# QUARTERLY

OF

## **APPLIED MATHEMATICS**

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### SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

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Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}]$$
 is preferable to  $e^{[a^2+b^2]^{1/2}}$ .

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\frac{x}{2b}}{\cos\frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t$$
 is preferable to  $\cos t(a + bx)$ .

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures that are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (for books—S. Timoshenko, Strength of Materials, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, On the flow of viscous liquids, especially in three dimensions, Phil. Mag. (5) 36, 354-372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, On the flow of viscous fluids is preferable to On the Flow of Viscous Fluids, but the corresponding German title would have to be rendered as Über die Stromung zaher Flüssigkeiten.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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Optimal Control Theory and Static Optimization in Economics. By Daniel Léonard, Cambridge University Press, 1992, x+353 pp., \$65.00 (cloth), \$29.95 (paper)

The author's approach emphasizes the links between the methods of classical programming and those of optimal control theory. They begin with a thorough exposition of static optimization techniques: unconstrained, equality- and inequality-constrained problems (chapter 1). After presenting some simple solution techniques for differential equations and their qualitative analysis through phase diagrams (chapter 2) they proceed with a short chapter introducing various concepts related to optimization in dynamic models (chapter 3). Chapter 4 describes the optimal control format for dynamic optimization problems and the core of its solution procedures, known as the maximum principle. Chapter 5 gives a brief account of the calculus of variations and dynamic programming. Chapter 6 deals with a much more general control problem, involving several types of constraints. Chapter 7 extends the results by allowing for various boundary conditions at the beginning or the end of the planning horizon. Chapter 8 concentrates on a special class of models that might elicit discontinuities in the controls. Chapter 9 considers infinite horizon problems and chapter 10 treats three separate topics. The book is intended to be a very detailed exposition of static and dynamic optimization, beginning at an elementary level.

The Mollification Method and the Numerical Solution of Ill-Posed Problems. By Diego A. Murio, John Wiley and Sons, 1993, xiii+254 pp., \$69.95

The primary goal of this monograph is to provide an introduction to a number of essential ideas and techniques for the study of inverse problems that are ill posed. It is intended to be a self-contained presentation of practical computational methods which have been extensively and successfully applied to a wide range of ill-posed problems. There is an emphasis on the mollification method and its applications when implemented as a space marching algorithm. As such, the book is intended to be an outline of the numerical results obtained with the mollification method and a manual of various other methods which are also used in arriving at some of these results. Although the presentation concentrates mainly on problems with origins in mechanical engineering, many of the ideas and methods can be easily applied to a broad class of situations.

Stochastic Analysis and Related Topics. Edited by H. Körezlioglu and A. S. Üstünel, Birkhäuser, 1992, 369 pp., \$79.50

This is volume 31 in the series Progress in Probability. It contains ten papers on subjects such as: super processes, Dirichlet forms, anticipative stochastic calculus, random fields and Wiener space analysis. The first part of the volume consists of two main lectures given at the third Silviri meeting in 1990: infinitely divisible random measures and superprocesses, by D. A. Dawson, and Dirichlet forms on infinite dimensional spaces and applications, by M. Röckner.

Time-Variant Systems and Interpolation. Edited by I. Gohberg, Birkhäuser Verlag, 1992, viii+299 pp., \$74.50

This is volume 56 in the series Operator Theory: Advances and Applications. It consists of six papers dealing with the theory of linear time-varying systems and time-varying analogues of interpolation problems. All papers are dedicated to generalizations to the time-variant setting of results and theorems from operator theory, complex analysis and system theory, well-known for the time-invariant case. They represent a cross-section of recent progress in this area.

Representation and Control of Infinite Dimensional Systems, Volume I. By Alain Bensoussan, Giuseppe Da Prato, Michel C. Delfour, and Sanjoy K. Mitter, Birkhäuser, 1992, xii+315 pp., \$74.50

The primary concern of this book is the control of systems whose state space is infinite-dimensional and whose evolution is typically described by a linear partial differential equation, linear functional differential equation or linear integral equation. The authors focus on two aspects of the control problem: (i) qualitative properties such as stability, controllability, and observability; (ii) optimal feedback control of such systems when the performance is measured by a quadratic cost criterion, both over a finite and an infinite interval. In the first volume of the work, a detailed investigation of the problem of representation of infinite-dimensional systems is carried out. The second volume is devoted to a study of the control problem. Chapter headings: 1. Semigroups of operators and interpolation; 2. Variational theory of parabolic systems; 3. Semigroup methods for systems with unbounded control and observation operators; 4. Differential systems with delays.

The Boundary Integral Approach to Static and Dynamic Contact Problems-Equality and Inequality Methods. By H. Antes and P. D. Panagiotopoulos, Birkhäuser, 1992, xiv+307 pp., \$99.50

This is volume 108 in the series International Series of Numerical Mathematics. This book examines both equality and inequality contact constraints in static and dynamic contact problems. The former lead to classical bilateral problems expressed as variational equalities, whereas the latter lead to unilateral problems expressed as variational inequalities in the case of monotone constraints, or as hemivariational inequalities in the case of nonmonotone constraints. The variational equalities give rise to classical boundary integral equations, whereas the variational and hemivariational inequalities lead to the multivalued boundary integral equations. Numerical examples help to elucidate the mathematical developments. There are 13 chapters and a bibliography of close to 500 items.

Meta-Analysis by the Confidence Profile Method-The Statistical Synthesis of Evidence. By David M. Eddy, Vic Hasselblad, and Ross Shachter, Academic Press, 1991, vii+428 pp., \$59.95

This is a volume in the series Statistical Modeling and Decision Science. The Confidence Profile Method is a set of quantitative techniques for interpreting and displaying the results of individual experiments: exploring the effects of biases that affect the internal validity of experiments; adjusting experiments for factors that affect their comparability or applicability to specific questions (external validity); and combining evidence from multiple sources. It can be used either as a Bayesian method to estimate a joint probability distribution for all the parameters of interest in a problem, or as a non-Bayesian or classical method to derive maximum likelihood estimates and covariances for the parameters in a problem. It formally incorporates both experimental evidence and, in a Bayesian analysis, the results of previous analyses or subjective judgements about specific factors that arise in the interpretation of evidence. There are 36 chapters, grouped into six parts: 1. Examples of problems; 2. Setting up the analysis; 3. Formulas of the confidence profile method; 4. Solutions to the example problems; 5. Implementation issues; 6. Conclusions. There is also a computer disk for IBM compatibles containing a tutorial of the method.

Computation and Control II. Edited by K. Bowers and J. Lund, Birkhäuser, 1991, 369 pp., \$78.00

These are the Proceedings of the Second Bozeman Conference, Bozeman, Montana, August 1–7, 1990. It contains the texts of 24 papers. They represent a cross section of the interdisciplinary blend of analytic and numerical techniques that occur between advanced control design and practical numerical solution of lumped and distributed parameter systems.

Introduction to the Calculus of Variation. By U. Brechtken-Manderscheid, translated by P. G. Engstrom, Chapman and Hall, 1991, 220 pp., \$79.95 (cloth), \$35.00 (paper)

This is a translation of the German introductory text first published in 1983. Its scope is apparent from the chapter headings: 1. Introduction to the problem; 2. The Euler differential equation; 3. Sufficient conditions for variational problems with convex integrands; 4. The necessary conditions of Weierstrass and Legendre; 5. The necessary condition of Jacobi; 6. The Hilbert independence integral and sufficiency conditions; 7. Variational problems with variable boundaries; 8. Variational problems depending on several functions; 9. The parametric problem; 10. Variational problems with multiple integrals; 11. Variational problems with side conditions; 12. Introduction to the direct methods of the calculus of variations.

Digital Control Systems. By Rolf Iserman, Vol. II, Springer-Verlag, 1991, xxi+325 pp., \$79.00

This volume is subtitled Stochastic Control, Multivariable Control, Adaptive Control, Applications. It is a translation of the second German edition, which was published in two volumes in 1987 and was a complete revision and enlargement of the first German edition, published in one volume in 1977. Both volumes are addressed to students and engineers in industry desiring to be introduced to the theory and application of digital control systems. The first volume (chapters 1–11) deals with the theoretical basis of linear sampled-data control and with deterministic control. In the second volume, chapters 12–31 are divided into the following parts: Part C: Control systems for stochastic disturbances; D. Interconnected control systems; E. Multivariable control systems; F. Adaptive control systems; G. Digital control with process computers and microcomputers.

A Guide to Computer Algebra Systems. By David Harper, Chris Woof, and David Hodgkinson, John Wiley and Sons, ix+148 pp., \$29.95

This is a volume in the series Wiley Professional Computing. In it, the author compares and takes a critical look at several popular computer algebra systems: REDUCE, MACSYMA, Maple, Mathematica, and Derive. There are tables comparing the systems from different points of view and case studies using the systems. There is also a brief history of such systems and a bibliography for further study.

Statistical Process Control for Quality Improvement. By James R. Thompson and Jacek Koronacki, Chapman and Hall, 1993, xx+391 pp., \$69.00 (cloth), \$29.95 (paper)

This text is the outcome of much experience in consulting and lecturing on quality improvement, as well as in mathematical modeling of practical situations. The earlier part of the book can be utilized by persons only interested in practical implementation of the paradigm of statistical process control. There is a mathematical and statistical appendix. Problems are given at the end of each of the seven chapters:

1. Statistical process control: a brief overview; 2. Acceptance-rejection SPC; 3. The development of mean and standard deviation control charts; 4. Sequential approaches; 5. Exploratory techniques for preliminary analysis; 6. Optimization approaches; 7. Multivariate approaches.

Design and Analysis of Reliability Studies. By Graham Dunn, John Wiley and Sons, 1993, viii+190 pp., \$59.95

This book is subtitled The Statistical Evaluation of Measurement Errors, and is a general and practically oriented introduction to the dependability or reliability of measurements, ranging from nominal classifications through simple ordinal rating scales to data assumed to be measured on an interval scale. Although primarily intended to be used by behavioural, medical and social scientists, it should be useful to physical scientists and biologists. The other group of potential readers of this text are applied statisticians who may be concerned with giving advice about the measurement of reliability but who will not be specialists in the field. The first three chapters cover the concepts required for the study of the rest of the text. Chapter 4 is concerned with design of a study to assess the performance of a measuring device or group of measuring instruments, and chapters 5–7 cover inferential methods useful in the analysis of data generated in a reliability study, covering techniques appropriate for both interval as well as categorical (including ordinal) measurements.

Mark Kac Seminar on Probability and Physics, Syllabus 1987–1992. Edited by F. den Hollander and H. Maassen, Centre for Mathematics and Computer Science, 1992, ix+179 pp., Dfl. 60.00

This is a volume in the series CWI Syllabi. It contains the texts of a selection of lectures delivered at the Seminar: Properties of general self-similar processes (W. Vervaat), conformal field theories on Riemann surfaces (O. Foda), entropy and Markov properties of equilibrium states of lattice models (J. M. Lindsay and H. Maassen), an invitation to probabilistic cellular automata (C. Maes).

Cluster Analysis. By Brian S. Everitt, John Wiley and Sons, 1993, viii+170 pp., \$59.95

This is the third, revised, edition of the book first published in 1974. The author's aim was to provide a convenient and readable introduction of clustering to workers in many fields: psychiatry, archeology, market research, anthropology, etc. In this edition more emphasis has been placed on examples in addition to describing recent work. Chapter headings: 1. An introduction to classification and clustering; 2. The initial examination of multivariate data; 3. Measurement of similarity, dissimilarity and distance; 4. Hierarchical clustering techniques; 5. Optimization methods for cluster analysis; 6. Mixture models for cluster analysis; 7. Other clustering techniques; 8. Some final comments and guidelines. Appendix: Software.

Chaos, Dynamics and Fractals. By J. L. McCauley, Cambridge University Press, 1993, xxi+323 pp., \$89.95

This volume 2 in the Cambridge Nonlinear Science Series is subtitled: an algorithmic approach to deterministic chaos. It develops deterministic chaos and fractals from the standpoint of iterated maps and is written to provide the reader with an introduction to more recent developments, such as weak universality, multifractals, and shadowing, as well as to older subjects like universal critical exponents, devil's staircases and the Farey tree. It tries to answer questions such as: How can a deterministic trajectory be unpredictable? How can one compute nonperiodic chaotic trajectories with controlled precision? Can a deterministic trajectory be random? What are multifractals and where do they come from? What is turbulence and what has it to do with chaos and multifractals? Why is it not merely convenient, but also necessary, to study classes of iterated maps instead of differential equations when one wants predictions that are applicable to computation and experiment? Chapter headings: 1. Flows in phase space; 2. Introduction to deterministic chaos; 3. Conservative dynamical systems; 4. Fractals and fragmentation in phase space; 5. The way to chaos by instability of quasiperiodic orbits; 6. The way to chaos by period doubling; 7. Introduction of multifractals; 8. Statistical mechanics on symbol sequences; 9. Universal chaotic dynamics; 10. Intermittence in fluid turbulence; 11. From flows to automata: chaotic systems as completely deterministic machines.

Optimal Control and the Calculus of Variations. By Enid R. Pinch, Oxford University Press, 1993, viii+234 pp., \$49.95

This is a volume in the series Oxford Science Publications. It aims to introduce readers who have a serious interest in its mathematical aspects to the subject of optimal control. It assumes an undergraduate background in analysis, algebra, and mathematical methods, but not measure theory; for instance, the author proves the central result of optimal control theory, Pontryagin's maximum principle, rigorously but by elementary methods, in chapter 6. For pedagogic reasons, he first introduces the optimization of functions and the methods of the calculus of variations: he starts by looking at the problem of finding the global minimum of a function of one variable and builds up from there. Chapter headings: 1. Introduction; 2. Optimization in  $\mathbb{R}^n$ ; 3. The calculations of variations; 4. Optimal control I: theory; 5. Optimal control II: applications; 6. Proof of the maximum principle of Pontryagin.

Annual review of Fluid Mechanics, Volume 26, 1994. Edited by John L. Lumley, Milton Van Dyke, and Helen L. Reed, Annual Reviews Inc, 1994, 704 pp., \$47.00

This year, the featured memoir is entitled "A. N. Kolmogorov as a fluid mechanician and founder of a school in turbulence research" and is written by one of Kolmorov's students and collaborators, A. M. Yaglom. It is distinguished by perceptive personal reminiscences, and by perspicacious insight into Kolmogorov's contributions as well as their place in the history of the problems he tackled. There are seventeen other papers on subjects such as turbulence, drop deformation in viscous flows, wave evolution, high Rayleigh number convection, vortex reconnection, long water waves, oceanography, supersonic turbulent boundary layers, combustion, climate dynamics and global change, Görtler vortices, pulmonary flow, vortex interactions with walls, ocean-atmosphere interaction, and others.