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CORRECTION TO "DIFFERENTIAL IDENTITIES IN PRIME RINGS WITH INVOLUTION"

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An example of Chuang [1] shows that the main results of [2] are false as stated. The purpose of this note is to state the correct versions of these theorems. We shall use the notation in [2], and all references to results are from that paper. We begin by noting that all of the results in [2] before Theorem 4 are correct as stated, and that the correction needed in Theorem 4 requires a subsequent change in Theorem 7 and in Theorem 9. All other results in the paper are correct.

The statement of Theorem 4 concerns a linear G*-DI f, all of whose exponents come from W, the ordered collection of k-tuples of outer derivations which are independent modulo the inner derivations. For any exponent w appearing in f and coming from W, let f_w be the sum of all monomials in f with exponent w. The error in the proof of Theorem 4 is the assumption that if $f_w(x,y)$ is a G*-PI for R, then $f_w(x^w, y^w)$ is also an identify for R. This is true when no involution is present, or equivalently, when y does not appear in f. However, given an exponent w appearing in f, a relation between r and r^* will not in general hold for r^w and $(r^*)^w$, unless * commutes in $\operatorname{End}(R)$ with w. Thus the induction used in Theorem 4 fails. The most important feature of Theorem 4 can be salvaged, using essentially the proof given.

For any w coming from $(d_1, \ldots, d_k) \in W$, let k be the length of w. If $f \in F$ is linear and has all its exponents coming from W, an exponent w appearing in f is said to be of longest length if no other exponent of f has longer length. The conclusion of Theorem 4 is correct for all exponents of longest length, and the following is what the statement of the theorem should be.

THEOREM 4. *Let R be a prime ring with *, and let $f \in F$ be linear and have all its exponents coming from W, so that $f = \sum_h \sum_i a_{hi} x^h b_{hi} + \sum_k \sum_j c_{kj} y^k d_{kj}$ with all h and k coming from W and all coefficients in N. Suppose that for some nonzero ideal I of R, $f(I) = 0$. Then for each exponent w appearing in f and of longest length, $f_w(x) = \sum_i a_{wi} x b_{wi} + \sum_j c_{wj} y d_{wj}$ is a G*-PI for R. In addition, if no y appears in f, or if each exponent appearing in f commutes with * in* $\operatorname{End}(R)$, *then $f_w(x)$ is a G*-PI for R for every exponent w appearing in f.*

The proof proceeds as in [2], except that one uses induction on the longest length of exponents appearing in f. One may still assume that R satisfies a GPI by Theorem 1, and if 0 is the longest length then $f = f_1$ is a G*-PI for R. As in [2], the expression $g(x) = f(cx) - cf(x)$ is a linear G*-DI which contains no basis monomial appearing in f, and has its exponents of longest length at most

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one less than the longest length for f. Thus induction can be applied to g. In the case that the longest length for f is 1, $f = f_1(x,y) + \sum f_d(x^d, y^d)$, $g = \sum c^d f_d(x)$, and the Vandermonde type argument given in [**2**] shows that each $f_d(x)$ is a G*-PI for R. For the general case, let w_1 be any exponent of f of longest length, and let w_1 come from $(d_1, m_2, \ldots, m_k) \in W$. Write $w_1 = d_1 v$ where v comes from $(m_2, \ldots, m_k)$. As in [**2**], by induction on k, $g_v(x)$ is a G*-PI for R, and as in [**2**] one sees that $g_v(x) = \sum q_s c^{d_s} f_{w_s}(x)$ where w_s represents any exponent of f of length k which comes from a k-tuple having some d_s inserted in the appropriate place in the ordered $k-1$ tuple $(m_2, \ldots, m_k)$. Also, q_s counts the number of occurrences of d_s in the k-tuple from which w_s comes. Since the collection of d_s appearing is independent modulo the inner derivations, the Vandermonde type argument shows again that each f_{w_s} is a G*-PI for R, so in particular, $f_{w_1}(x)$ is.

To see how Theorem 7 needs to be changed in light of the change to Theorem 4, we recall that for $f \in F$ which is multilinear and homogeneous of degree n, and having all exponents coming from W, $W(f)$ is the set of all n-tuples $\overline{w} = (w_1, \ldots, w_n)$ for which there is a monomial in f having each w_i as the exponent of x_i, or y_i. Then for any $\overline{w} \in W(f)$, $f_{\overline{w}}(x_1^{w_1}, \ldots, x_n^{w_n}, y_1^{w_1}, \ldots, y_n^{w_n})$ is the sum of all such monomials. Theorem 7 asserts that if f is a G*-DI for an ideal I, then each $f_{\overline{w}}(x_1, \ldots, x_n, y_1, \ldots, y_n)$ is a G*-PI for R. Using the correct statement of Theorem 4 above requires a restriction on which $\overline{w} \in W(f)$ one can use. Call $\overline{w} \in W(f)$ *special* if after some reordering of subscripts, the length of w_1 is maximal among the lengths of exponents in f appearing with either x_1 or y_1, the length of w_2 is maximal among the lengths of exponents of either x_2 or y_2, appearing in any monomial in which the exponent of x_1, or of y_1, is w_1, and in general, the length of w_i is maximal among the lengths of exponents of x_i or y_i which appear in monomials for which $(w_1, \ldots, w_{i-1})$ is the exponent sequence of the variables with subscript smaller than i. The proof of Theorem 7 is valid for all $\overline{w} \in W(f)$ which are special, and the following is the correct statement.

THEOREM 7. *Let R be a prime ring with involution, $*$, and let $f \in F$ be multilinear and homogeneous of degree n with all exponents coming from W and all subscripts of variables in $\{1, 2, \ldots, n\}$. For any special $\overline{w} = (w_1, \ldots, w_n) \in W(f)$, let $f_{\overline{w}}(x_1^{w_1}, \ldots, x_n^{w_n}, y_1^{w_1}, \ldots, y_n^{w_n})$ denote the sum of all monomials in f in which x_i or y_i appears with exponent w_i. If f is a G*-DI for some nonzero ideal I of R, then $f_{\overline{w}}(x_1, \ldots, x_n, y_1, \ldots, y_n)$ is a G*-PI for R, and R satisfies a GPI, unless $f = 0$ in F. Furthermore, the same conclusion holds for every $\overline{w} \in W(f)$ if either no y appears in f, or if each exponent appearing in f commutes with * in* End(R).

We note for Theorem 7 that in the case when every exponent appearing in f is either a derivation or is 1, then an exponent sequence is special if it contains a maximal number of derivations, although other sequences may be special. For example, if $W(f)$ consists of the sequences $(d, 1, d)$, $(1, d, 1)$, and $(h, 1, 1)$, then each would be special. Finally, it is important to observe that the applications we have made of Theorem 4 and Theorem 7 [**3** and **4**] are valid as given, using the corrected versions of these two theorems as they appear here.

The last correction needed in [**2**] is to change the hypothesis of Theorem 9 to assume that for each i, h_i is inner on Q exactly when k_i is inner on Q. With this modification and the comments above, the proof as given in [**2**] holds.

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