Convergence of Solutions of Stochastic Integral Equations. Preliminary report.

A theory of stochastic integral equations is developed for the integrals of Kunita, Watanabe, and P. A. Meyer. We prove the existence and uniqueness of solutions of systems of equations with semi-martingale(or quasi-martingale) differentials. That is, the equations are of the form:

dX(s) = f(s,X(s))dL(s) + g(s,X(s))dA(s)

where L(s) is a continuous local martingale and A(s) is a continuous, adapted process of bounded variation. We improve upon the customary results by eliminating the growth conditions on the coefficients:  $f(t,x) \leq K(1+|x|^2)$ ; and we still obtain the same conclusions. By requiring only that the coefficients satisfy the Lipschitz condition on compact sets we prove the existence and uniqueness with explosions. We obtain a sufficient condition for explosions to be impossible for solutions of systems of equations, and we generalize Feller's test for explosions. We extend the convergence results of Gihman and Skorohod and prove a local martingale version of the convergence of solutions of ordinary integral equations to the solutions of stochastic integral equations, a result of Wong and Zakai. We also show that when the (random) coefficients and/or the differentials converge, the solutions converge to the solution of the limiting equation. (Received April 17, 1975.) (Author introduced by Professor Ronald Getoor.)

### Topology

*75T-G42 J. W. BALES, Auburn University, Auburn, Ala. 36830. <u>Representable Spaces and</u> Strongly Locally Homogeneous Spaces.

<u>Definition</u>: A topological space X is said to be representable (resp. strongly locally homogeneous) provided it is true that if P is a point of the open set U in X then U contains an open subset V containing P such that if Q is a point of V then there is a homeomorphism taking X onto X which takes P onto Q and leaves each point of X outside U (resp. outside V) fixed.

In their paper <u>Isotopy Galois Spaces</u> (Pacific Math. Journal, Vol. 42, No. 2, 1973) Duvall, Fletcher and McCoy raise the question whether every representable space is strongly locally homogeneous. This question is answered in the affirmative. (Received February 21, 1975.)

75T-G43 WILLIAM PARDON, Columbia University, New York, N.Y. 10027. <u>The</u> <u>Exact Sequence of a Localization in L-Theory</u>.

Let  $L_n(\Lambda)$  denote the Wall group in dimension n for the ring with involution  $\Lambda$ . Then the exact sequence of a localization in L-theory is a long exact se-

quence of the form  $\ldots \longrightarrow L_n^t(2\pi) \longrightarrow L_n(2\pi) \longrightarrow L_n(Q\pi) \longrightarrow L_{n-1}^t(2\pi) \longrightarrow \ldots$  where  $\pi$  is a finite group. In geometric form,  $L_n^t(2\pi)$  is the group of bordism classes of normal maps f:  $\mathbb{N}^n \longrightarrow \mathbb{Y}^n$  with  $f_*: \mathbb{H}_*(\mathbb{N}; \mathbb{Q}) \xrightarrow{\cong} \mathbb{H}_*(\mathbb{I}; \mathbb{Q})$  and with normal bordisms satisfying a similar condition. By developing the properties of a stratified space  $\mathcal{L}$ , a generalized  $\mathbb{Z}_n$ -manifold, one may develop the surgery theory  $L_n^t(2\pi)$ along the lines of "ordinary" surgery theory,  $L_n(2\pi)$ , where the  $\mathcal{L}$  play the role of spheres. This allows one to give an algebraic description of  $L_n^t(\pi)$  which is entirely analogous to that of  $L_n(\pi)$ , except that where  $L_n(\pi)$  is based on hermitian forms over free Z -modules,  $L_n^{t(\pi)}$  is based on forms over Z-torsion  $2\pi$ modules with short free resolution. Calculations can be made and the sequence can be derived algebraically for more general rings. This extends similar work of Karoubi and Connoly, and calculations of R.Lee. (Received February 27, 1975.)

*75T-G44 JOHN GINSBURG and R.GRANT WOODS, Univ. of Manitoba, Winnipeg, Canada R3T 2N2 On the cellularity of  $\beta X - X$ .

Let c(Y) denote the cellularity of a Tychonoff space Y, and let k(Y) denote the least cardinality of a cobase for the compact subsets of Y. <u>Theorem 1</u>.  $c(\beta Y - Y) \le 2^{c(Y)k(Y)}$ . <u>Theorem 2</u>. If X is an extremally disconnected Tychonoff space and if  $\alpha$  is a cardinal no less than k(X), then  $\beta X - X$  contains a discrete  $C^*$ -embedded subspace of cardinality  $\alpha^+$ whenever  $c(\beta X - X) > 2^{\alpha}$ . Examples are given to show that the inequality given in Theorem 1 is sharp. (Received February 28, 1975.)

75T-G45 R. C. SOLOMON. University of Lancaster, Lancaster, England. A Scattered Space that is not O-dimensional

There is a completely regular scattered space which is not O-dimensional. This answers a question raised by Z. Semadeni in Sur les Ensembles Clairsemes. Rozpr. Matem. 19 (1959). As every compact scattered space is O-dimensional, this space is another example of a completely regular scattered space which has no scattered compactification. The space concerned is 1st-countable and there is only one point in the second derived set. (Received February 28, 1975.) (Author introduced by G. H. Bailey.)

### *75T-G46 V.KANNAN, Madurai University, Madurai-625021, INDIA. <u>A note on a preperty of Urysohn spaces</u>

The following is the review by S.P.Franklin (Math.Reviews (1973)  $\neq$  9931) of a paper by S.Mrowka and H.P.Tan (Czech.Math.Jour. <u>22</u> (97) (1972) 517-521) 'Spaces having the preperty that to each infinite subset A, there corresponds a sequence of pairwise disjoint open sets, each of which interesects A, are studied. The question whether every Urysohn space pessesses this preperty is left open'. Here we give an affirmative answer for this question. (Received February 28, 1975.) (Author Introduced by Professor M. Rajagopalan.)

*75T-G47 JOHN J. WALSH, Institute for Advanced Study, Princeton, NJ 08540. <u>Monotone and Light Open Mappings on Manifolds</u>, Preliminary report.

Let  $M^{m}$  be a compact, connected (metric) p.L. manifold of dimension  $m \geq 3$ . Theorem: If f is a monotone mapping of  $M^{m}$  onto a space Y, then f can be approximated by an open

mapping g from  $M^{m}$  onto Y satisfying for each  $y \in Y$ : (i)  $g^{-1}(y)$  is not a point and (ii)  $g^{-1}(y)$  and  $f^{-1}(y)$  have the same shape.

Let  $S^m$  denote the m-sphere and assume that  $M^m$  has no boundary. Let C be a closed, proper subset of  $M^m$  such that if U is an open connected subset of  $M^m$ , then either  $U - (C \cap U)$  is connected or  $C \cap bd(U) \neq g$ . <u>Theorem</u>: There is a monotone mapping f of  $M^m$ onto  $S^m$  such that each component of C is a point inverse of f. If M is oriented, then f can be taken to have any degree desired. (The preceding theorem generalizes a result of R. H. Bing in "Extending Monotone Decompositions of 3-Manifolds," <u>Trans. of AMS</u>, Vol. 149 (1970).) <u>Theorem</u>: There are light open mappings f,g of  $M^m$  onto  $M^m$  such that: (i) dim(B_f) = dim(f(B_f)) = m-1 and (ii) dim(B_g) = m-1 and g(B_g) =  $M^m$  (B_f is the branch set of f). (Received March 3, 1975.)

### *75T-G48 SAMUEL BROVERMAN, Univ. of Manitoba, Winnipeg, Manitoba, Canada, R3T 2N2 Homomorphisms between lattices of zero sets. Preliminary Report

For a completely regular Hausdorff topological space X, let Z(X) denote the lattice of zero-sets of X. A lattice homomorphism is called a  $\sigma$ -homomorphism if it preserves countable meets. If  $\tau: X \rightarrow Y$  is a continuous map then there is a lattice homomorphism  $\tau'$  induced by  $\tau$  which is defined by  $\tau'(A) = \tau'(A)$ . <u>Theorem</u>. Let Y be a realcompact space. A lattice homomorphism  $t: Z(Y) \rightarrow Z(X)$  is induced by a continuous map  $\tau$  from X to Y (such that  $t = \tau'$ ) if and only if t is a  $\sigma$ -homomorphism (such that t(Y) = X and  $t(\phi) = \phi$ ). (Received March 3, 1975.)

### *75T-G49 A. MUKHERJEA, University of South Florida, Tampa, Florida 33620. Limit Theorems for Probability Measures on Non-Compact Groups and Semi-Groups.

In this paper, theorems on the vague convergence of averaged and unaveraged convolution sequences of a probability measure are given. Typical results are: (1) In every locally compact non-compact second countable group generated by the support of a probability measure  $\pi$ ,  $\pi^n$  converge to zero vaguely as n goes to infinity. (2) In every locally compact semigroup generated by the support of a probability measure  $\pi$  and satisfying the condition  $-Kx^{-1}$  compact for each compact K and every element x,  $\pi^*\beta=\beta^*\pi=\beta$  for some probability measure  $\beta$  iff 1/n.  $\sum_{k=1}^{n} \pi^k + \beta=\beta^2$  vaguely as n goes to infinity. The compactness condition is a necessary condition for this result. Some open questions are also discussed. (Received November 11, 1974,) (Author introduced by Dr. M. N. Manougian.)

### *75T-G50 MILLER, JOHN L., University of Hawaii, Honolulu, Hi 96822. Realizing 1-homology classes on orientable surfaces.

Let  $(u_1, \ldots, u_k)$  denote the greatest common divisor of not all zero integers  $u_1, \ldots, u_k$ . If  $\xi = \sum_{i=1}^{\infty} (p_i \alpha_i + q_i \beta_i) \neq 0 \in H_1(T_m; Z)$  where  $T_m$  is an orientable surface of genus m with  $\alpha_i, \beta_i$  standard generators of  $H_1(T_m; Z)$ ; then there exists an embedding f:  $S^1 \rightarrow T_m$  such that  $f_*([S^1]) = \xi$  if and only if  $(p_1, q_1, \ldots, p_m, q_m) = 1$ . (Received March 6, 1975.)

#### *75T-G51 GARY M. HUCKABAY, Cameron University, Lawton, Oklahoma 73501 Expansive homeomorphisms on separable metric spaces.

The following result is a generalization of a theorem by W. Reddy (Math Systems Theory, Vol.2, No. 1, pp 91 - 92). Let Z be the set of integers and X a separable metric space. For each x in X let  $B(x;\epsilon)$  be the open ball centered at x with radius  $\epsilon$ . For any dense subset  $\{x(i): i\epsilon Z\}$  define  $S(\epsilon)$  to be the open cover  $\{B(x(i);\epsilon): i\epsilon Z\}$ . Define SL to be  $\mathbb{R}\{Z(i): i\epsilon Z\}$  where A-479 Z(i) is Z for each i. For each homeomorphism h of X onto X define  $\epsilon$ (h) from SL to P(X) by  $\epsilon$ (h)( $\langle s_1 \rangle$ ) =  $\cap\{h^{-1}(clB(x(s_1);\epsilon)) : i \in Z\}$ . Theorem. Let X be a separable metric space and h a homeomorphism of X onto X. Then h is expansive if and only if for some  $\epsilon > 0$  h $\circ\epsilon$ (h) =  $\epsilon$ (h) $\circ\sigma$  where  $\epsilon$ (h), defined as above, is a continuous map of some subset of SL onto X and  $\sigma$  is the ordinary shift. (Received March 17, 1975.)

75T-G52 CHARLES L. HAGOPIAN, California State University, Sacramento, California 95819. Homogeneous star-like continua.

A <u>star</u> is a tree with one branch point. A continuum is <u>star-like</u> if for each  $\varepsilon > 0$ it can be  $\varepsilon$ -mapped onto a star. Theorem 1. Every homogeneous star-like continuum has the fixed-point property for homeomorphisms. Theorem 2. Every proper subcontinuum of a homogeneous star-like continuum is a pseudo-arc. (Received April 2, 1975.)

*75T-G53 JAMES T. LOATS, University of Colorado, Boulder, Colorado 80302. <u>On Compactness and Ordinals</u>.

For each topological space X, define a function  $\mu_X$ : Ord  $\rightarrow$  Ord by  $\mu_X(\alpha) = \text{least}$ ordinal  $\beta$  such that every open cover of X of power less than  $\aleph_{\alpha}$  has a subcover of power less than  $\aleph_{\beta}$ . Knight [Glasgow Math. J. <u>13</u>(1972), 153-158, MR <u>48</u> #1138] proved that such a function  $\mu_X$  must satisfy these conditions: (1)  $\mu_X$  is continuous, (2)  $\mu_X(0) = 0$  and for all  $\alpha$ ,  $\mu_X(\alpha \div 1) = \alpha \div 1$  or  $\mu_X(\alpha)$ , (3) If  $cf(\aleph_{\beta}) = \aleph_Y$  and  $\mu_X(\gamma \div 1) \leq \gamma$ , then  $\mu_X(\beta \div 1) \leq \beta$ , and (4)  $\mu_X$  is eventually constant. <u>Theorem</u> For every function f: Ord  $\rightarrow$  Ord satisfying conditions (1) thru (4), there is a completely regular space X with  $\mu_X = f$ . The proof uses a generalization of an example due to Miščenko [Soviet Math. Døkl. <u>145</u>(1962), 1199-1202]. (Received March 20, 1975.)

75T-G54 PAUL BANKSTON, McMaster University, Hamilton, Ontario L85-4Kl Baire Category and Uniform Boundedness For Topological Ultraproducts Preliminary Report

(See these Notices, Feb., '74, 74T-G42). The topological ultraproduct of non-discrete Hausdorff spaces is neither complete metrisable nor locally countably compact, these being two of the conditions sufficient for a space to be Baire. We show that if  $\kappa$  is any infinite cardinal and if U is a  $\kappa$ -good ultrafilter then  $\Pi_{U}X_{i}$  is always  $\kappa$ -Baire (i.e. intersections of  $<\kappa$  dense open sets are dense (even open)). Since countably incomplete ultrafilters are  $\omega_{1}$ -good, all corresponding ultraproducts are Baire spaces. We use our results to prove some new "uniform boundedness" theorems for set-valued maps as well as for families of continuous operators between normed vector spaces over ordered fields. (Received March 21, 1975.) 75T-G55 WITHDRAWN

*75T-G56 L. D. Loveland, Utah State University, Logan, Utah 84321. Continuous finite Apollonius sets in metric spaces

The midset M(a,b) of two distinct points a and b in a metric space is the set of all points equidistant from a and b. The midset function M is said to be continuous if  $\{M(a_i,b_i)\}$  converges to M(a,b) whenever  $\{(a_i,b_i)\}$ converges to (a,b) and these point-pairs lie in the domain of M. Among continua having continuous midset functions, the arc is characterized by the property that each of its midsets is a singleton set. If a continuum X with a continuous midset function has the property that each of its midsets consists of n points, then either n=1 and X is an arc or X is a simple

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closed curve. A 1-dimensional continuum containing a triod and having the continuous finite midset property is exhibited, and in contrast it is shown that no triod can have the continuous finite midset property. These theorems are also proved with the more general "Apollonius set" or " $\lambda$ -set" replacing "midset", and a continuum is shown to be a simple closed curve if it has the continuous double  $\lambda$ -set property. (Received March 24, 1975.)

*75T-G57 FRANKLIN D. TALL, University of Toronto, Toronto, Canada, M5S 1A1. The density topology, II.

This note contains the results cited in these Notices 22, Jan. 1975, 720-54-43 as well as the following new results. <u>Theorem</u>. The density topology is not countably paracompact. <u>Theorem</u>. The assertion that the density topology is metalindelöf is consistent with and independent of the usual axioms for set theory. (Received March 25, 1975.)

#### *75T-G58 VICTOR SAKS, University of Costa Rica Density character of closed subgroups

Let  $m \ge \aleph_0$ . Theorem. There exists an Abelian group G and a closed subgroup H of G such that d(G) = m and d(H) > m, where d(G) is the density character of G, i.e., the smallest cardinality of a dense subset of G. There is a separable Abelian group with a closed subgroup H such that d(H) = c. (Received April 9, 1975.) (Author introduced by W. W. Comfort.)

75T-G59 WITHDRAWN

*75T-G60

SURREDRA-NATE PARLATE, Indian Institute of "echnology, Hauz Khas, New Delhi-29, India. Fixed points and selections. Preliminary report.

Let X and Y be topological spaces and let  $2^{Y}$  denote the space of nonempty closed subsets of Y. If F:X. is a function, then a selection for F is a continuous function f:X-Y such that  $x \in F(x)$ . A fixed point of a function F:X-2 is a point x X such that  $x \in F(x)$ . In problems concerning fixed points or selections, the set-valued function is usually accumed to be upper semi-continuous(use) or lower semi-continuous(lee). A form of the classical Kasutani fixed point theorem states: Let X be a closed n-simplex in the Euclidean space Rⁿ and let a use function F:X-2^A be such that for every  $x \in X$ , F(x) is convex, then F has a fixed point. In the present paper, we study the problem of selection for a convex-valued use function  $X \rightarrow 2^{X}$ , where X is a closed n-simplex or a compact polyhedron. A imilar problem is considered when the convexity assumption on the image sets is relaxed, but under the added condition on F being less. Finally, the inter-relationships with convexity, fixed points of set-valued maps, via generalizations of the Kasutani fixed point theorem, and the selection problem is discussed. (Received March 18, 1975.)

*75T-G61 RICHARD Z. GOLDSTEIN, Mathematics Institute der Universität Würzburg, 87 Würzburg Am Hubland, West Germany and EDWARD C. TURNER, State University of New York at Albany, Albany, NY 12222. <u>A Formula for the Stiefel Whitney Homology Classes</u>.

The Stiefel Whitney homology classes  $\omega_{p}(K)$  of a simplicial complex K are represented by the skeleta of the barycentric subdivision K', when they are chains. We prove the following formula for  $\omega_{p}(K)$  in terms of the simplices of K: order the vertices of K and say that a pair of simplices s Ct is regular if in the given ordering,  $s = \langle v_{0}, \dots, v_{p} \rangle | v_{i} \langle v_{i+1} \rangle$ and there are no vertices of t before  $v_{0}$  or between  $v_{2i-1}$  and  $v_{2i}$  for each  $i - \partial_{p}(t)$  denotes the set of all p-simplices regular in t. Theorem:  $\omega_{p}(K) = \Sigma \partial_{p}(t)$ . (Received April 11, 1975.) *75T-G62 J. W. CANNON, University of Wisconsin, Madison, Wisconsin 53706. <u>Cell-like decompositions of manifolds arising from mismatched sewings:</u> <u>Applications to</u>

The work of R. J. Daverman announced in Abstract 717-G2, these Notices 21(1974), A-617, and in Abstract 723-G7, these Notices 22(1975), A-424, may be reformulated so as to depend only on the generalization of a weak version of the well-known taming theorem which states that an (n - 1) -sphere S in Sⁿ  $(n \neq 4)$  is flat if Sⁿ - S is 1-ULC. One obtains thereby a unified proof of the Eaton-Daverman Mismatch Theorem in dimensions  $n \neq 4$ . Furthermore, since the weak taming theorem can often (possibly always) be verified for decompositions of S⁴, one obtains by the Eaton-Daverman techniques many results on cell-like decompositions of S⁴. For example, one proves the following theorems. Theorem. If C, and C₂ are crumpled 3-cubes and

h:  $BdC_1 \rightarrow BdC_2$  is a homemorphism, then  $(C_1 \cup_h C_2) \ge L^1 \not\cong S^3 \ge L^1$ . Theorem. If C is a crumled 3-cube, then the spun decompositions arising from C are either all Euclidean or all non-Euclidean. Theorem. All of the endspins of the dogbone space except the zero-spin are Euclidean. The first of these three theorems has previously been noted by R. J. Daverman, and W. T. Eaton in these Notices 15 (1968), 542. The high dimensional analogues of the latter two theorems have been proved by Daverman (to appear). (Received April 14, 1975.)

*75T-G63 TALAMO RODOLFO,Ist.Geom.Univ.Torino-V.Amedeo,8-10123 TORINO On prerealcomplete uniform spaces and a criterium for pseudocompactness

A topological space is pseudocompact if and only if every admissible uniformity is prerealcomplete. Thus it is immediately clear that the real line has an admis= sible uniformity that is not generated by a family of continuous real-valued functions on the space. The nature of this uniformity is presently not known. Howewer we construct on the subspace N of natural numbers an admissible pseudometric uniformity which cannot be generated by mappings. (Received April 14, 1975.)

*75T-G64 G. H. TOOMER, Ohio State University, Columbus, Ohio 43210. <u>The comparison theorem</u> for weak fibrations. Preliminary report.

By a weak fibration is meant a fibration in the sense of Dold [Ann. of Math. 78(1963), 223-255]. We show that if  $h/g: (E \xrightarrow{P} B) \rightarrow (\overline{E} \xrightarrow{\overline{P}} \overline{B})$  is a map of weak fibrations (that is  $\overline{p}h = gp$ ) and  $g: B \rightarrow \overline{B}$ ,  $h: E \rightarrow \overline{E}$  are homotopy equivalences, then h is actually a fibre homotopy equivalence. (Hence h|, h restricted to any fibre, is a homotopy equivalence. Conversely if B and  $\overline{B}$  are numerably locally contractible, and h| and h (or h| and g) are homotopy equivalences, then so is g (or h). There are corresponding results in the based category for well-pointed spaces. (Received April 18, 1975.)

75T-G65 ALI A. FORA, State University of New York at Buffalo, Amherst, N.Y. 14226 <u>The Dimension Conjecture for Products of Modified Sorgenfrey Spaces</u>. Preliminary Report

It has been know for some time that the product of Sorgenfrey Spaces is strongly zerodimensional. In 1972 Mrowka and Tan [Springer Lecture Notes in Mathematics, no. 378, p. 513-516] raised the question of whether the product of modified Sorgenfrey Spaces  $S_{\star}$  is still strongly zero-dimensional.

As a matter of fact, if  $S_{\star} = \mathbb{R} \times [0,1]$  is topologized in the following way,  $\mathbb{N}_{\subset}(\varkappa, 1) = (\varkappa, \varkappa + \in) \times [0,1] \cup \{(\varkappa, 1)\}$ ,

 $\mathbb{N}_{C}(\varkappa,\lambda) = \{(\varkappa,\lambda') : \lambda \leq \lambda' < \lambda + \in\}$  where  $\in$  is small positive real

number, and  $\lambda < 1$ , then S_{*} has almost the same properties as Sorgenfrey topology S (for example S_{*} is Lindelöf first countable, and N-compact.

Since  $N_{\in}(\varkappa,\lambda)$  is clopen for each  $\lambda$ , ind  $S_{\star} = 0$ . But  $S_{\star}$  is Lindelof so dim  $S_{\star} = 0$ . Now if dim  $S_{\star} x S_{\star} > 0$ , one would have a counterexample to the dimension conjecture for the product of Tychonoff strongly zero-dimensional spaces.

We show, however, that dim  $S_*xS_* = 0$ . In fact, the proof can be easily generalized to  $S_*^{\aleph_0}$ . (Received April 21, 1975.)

75T-G66 Ernest Lane, Appalachian State University, Boone, North Carolina 28608. <u>An insertion theorem for perfectly normal spaces</u>. Preliminary report.
A space X is perfectly normal if and only if for real-valued functions g and f defined on X such that g is upper semicontinuous, f is lower semicon-

tinuous, and g(x) < f(x) for each x in X, then there exists a continuous

function h defined on X such that  $g(x) \le h(x) \le f(x)$  for all x in X and if g(x) < f(x), then g(x) < h(x) < f(x). Thus if X is perfectly normal, if F is a closed subset of X, if  $\phi$  is a continuous real-valued function defined on F, if g and f are real-valued functions defined on X such that g is upper semicontinuous, f is lower semicontinuous,  $f(x) = \phi(x) = g(x)$  for all x in F, and g(x) < f(x) for all x in X - F, then there exists a continuous function h defined on X such that  $h(x) = \phi(x)$  for all x in F and g(x) < h(x) < f(x) for all x in X - F. (Received April 10, 1975.)

### **Miscellaneous** Fields

75T-H3 ROBERTO MACCHIA, Stevens Institute of Technology, Hoboken, New Jersey, 07030. An extension of the Inverse Function Theorem. Preliminary report. Theorem: Let  $f \in C^{(m+1)k-1}(-h,h)$  (h > 0), where  $m \ge 0$ , k > 0 are integers such that  $f^{(j)}(0) = 0$  for  $0 \le j \le k-1$ ,  $f^{(k)}(0) \ne 0$ . Then: (i) if k is odd, there exists  $\delta > 0$  such that for  $y \in (-\delta, \delta)$  the inverse function  $x = f^{-1}(y)$  exists and  $x = f^{-1}(y) = a_1 y^{1/k} + a_2 y^{2/k} + \ldots + a_{mk} y^m + R_m(y)$ where  $R_m \in C^m(-\delta, \delta)$  and  $R_m^{(j)}(0) = 0$  for  $0 \le j \le m$ ; (ii) if k is even, an analogous statement holds, provided we restrict ourselves to either [0,h) or (-h,0]. Various generalizations and applications of this theorem are discussed. (Received April 14, 1975.)

### The April Meeting in St. Louis, Missouri April 11 - 12, 1975

### Analysis

***723-**Вб5

HERBERT H. SNYDER, Southern Ill. Univ., Carbondale, IL 62901 Exact calculation of certain Fourier-Bessel expansion coefficients

After Fourier series and series of orthogonal polynomials, expansions in series of orthogonal sequences of Bessel functions are perhaps of the most frequent occurrence. Yet, there are very few such expansions known for which the coefficients are given exactly and explicitly. Our theorem affords a modest advance in this direction for expansions in functions of zero order. For definitions, we refer to Watson's treatise (pp. 576-581; 618-621), and only remark here that the Schlömilch coefficients are given by an integral which is often elementary. Theorem: Let f'(x) be both continuous and BV on [0,a], a>0, and let f(x) have a uniformly convergent Schlömilch series, f(x) =  $A_0/2 + \sum A_m J_0(m \pi x/a)$  on [0,a]. Then the coefficients  $a_n$  in the Fourier-Bessel series, f(x) =  $\sum a_n J_0(j_n x/a)$  on [0,a], are given by

$$a_{n} = [j_{n}J_{1}(j_{n})]^{-1}[A_{0} + 2j_{n}^{2} \sum_{m} A_{m}J_{0}(m\pi)(j_{n}^{2} - (m\pi)^{2})^{-1}. (j_{n} = n-th)$$

positive zero of  $J_0(x)$ ; m,n = 1,2,3,...) (Received February 20, 1975.)

### Topology

### 723-G12 SHOU-JEN HU LUH, University of Chicago, Chicago, Ill. 60637 Classifications of G-foliations, Freliminary Report

Let G be a finite group and  $\Gamma$  be a topological groupoid. A G- $\Gamma$ -structure on a G-space M is defined and a classifying space  $B(\Gamma,G)$  with a universal G- $\Gamma$ -structure  $\omega$  is constructed. <u>Theorem</u> (i) Let M be a 2nd countable locally euclidean G-space. For any G- $\Gamma$ -structure  $\sigma$  on M, there is a continuous G-map  $f:\mathbb{M} \to B(\Gamma,G)$  such that  $f^*\omega = \sigma$ . (ii) Let  $f_0, f_1:\mathbb{M} \to B(\Gamma,G)$  be two G-maps. The G--structure  $f_0^*\omega$  is homotopic to  $f_1^*\omega$  if and only if  $f_0$  is G-homotopic to  $f_1$ . An equivariant Gromov's theorem proved by E. Bierstone is used to prove a classification theorem for G-foliations on G-manifold. Theorem Let M be a paracompact open differentiable manifold with G action. If M has all its orbit bundle  $M_{(H)}$  non-closed, then there is a one to one correspondence between intergrable homotopy classes of G-foliations on M and homotopy classes of G-cpimorphisms TM  $\rightarrow \nu(\omega)$ . (Received February 24, 1975.)

### The April Meeting in Monterey, California April 19, 1975

### Algebra & Theory of Numbers

*724-A7 DEBORAH GALE, University of California, Davis, California 95616.  $\underline{Z^n} - \overline{n}$ -Semigroups.

If S is an  $\overline{\mathbb{N}}$ -semigroup then the following are equivalent: (1) S is a  $\mathbb{Z}^n$ - $\overline{\mathbb{N}}$ -semigroup. (2) The quotient group Q(S) of S is isomorphic to  $\mathbb{Z}^n$ . (3) Every structure group of S is isomorphic to  $\mathbb{Z}^n \times \mathbb{Z}_m$  for some  $m \in \mathbb{Z}_+$  and every  $\mathbb{Z}^n \times \mathbb{Z}_m$  (m=1,2,...) is isomorphic to a structure group of S. (4) S is isomorphic to a subsemigroup S' of  $\mathbb{Z}^{n+1}$  such that S' contains a base  $\{\alpha_1, \ldots, \alpha_{n+1}\}$  for  $\mathbb{Z}^{n+1}$  considered as a Z-module. (5) There are elements  $a_1, a_2, \ldots, a_{n+1} \in S$  such that for each  $x \in S$  there is a unique expression  $a_1^{\ell_1} a_2^{\ell_2} \cdots a_{n+1}^{\ell_{n+1}} x =$   $a_1^{\ell_1} a_2^{\ell_2} \cdots a_{n+1}^{\ell_{n+1}}$  where  $\ell_k, \ \ell_k \in \mathbb{Z}_{+,0}$  for k=1,2,...,n+1 and if  $\ell_k \neq 0$  then  $\ell_k' = 0$  and  $\ell_1' + \ell_2' + \cdots + \ell_{n+1}' > 0$ . (Received April 14, 1975.)

### *724-A8 TAKAYUKI TAMURA, University of California, Davis, California 95616. <u>Putcha's</u> <u>Problem on Maximal Cancellative Subsemigroups</u>.

Putcha gave the following question in his recent paper (Proc. Amer. Math. Soc. 47, 1975, January, 49-52). Let S be a commutative archimedean semigroup without idempotent. Is a maximal cancellative subsemigroup of S necessarily an  $\mathbb{R}$ -semigroup? The author gives a counter-example by using the forest structure theorem (Math. Nacht. 36, 1968, 257-287). The example is not archimedean but an  $\mathbb{R}$ -semigroup. The following question remains: Is a maximal cancellative subsemigroup of S necessarily an  $\mathbb{R}$ -semigroup? (Received April 14, 1975.)

### Analysis

724-B8 CHARLES TUCKER, University of Houston, Houston, Texas, 77004 <u>A</u> theorem related to Baire Functions

<u>Theorem</u> Suppose each of H and K is a vector lattice of real valued functions, H contains the constant functions, G is the set of all functions which are the pointwise limits of sequences of functions in H, and  $\phi$  is a linear lattice homomorphism from G into K. Then if  $f_1, f_2, f_3, \ldots$  is a non-increasing sequence of functions in G converging pointwise to zero,  $\phi(f_1), \phi(f_2), \phi(f_3), \ldots$  is a non-increasing sequence of functions in K converging pointwise to zero. (Received February 20, 1975.)

*724-B9 BANSHI D. MALVIYA, North Texas State University, Denton, Texas, 76203, Modular Annihilator A* - algebras. Preliminary report.

It was proved in (Amer. Math. Monthly, Vol. 81, pp. 267-268) that a modular annihilator B* - algebra is weakly completely continuous. We discuss the question: Whether a modular annihilator A* - algebra is weakly completely continuous. We also discuss some properties of ideals of modular annihilator A* - algebras. (Received February 24, 1975.)

### Geometry

724-D3 John C. Nash, Stanford University, Stanford, Ca. 94305, Positive Ricci curvature on exotic spheres and fibre bundles, Preliminary report

Complete metrics of positive Ricci curvature are constructed on a class of fibre bundles. All bundles are assumed to have a compact semi-simple structural group, G, and a compact base space, M, that admits a metric of positive Ricci curvature. A proof is given that any such principal bundle admits a metric of positive Ricci curvature.

This is used to construct such metrics on bundles with more general fibre. For example, if G/H is a Riemannian homogeneous space with positive Ricci curvature, then any G/H bundle (with the base space restriction as above) having structural group, G, admits a metric of positive Ricci curvature. In particular, all Sⁿ (n>1) bundles over spheres have such metrics. Since a number of exotic 7 and 15-spheres occur as such bundles, these are new examples of exotic spheres admitting metrics of positive Ricci curvature.

Finally, using similar techniques, a proof is given that any vector bundle with fibre dimension  $\neq$  2, and having the previous base space restrictions, admits a complete metric of positive Ricci curvature. (Received April 7, 1975.) (Author introduced by Professor Robert Osserman.)

### Topology

*724-G3 ERIC LANGFORD, University of Maine, Orono, Maine 04473. Problems of Kuratowski type involving unions and intersections.

Let Xc, Xk, Xi, and Xb denote the closure, complement, interior, and boundary of the set X in a topological space. Kuratowski's famous closure-and-complement problem asserts that the family of sets CK(X) (notation obvious) contains a maximum of 14 members; if |CK(X)| = 14, we say that X is CK-14. It is known that X is CK-14 iff it is CI-7 and that this holds iff the following five sets are nonempty: Xbi, X\Xcic, Xici\X, Xcib, Xicb. (See E. Langford, Characterization of Kuratowski 14-sets, Amer. Math. Monthly 78 (1971), 362-367.

We investigate the situation when unions and intersections are also allowed. We show that if X is CI-7, then  $8 \leq |CIU(X)|$ ,  $|CIN(X)| \leq 13 \leq |CIUN(X)| \leq 35$  and we characterize those sets X for which the maximum is attained. For example, X is CIUN-35 iff all of the following sets meet both X and Xk: Xbi, Xc\Xcic, Xici\Xi, Xic\Xci, Xcic\(XciUXic), (XciNXic)\Xici. (The fact that  $|CIU(X)| \leq 13$  was noted by Smith, Problem 5996, Amer. Math. Monthly 81 (1974), p. 1034.) (Received February 20, 1975.)

### The June Meeting in Pullman, Washington June 21, 1975

### Algebra & Theory of Numbers

### 725-Al WILLIAM A. WEBB, Washington State University, Pullman, Wa. 99163, On the unsolvability of k/n = 1/x + 1/y + 1/z, Preliminary report. Let $\lambda_k$ be the largest value of n for which the equation k/n = 1/x + 1/y + 1/zis not solvable in positive integers x, y, z. A well known conjecture states that $\lambda_4 = 1$ , $\lambda_5 = 1$ , $\lambda_6 = 1$ , $\lambda_7 = 2$ , ... and in general, $\lambda_k$ is finite for all $k \ge 4$ .

The following result gives a lower bound for  $\lambda_k$  which is of interest for large values of k. Theorem: There is a constant c > 0 such that  $\lambda_k > \exp(c \log k \log \log k)$ . (Received April 4, 1975.)

*725**-**A2

J. M. GANDHI, Western Illinois University, Macomb, Illinois 61455. Fermat's Last Theorem I. Some Interesting Observations for the First Case.

It is well known [See Dickson, Theory of Numbers, (Chelsea Pub. Co. 1952) Vol. II, P. 771-772.] that if  $x^{p}+y^{p}+z^{p}=0$  has an integral solution with (xyz,p)=1 then there should exist an integer s such that (1)  $(s+1)^{p} \equiv s^{p}+1 \pmod{p^{3}}$ . As far as I know nobody tried to use this condition to verify the first case of Fermat's Last Theorem. Numerous tables do exist for the congruence  $(s+1)^{p} \equiv s^{p}+1 \pmod{p^{2}}$ . We note that if p is a prime of the form 6m-1, then the congruence (1) has no integral solution for the first case for all primes < 3511, while for primes of the form 6m+1, there are solutions only if  $s^{2}+s+1 \equiv 0 \pmod{p}$ . (In the range  $1 \leq s \leq \frac{p-1}{2}$ , there is only one solution for (1) for primes of the form 6m+1). However using Pollaczek's Theorem

[Sitzungsber. Akad. wiss. wien (Math.) 126, IIa, 1917, 45-49] it is easy to derive that  $s^{2}+s+1 \ddagger 0 \pmod{p}$ . Thus the first case of FLT is verified by this method for all primes  $p \le 3511$ . We believe the range can be easily extended. It will be interesting to find the first prime where the above method will fail to verify the first case of FLT. (Received April 14, 1975.)

725-A3 G. V. CHOODNOVSKY, Tarasovskaya, 10a, ap.17, Kiev, 252033, USSR. <u>Algebraic</u> independence of constants connected with the functions of analysis. Preliminary report.

We consider exp(z), Weierstrass elliptic function  $\mathscr{P}(z)$  with invariants from the field Q(j), where j is corresponding modular invariant, function  $\zeta(z)$ ,  $\zeta'(z) = -\mathscr{P}(z)$ ;  $\omega_1, \omega_2$  are basic periods of  $\mathscr{P}(z)$ ,  $\eta_i = \zeta(\omega_i/2)$ . If  $\mathscr{P}(z)$  has algebraic invariants, then for any nonzero complex  $\varphi$  the following results are valid: <u>Theorem 1</u>. There are two algebraically independent (a.i.) among the numbers  $\omega_1, \omega_2, \varphi, e^{\varphi \omega_1}$ ,  $e^{\varphi \omega_2}$ . <u>Theorem 2</u>. There are two a.i. among the numbers  $\eta_1, \eta_2, \varphi, e^{\varphi \omega_1}, e^{\varphi \omega_2}$ . <u>Theorem 3</u>. There are two a.i. among the numbers  $\pi/\omega_1, \eta_1/\omega_1, e^{\pi_1 \omega_2/\omega_1}$ . <u>Corollary</u>. If  $\mathscr{P}(z)$  has complex multiplication in Q(i), then  $\pi/\omega_1$  and  $e^{\pi}$  are a.i. All the theorems remain true if we change algebraicity of j to adding j to the considered sets of numbers. (Received April 23, 1975.)

*725-A4 IRVIN ROY HENTZEL, IOWA State University, Ames, IOWA 50010 GIULIA MARIA PIACENTINI CATTANEO*, University of Rome, Rome, Italy Semi-prime Generalized Right Alternative Rings

We define a generalized right alternative ring to be a non-associative ring R satisfying the hypotheses

1) (ab,c,d)+(a,b,[c,d]) = a(b,c,d)+(a,c,d)b

2) (a,a,a) = 0

for all a,b,c,d in R. We furthermore assume weakly characteristic not n for n=2 and n=3.

We prove that any semi-prime generalized right alternative ring is a right alternative ring. (Received April 29, 1975.)

### Analysis

*725-B1 ANDREW M. BRUCKNER, JACK G. CEDER, and MELVIN ROSENFELD. University of California, Santa Barbara, California 93106. <u>On invariant sets</u>.

Let f be a continuous function from a set X into X. A closed proper subset A of X is said to be invariant with respect to f if  $f(A) \subseteq A$ . This paper initiates a study of invariant sets. For example, it is shown that an invariant set with non-void interior exists iff no point has a dense orbit. The pathology of functions having a point with dense orbit is investigated in detail. For instance, if X is a non-compact real interval any such f admits a dense set of cyclic points (i.e., a fixed point for some iterate of f). (Received April 24, 1975.)

725-B2 JOHN J.F. FOURNIER, Department of Mathematics, University of British Columbia, Vancouver, Canada V6T 1W5. <u>Sharpness in the Hausdorff-Young Theorem on uni-</u> modular groups. Preliminary report.

Let G be a unimodular, locally compact group, and let  $f \mapsto L_f$  be the operator considered by R.A. Kunze (Transactions A.M.S. 89(1958), 519-540), in his generalization of the Hausdorff-Young theorem. Let 1 , and let p' be the index conjugate to p. If $G has a compact open subgroup, then there exist functions f, in <math>L^{p}(G)$ , for which the inequality  $||L_{f}||_{p'} \leq ||f||_{p}$  is sharp. It is shown here that there exists a constant  $B_{p} < 1$  such that, if G has no compact open subgroups, then  $||L_{f}||_{p'} \leq B_{p'}||f||_{p}$  for all f in  $L^{p}(G)$ . This settles a conjecture of Bernard Russo (Transactions A.M.S. 192(1974), 293-305). (Received April 25, 1975.)

### 725-B3 TAI-CHI LEE, University of Utah, Salt Lake City, Utah 84112. Boundary value problems for second order ordinary differential equations and applications to singular perturbation problems. Preliminary report.

Appropriate conditions are given which imply the existence of solutions to two-point boundary value problems

$$\begin{split} &Ly = y'' + p(t)y' + q(t)y = f(t,y,y',y(\cdot)), \quad a < t < b, \\ &a_1y(a) + a_2y'(a) = A, \qquad b_1 \mathcal{L}^0 y(b) + b_2 \mathcal{L}^0 y(b) = B, \end{split}$$

where  $Q^{0}y(b)$  and  $Q^{1}y(b)$  are the generalized derivatives at b, and b is possibly a finite singular point or  $\infty$ .

Solutions are obtained by application of the Tonelli procedure for demonstrating the existence of solutions to initial value problems and the intermediate value theorem for continuous functions. Solutions to the boundary value problem are found between the upper and lower solutions of the equation. These results generalize most of the existence theorems for the solutions of two-point boundary value problems on the finite interval and also apply to nonlinear boundary conditions as well. Finally, the results are used to deduce the existence of solutions to singular perturbation problems on  $[a, b] \subset (-\infty, \infty]$ . Examples are given to illustrate these results. (Received April 28, 1975.)

#### *725-B4 ROY A. JOHNSON, Washington State University, Pullman, Washington 99163. <u>A Decomposition Theorem for Product Measures</u>.

If  $\mu$  and  $\lambda$  are (non-negative countably additive) measures on a  $\upsilon$ -ring g, then  $\mu S\lambda$ means that given Eeg, there exists Feg such that  $\mu(E) = \mu(E\cap F)$  and  $\nu(E\cap F) = 0$ . Suppose  $\mu$  and  $\mu'$  are semifinite measures on S and  $\nu$  and  $\nu'$  are semifinite measures on S such that  $\nu < c \nu'$ . Let  $\mu x_{\perp} \nu [\mu' x_{\perp} \nu']$  denote the largest product of  $\mu$  and  $\nu$ [ $\mu'$  and  $\nu'$ ]. It is shown that there exist product measures  $\mu_1 \cdot \nu$  and  $\mu_2 \times \nu$  such that (1)  $\mu x_{\perp} \nu = \mu_1 \times \nu + \mu_2 \times \nu$ , (2)  $\mu_1 \times \nu < c \mu' x_{\perp} \nu'$  and (3)  $(\mu_2 \times \nu) S(\mu' x_{\perp} \nu')$ . An example is given to show that  $\mu_1 \times \nu [\mu_2 \times \nu]$  cannot in general be taken as the largest product of  $\mu_1$  and  $\nu [\mu_2$  and  $\nu$ ]. Moreover, it is possible to have  $\mu S\mu'$  and not have  $(\mu x_{\perp} \nu) S(\mu' x_{\perp} \nu)$ . (Received April 28, 1975.)

### **Applied Mathematics**

### 725-Cl ARNOLD JOHANSON, University of Toledo, Toledo, Ohio 43606. <u>A symmetric two-body</u> problem in Whitehead's theory of relativity.

A rigorous solution of the following problem is given. Suppose a particle is constrained to move in a circular orbit. Find a solution of Whitehead's equations for a second particle which moves in a circular orbit a fixed distance from the first particle and has a mass equal to the first particle. The motion of the two particles is symmetric up to terms of the third order but the symmetry is not exact. Since the solution is unique it shows that there does not exist an exact solution in Whitehead's theory in which two bodies keep a fixed distance from each other and revolve in a circular orbit. However, the solution found gives nearly exact relativistic corrections to the Newtonian solution of the symmetric two-body problem. (Received April 22, 1975.) (Author introduced by Dr. L. Bentley.)

### Geometry

*725-D1 DAVID BARNETTE, University of California, Davis, California 95616. From convex polytopes to manifolds.

There are many properties of convex polytopes that are also possessed by more general topological structure. For example Euler's equation holds for triangulated spheres, and similar equations hold for other manifolds. The lower bound theorem, which gives a lower bound on the number of facets of a simplicial polytope in terms of the number of vertices, is valid for all triangulated manifolds. Also several properties of the graphs of convex polytopes can be extended to triangulated manifolds, and in fact in the general setting of homology manifolds, these properties are easier to prove. In studying combinatorial properties of polytopes it may be profitable to find a more general topological setting. We shall examine several of these settings including one that is a generalization of homology manifolds. One is led to combinatorial questions about manifolds that apparently have never been investigated by topologists, such as: What is the largest  $n_d$  such that every triangulated d-manifold with  $n_d$  vertices is a sphere? We shall give interesting lower bounds on  $n_d$  and also indicate some other combinatorial questions about manifolds that do not appear to have been investigated. (Received March 28, 1975.)

*725-D2 MICHAEL J. KALLAHER, Washington State University, Pullman, Washington 99163 Semi-field planes with autotophism groups having large orbits.

Let  $\pi$  be an affine plane of order p^r coordinatized by a semi-field D and let G be the autotophism group of  $\pi$  . s(G) is the number of affine point orbits of G in  $\pi$  . s(G)-3 is the largest number of distinct non-isotopic semi-fields coordinatizing  $\pi$  . The smallest possible value for s(G) is 4. s(G) = 4 if and only if  $\pi$  is Desarguesian (and D is a field). This confirms a conjecture by D. R. Hughes and F. C. Piper. Let H be a solvable subgroup of G and assume H is transitive on one of the coordinatizing axes of  $\pi$  with respect to D. For every integer  $s \ge 4$  there exists a positive integer n(s)such that  $\pi$  of order n(s) and s(H) = s implies  $\pi$  is Desarguesian. In particular, if  $\pi$  has non-square order and s(H) = 5 or 6, then  $\pi$  is Desarguesian. (Received April 14, 1975.)

### Logic and Foundations

725-El

CARLOS A. INFANTOZZI, Universidad de la República, Atléntico 1514, Montevideo, Uruguay. "Axiomatic Systems for complex numbers, quaternions and octonions".

Axioms: <u>1)</u>-Let  $(L,+,\cdot)$ ,  $(K,+,\cdot)$ ,  $(F,+,\cdot)$ ,  $(F,+,\cdot)$  be algebraic structures such that each one is a whole substructure of the precedent, L is a non-associative ring and R is an ordered field:<u>11</u>)a whole substructure of the precedent, L is a non-associative ring and R is an ordered field; II]-a).-There exists, at least, an element i, is P, such that: a, bi=ab.i, ai. b=ab.i, ai. bi=-ab, where a.R and b_P; b).-For every  $\alpha, \alpha \in P$ , there exist a and a', at R and  $\alpha \in R$ , such that:  $\alpha = a + ai; III)_{\alpha} = a$ ).-There exists at least, an element j, jt K, such that:  $\alpha, \beta j = \alpha \beta, j, \alpha j, \beta j = \alpha \beta, where \alpha \in P, \beta \in P$  and  $\alpha = a - ai$  if  $\alpha = a + ai$  (Id. for  $\beta$ ); b).-For every  $A, A \in K$ , there exists  $\alpha$  and  $\alpha', \alpha \in P$  and  $\alpha' \in P, \beta \in P$  and  $\alpha = a - ai$  if  $\alpha = a + ai$  (Id. for  $\beta$ ); b).-For every  $A, A \in K$ , there exist  $\alpha$  and  $\alpha', \alpha \in P$  and  $\alpha \in P, \beta \in P$  and  $\alpha = a - ai$  if  $\alpha = a - a'$  if  $A = a + \alpha' j$  (Id. for  $\overline{B}$ ); b).-For every  $A, A \in K$ , there exist A and  $A, A \in K$  and  $A \in K$  such that:  $A = a + \alpha' j$  if  $A = a + \alpha' j$  (Id. for  $\overline{B}$ ); b).-For every  $A, A \in L$ , there exist A and  $A, A \in K$  and  $A \in K$  such that:  $A = A + A^{-1} j$  if  $A = a + \alpha' j$  (Id. for  $\overline{B}$ ); b).-For every  $A, A \in L$ , there exist A and  $A, A \in K$  and  $A \in K$  such that:  $A = A + A^{-1} j$  if  $A = a + \alpha' j$  (Id. for  $\overline{B}$ ); b).-For every  $A, A \in L$ , there exist A and  $A, A \in K$  and  $A \in K$  such that:  $A = A + A^{-1} j$  if  $A = a + \alpha' j$  (Id. for  $\overline{B}$ ); b).-For every  $A, A \in L$ , there exist A and  $A, A \in K$  and  $A \in K$  such that:  $A = A + A^{-1} j$  if  $A = a + \alpha' j$  (Id. for  $\overline{B}$ ); b).-For every  $A, A \in L$ , there exist A and  $A, A \in K$  and  $A \in K$  such that:  $A = A + A \in L$ . If we drop the axion IV and the structure (L, +, \cdot) (or the axioms IV, III and the structures (L, +, \cdot), (K, +, \cdot)) and we postulate that K (or P, respectively) is a ring, we obtain an axiom systemfor the quaternions (or for the complex numbers). Also axiomatic theories are obtained for the natio- ral octonions or quaternions, for ordinary octonions or quaternions, for integer octonions or quaternions.

nal octomions or quaternions, for ordinary octonions or quaternions, for integer octonions or quaternions of Lipschitz'type, for Gauss' numbers field, common complex numbers and Gauss' integers. (Received February 20, 1975.)

### Statistics and Probability

*725-F1 THEODORE E. HARRIS, University of Southern California, Los Angeles, California 90007. Some results on Markov interaction processes.

Let Z be a countable set,  $\Xi$  the set of subsets  $\xi$  of Z; each  $\xi$  may also be considered as a map from Z into  $\{0,1\}$ . We consider Markov processes  $\{\xi_t\}$  with state space  $\Xi$ . We interpret  $\xi_t(x) = 1$ (resp. 0) to mean that the site x is occupied (not occupied) by a particle at time t, or that the site x has say, an up (down) spin. The following topics are considered. (a) conditions for existence of a process with a given generator; (b) extremals of the set of invariant distributions; (c) conditions for extinction, i.e.  $\lim_{t\to\infty} q_t(\xi) = 1$  for all finite  $\xi$ , where  $q_t(\xi) = \operatorname{Prob}[\xi_t = \emptyset | \xi_0 = \xi]$ ; (d) monotonicity and submodularity of  $q_t$  as a function of  $\xi$ ; (e) the relation of association between pairs of processes. (Received April 28, 1975.)

### Topology

*725-G1 H.B. KEYNES, University of Minnesota, Minneapolis, Minn. 55455 and DAN NEWTON, University of Sussex, Sussex, Eng. Ergolic measures for compact group extensions.

In this paper, we study the structure of ergoic measures in the setting of two compact metric transformation groups (X,T) and (Y,T) such that a compact topological group G acts freely and continuously on X, induces T-equivariant maps, and Y = X/G. If m is an ergodic Borel measure on Y, and  $\pi: X \in Y$  is the canonical map, we show that any ergolic measure  $\nu$  on X with  $\pi(\nu) = m$  can be obtained in a canonical way via an isomorphism with m and a "Haar Lift," under the assumption that T is locally compact separatile. In addition, given  $\chi: T \to G$  a homomorphism, and both T and G abelian, we can define a new action  $\varphi_{\chi}(x,t) = \chi(t)xt$  so that  $(X,T,\phi_{\chi})$  is still a group extension of (Y,T). We show that for "most"  $\nu$  (in the sense of category), the Haar Lift  $\tilde{\pi}_{\chi}$  of m is ergodic in m is engodic and is veakmixing if m is weak-mixing, under the same assumption on T (thus including the reals). (Received February 21, 1975.)

### 725-G2 PHILIP D. STRAFFIN, Beloit College, Beloit, Wisconsin 53511. <u>Identities for conjugation</u> in the Steenrod algebra.

Donald Davis and the author recently used Milnor's calculation of  $\chi(\mathrm{Sq}^n)$  in the Milnor basis for the Steenrod algebra to prove a technical <u>Proposition</u>  $\mathrm{Sq}^m \chi(\mathrm{Sq}^n) = \sum \begin{pmatrix} \sum 2^i r_i \\ m \end{pmatrix} \mathrm{Sq}^{(r_1, r_2, \ldots)}$  where the outer sum is taken over all elements in the Milnor basis such that  $\sum (2^i - 1) r_i = m + n$ . A similar result holds for the mod p Steenrod algebra. This proposition makes it possible for the first time to obtain a wealth of relations involving  $\chi$ , simply by working with binomial coefficients. Typical examples: <u>Theorem 1</u>.  $\mathrm{Sq}^{2^n} + \chi(\mathrm{Sq}^{2^n}) = \mathrm{Sq}^{2^{n-1}} \chi(\mathrm{Sq}^{2^{n-1}})$ . <u>Theorem 2</u>.  $\mathrm{Sq}^m \chi(\mathrm{Sq}^m) = 0$  if  $m = 2^n - k$  for  $1 \leq k \leq (n+1)/2$ . (Received April 24, 1975.)

725-G3 ROMAN FRIČ, Univ. of Transport Engineering, Žilina, Czechoslovakia; KELLY McKENNON, Washington State Univ., Pullman, WA 99163; and GARY D. RICHARDSON, East Carolina Univ., Greenville, NC 27834. Sequential Convergence in C(X),II. Preliminary Report.

The third author proved (Sequential Convergence in C(X), these Notices 22(1975), A-117) that a sequential convergence space X is sequentially regular iff it is homeomorphic to a subspace of  $C^2(X)$  endowed with the continuous convergence (in the sense of Kuratowski). We prove that X is sequentially complete, i.e. X is a sequential envelope (in the sense of Novák) of itself, iff it is homeomorphic to a closed subspace of  $C^2(X)$ . In the original representation theorems  $C^2(X)$  is replaced by  $R^{C(X)}$  endowed with the pointwise convergence. While the original technique is similar to that used for E-completely regular and E-compact spaces, the latter resembles the functional analysis technique used by E. Binz for filter convergence spaces. We also give a new construction of the sequential envelope  $\sigma(X)$  of X and prove that  $f_n$  converges continuously to f in C(X) iff  $\overline{f_n}$  converges continuously to  $\overline{f}$  in  $C(\sigma(X))$ , where  $\overline{g}\in C(\sigma(X))$  denotes the extension of  $g\in C(X)$ . (Received April 28, 1975.)

%725-di R. GAZIK, Arkansas State Univ., State University, AR 72467; G. RICHARDSON, East Carolina Univ., Greenville, NC 27834; and D. KENT, Washington State Univ., Pullman, WA 99163. <u>T-Regular-Closed Convergence Spaces</u>.

A convergence space is T-regular if each convergent filter has a filter base of closed sets. A T-regular space is T-regular-closed if it is a closed subspace of every T-regular space in which it is embedded. A T-regular-closed space can be considered as an "approximation" to a compact regular space.

<u>Theorem</u> Let X be a T-regular convergence space whose topological modification is Hausdorff and locally compact. Then X can be densely embedded in a T-regular-closed space X*, and each continuous map from X into a compact regular  $T_2$  space can be lifted to X*. (Received April 28, 1975.)

### ERRATA Volume 22

GUTTALU R. VISWANATH, On convex univalent functions. Preliminary report. Abstract 75T-B65, Page A-317. Line 4, for "zf"(z) f'(z)" read zf"(z)/f'(z).

BRUCE M. HOROWITZ, <u>Set productive and arithmetically set productive sets</u>. Preliminary report. Abstract 722-E1, Page A-358. Line 4, for "be" read "by". Line 6, for "\Sum" read "\Sum" (both occurrences).

HILARY PUTNAM, Minimality and gap ordinals, Abstract 723-E3, Page A-420. Line 3, for " $\gamma$ " read " $\gamma > 0$ ".

LOUIS M. FRIEDLER and DIX H. PETTEY, <u>Inverse limits and mappings of minimal topological spaces</u>, Abstract 723-G5, Page A-424. The sentence beginning on line 5 and ending on line 6 should read, "The inverse limit of nonempty H-closed spaces, over an ordered set, is proved to be a nonempty Hausdorff space."

JACK SEGAL and GEORGE KOZLOWSKI, Local behavior and the Vietoris and Whitehead theorems in shape theory. Abstract 721-G2, Page A-342. Line 9, for " $(0 \le k \le n + 1)$ " read " $(0 \le k \le n)$ ".

MICHAEL W. POOLE, <u>Structure properties of cones of positive operators</u>. Preliminary report. Abstract 721-A1, Page A-335.
The terms "full and pointed" should precede the terms "polyhedral cone" or "polyhedral cones" in each place that they appear, i.e., lines 1, 4, and 7.

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RESEARCH MATHEMATICIAN, STATISTICIAN. Ph.D. Candidate, Age 25, Specialty: probability and statistics. Three years experience as teaching assistant. Experience in administrative analysis and programming. Peace Corps Ghana. References and resume available upon request, Western U.S. preferred, Available July, 1975. Larry Hall, 7611 Kirwin Lane, San Jose, California 95129.

TEACHING AND/OR RESEARCH MATHEMATICIAN. Ph.D. 1971. Age 34. Specialty: Categorical Topology. Four papers submitted. Excellent teacher. Ability to excite and motivate students. Involved in undergraduate research; ten student papers have or will appear. Will consider visiting/temporary position USA/CANADA. References on request. Temple H. Fay, 402 Herford, Conway, Arkansas 72032. MATHEMATICS TEACHING AND RESEARCH. Ph.D. Wisconsin, 1974, algebra. Age 28. Four papers accepted, 2 submitted. Research on incidence algebras, matrix theory and applications, graph theory. Background in combinatorial mathematics, logic, Teaching experience, industrial work, including computing, and postdoctoral experience. Available Sept. 1975. Resume and references on request. Robert Feinberg, Applied Math. Div., National Bureau of Standards, Washington, D.C. 20234.

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