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NEW BOOKS

Pattern Theory: The stochastic analysis of real-world signals. By David Mumford and Agnes Desolneux, A K Peters, Ltd., 407 pp., $79.00

This is a remarkable work. It is full of sophisticated mathematical models dealing with all sorts of phenomena from the physical as well as from the man-made world. Or, to use the author's parlance, "analysis of real-world signals". Much work in statistics, pattern recognition, etc. is based on highly simplified models and reflect more the researcher's statistical expertise than an attempt at real understanding of the phenomena studies. In contrast, this book goes deeper and presents serious attempts to really dig deeply into the subject matter of the signals being analyzed.

At the same time it does not avoid the mathematical difficulties in the construction of reality-based models. Indeed, each chapter contains a section called Basics, in which the reader is being prepared to use the mathematical tools suitable for a particular application. These tools are often of an advanced character. Even elementary ideas are presented from a higher and unifying perspective. Or to quote Felix Klein: vom höheren Standpunkt aus. While expertly guiding the reader to pattern theoretic thinking, this book also entertains by presenting additional material in the form of some delightful mathematics in the Basics sections.

One of the first types of signals to be studied in an early chapter is English text (written) beginning with Claude Shannon's thought experiment with simulated text. This gives the authors a natural occasion to introduce the Markov chains following the eponymous mathematician's frequency study of letters in Pushkin's *Yevgeny Onegin*. The natural tool for such analysis turns out to be Bayes' theorem which is fully described, also with the help of dynamic programming. The corresponding problem for spoken language is studied in a subsequent chapter, leading up to a state-of-the-art algorithm for speech recognition.

Music presents many problems to the pattern analyst. Simple sound patterns can be described by Fourier analysis, and the authors lead the reader painlessly through the labyrinth of harmonic analysis. Applied to sound, this can be traced back to Pythagoras, but more complicated sound schemes require different ideas. How about a Bach fugue? Musicologists have wrestled with such difficulties for centuries, searching for order in apparent complexity. In the book some models involve Poisson processes for the breaks in the sound stream, but do not attempt to penetrate the more intricate forms of musical composition. In this connection it should be pointed out that the authors pay little attention to what is sometimes named compositionality, or Frege's principle: taking an architectural approach combining building blocks into structures. This is surprising since many of the models in the book seem expressible through this principle.

Character recognition is one of the most intensively studied tasks, and a chapter is devoted to it, covering edge detection, level curves and medial axes. This is of course related to segmentation in 2D and analyzed in terms of Markov random fields. This has become a standard tool for the analysis of plane images since the seminal work of Geman and Geman employing simulated annealing for image reconstruction. The authors also

point out that Gaussian models have limited power to generate textures, a fact that was overlooked in the early attempts: clutter is not just noise.

Chapter 5 deals with face recognition and is a beautiful example of the book's general tendency: to really penetrate the subject matter of the underlying patterns. This beautifully illustrates the authors' attitude to Pattern Theory. This continues the work by the senior author and his group on face recognition. As a bonus the reader will also learn about Lie groups and algebras presented with the didactic skill that is typical of this book.

The most significant part of the book is Chapter 6, Natural Scenes and their Multiscale Analysis. As Mumford has pointed out repeatedly, one should go out and measure the world, the real world, the real scenes. Only when in possession of such knowledge can one hope to design algorithms for image understanding of natural scenes. This has not always been acknowledged; instead many have tried to rely on clever, or not so clever, statistical and image processing methods. Chapter 6 shows convincingly that such scenes can be analyzed, although so far our knowledge is far from complete. It contains a series of exciting studies, viewing the problem from different angles. The authors succeed in making the reader feel truly involved in the quest for scene theory, the Pattern Theory of visual scenes. The only point where this reviewer hesitates to agree is in the discussion of scale invariance. Theorem 6.5 indicates that, to establish scale invariance, one is forced to go outside the domain $L(Rn)$ to a space of Schwartz distributions. Does this not argue against the hypothesis of scale invariance?

It should be clear from the above that the book is a striking report of remarkable research, an admirable success story. But it is more than that. It points forward, pointing the way for mathematicians to follow, enthusiastically suggesting future work.

It is indeed brilliant!

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