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The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2+b^2)^{1/2}] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\frac{x}{2b}}{\cos\frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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Vol. L, No. 1 **March 1992**

Non-Life Insurance Mathematics. By Erwin Straub. Springer-Verlag, 1988. 136 pp., $49.50.

This text is based on lecture notes written between 1975 and 1985 when the author taught the subject at the University of Berne. It is designed to be a textbook for University students and for nonexpert practitioners. Its main purpose is to provide insight into some pertinent practical problems and their possible theoretical solutions. There are eight chapters. In Chapter 1 some of the problems are defined. Chapter 2 gives some of the main results of the collective theory of risk, including Cramèr's inequality. Chapter 3 discusses premiums, under the headings Pragmatic Principles, Theoretical Principles, and Experience Rating. The titles of the other chapters are: 4. Reinsurance; 5. Retentions; 6. Statistics; 7. Reserves; 8. Solutions.

An Introduction to the Theory of Point Processes. By D. J. Daley and D. Vere-Jones. Springer-Verlag, 1988. xxi+702 pp., $79.00

The authors' intention in producing this book was to write a text that would provide a survey of point process *theory* accessible to beginning graduate students and workers in applied fields. With this in mind they adopted a partly historical approach, starting with an informal introduction followed by a more detailed discussion of the most familiar and important examples, and then moving gradually into topics of increased abstraction and generality. Chapters 1–4 provide historical background and treat fundamental special cases (Poisson processes, stationary processes on the line, and renewal processes). Chapter 5, on finite point processes, has a bridging character, while Chapters 6–14 develop aspects of the general theory. The general results from Chapter 6 onwards are couched in the setting of complete separable metric spaces, although the examples continue to be drawn mainly from one- or two-dimensional Euclidean spaces. Two appendices collect together the main results needed from measure theory and the theory of measures on metric spaces. A third appendix summarizes the main ideas needed for Chapter 13, on the martingale approach, from martingale theory and the general theory of processes. The authors have taken as their model the texts by Feller with their format of motivating and illustrating the general theory with a range of examples, many taken from real applications. They have tried to strike a mean between the rigorous, abstract treatments and the practically motivated but informal ones available in the literature. Chapter headings: 1. Early History; 2. Basic Properties of the Poisson Process; 3. Simple Results for Stationary Point Processes on the Line; 4. Renewal Processes; 5. Finite Point Processes; 6. Introduction to the General Theory of Finite Measures; 7. Introduction to the General Theory of Point Processes; 8. Cluster Processes, Infinitely Divisible Processes, and Doubly Stochastic Processes; 9. Convergence Concepts and Limit Theorems; 10. Stationary Point Processes and Random Matrices; 11. Spectral Theory; 12. Palm Theory; 13. Conditional Intensities and Likelihoods; 14. Exterior Conditioning.

Numerical Techniques for Stochastic Optimization. Yuri Ermoliev and Roger J.-B. Wets (editors). Springer-Verlag, 1988. xiii + 571 pp., $74.00.

This is Volume 10 in the Springer Series in Computational Mathematics. It is the result of a project on "Numerical Methods for Stochastic Optimization Problems" by the International Institute for Applied Systems Analysis, Laxenburg, Austria. The book is divided into four parts. Part I is an introduction to some general and particular stochastic programming problems as models for decision making under uncertainty. Part II consists of a number of chapters, each covering some of the numerical questions that must be dealt with when developing solution procedures for stochastic programming problems. Part III describes the implementation of a number of methods. Part IV is a collection of selected applications and test problems. Altogether there are 30 chapters in these four parts.

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The Elements of Stochastic Processes, with Applications to the Natural Sciences. By Norman T. J. Bailey. John Wiley & Sons, 1990. xi+249 pp., $34.50.

This is the Wiley Classics Library Edition of the monograph first published in 1964.

Fourier Analysis on Groups. By Walter Rudin. John Wiley & Sons, 1990. ix+285 pp., $26.95.

This is the Wiley Classics Library Edition of the treatise first published in 1962.

Sample Design in Business Research. By W. Edwards Deming. John Wiley & Sons, 1990. xx+517 pp., $29.95.

This is the Wiley Classics Library Edition of the standard text first published in 1960.

Finite Temperature Field Theory. By Joseph I. Kapusta. Cambridge University Press. x+219 pp., $59.50.

This is a volume of the Cambridge Monographs on Mathematical Physics. The monograph is based on lectures given in a graduate course on relativistic many-body theory and assumes that the reader has had graduate level courses on thermal and statistical physics and in relativistic quantum field theory. It explores questions such as what happens when ordinary matter is compressed so that the electrons form a relativistic degenerate gas (as in a white dwarf), or so that atomic nuclei overlap to form superdense nuclear matter (as in a neutron star), or heated such that nucleons and pions melt into quarks and gluons (as in high-energy nucleus-nucleus collisions), etc. It is the purpose of the monograph to present a coherent picture of the field and to prepare the reader to read the original and current literature. Chapter headings: 1. Review of quantum statistical mechanics; 2. Functional integral representation of the partition function; 3. Interactions and diagrammatic techniques; 4. Renormalization; 5. Quantum electrodynamics; 6. Linear response theory; 7. Spontaneous symmetry breaking and restoration; 8. Quantum chromodynamics; 9. Weak interactions; 10. Nuclear matter.

The Econometric Analysis of Time Series. By Andrew Harvey. The MIT Press, 1990. xii+387 pp., $47.50.

This volume in the series London School of Economics Handbooks in Economics is the second edition of a monograph first published in 1980. It includes new material on a number of topics and several sections have been completely rewritten. It focuses on the statistical aspects of model building, with an emphasis on providing an understanding of the main ideas and concepts in econometrics rather than presenting a series of rigorous proofs. It explores the way in which recent advances in time series analysis have affected the development of a theory of dynamic econometrics, sets out an integrated approach to the problems of estimation and testing based on the method of maximum likelihood and presents a coherent strategy of model selection. Chapter headings: 1. Introduction; 2. Regression; 3. The method of maximum likelihood; 4. Numerical optimisation; 5. Test procedures and model selection; 6. Regression models with serially correlated disturbances; 7. Dynamic models I; 8. Dynamic models II: stochastic difference equations; 9. Simultaneous equation models.

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Finding Groups in Data: An Introduction to Cluster Analysis. By Leonard Kaufman and Peter J. Rousseeuw. John Wiley & Sons, 1990. xiv+342 pp., $49.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. The authors' purpose was to write an applied book for the general user who does not necessarily have a strong mathematical or statistical background. The selection of methods was based on a combination of methodological aims (mainly robustness, consistency, and general applicability) and the authors' experience in applying clustering to a variety of disciplines. Chapter headings: 1. Introduction; 2. Partitioning around mediods (Program PAM); 3. Clustering large applications (Program CLARA); 4. Fuzzy analysis (Program FANNY); 5. Agglomerative nesting (Program AGNES); 6. Divisive analysis (Program DIANA); 7. Monothetic analysis (Program MONA). The programs described here are for the IBM PC, but the source code is very portable and has been run on several types of mainframes.

Statistics in Scientific Investigation: Its Basis, Application, and Interpretation. By Glen McPherson. Springer-Verlag, 1990. xxvi+666 pp., $49.95.

This is a volume in the series Springer Texts in Statistics. The author's aim is to provide an insight into statistics and a blueprint for statistical applications for a wide audience. It is the first of three projected volumes of a comprehensive treatise. Mathematical prerequisites are minimal and the emphasis is on the role of statistics as the foundation of scientific investigations. However, the role of probability, and of mathematics in general, in the development of statistics is also stressed. There are many examples, and the place of computers in statistical analysis is discussed.

Relativistic Fluids and Magneto-Fluids, with Applications in Astrophysics and Plasma Physics. By A. M. Anile. Cambridge University Press, 1990. xii+336 pp., $85.00.

This is a volume in the series Cambridge Monographs on Mathematical Physics. It is its aim to provide a unified and systematic treatment of the main results and techniques of relativistic fluid dynamics with an emphasis on waves and shock waves. The first four chapters provide an introduction to the fundamental principles of the subject. The remaining four chapters present more specific topics, including relativistic simple waves in fluids and magneto-fluids, nonlinear electromagnetic waves in relativistic cold plasmas, relativistic asymptotic waves, and relativistic shock waves. Examples are given of the applications of the theory to plasma physics, nuclear physics, and astrophysics.

Ramsey Theory. By Ronald L. Graham, Bruce L. Rothschild, and Joel H. Spencer. John Wiley & Sons, 1990. x+196 pp., $49.95.

This is the second edition of a volume in the Wiley-Interscience Series in Discrete Mathematics and Optimization, first published in 1980. It gives a complete treatment of Saharon Shelah's totally elementary 1987 proof of van der Waerden's Theorem, a cornerstone of Ramsey Theory: if the positive integers are finitely colored then one color class contains arithmetic progressions of arbitrary length. The other classic theorems are: Schur—If the positive integers are finitely colored then one class contains x, y, z with $x+y=z$; Ramsey—If a graph contains sufficiently many vertices (depending on k) then it must contain either a complete set or an independent set of vertices of size k. The authors' aim is to give a clear, selfcontained exposition of the central results of Ramsey theory, showing it to be a cohesive subdiscipline of Discrete Mathematics.

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Graphs, An Introductory Approach—A First Course in Discrete Mathematics. By Robin J. Wilson and John J. Watkins. John Wiley and Sons, 1990. x+340 pp., $36.95.

This book arose out of a British Open University course on Graphs, Networks and Design, first given in 1981. It consists of two parts. Part 1 (five chapters) contains the basic definitions relating to graphs and digraphs, together with a large number of examples and applications. Part II (ten chapters) contains a number of different topics: Eulerian and Hamiltonian graphs and digraphs, algorithms, connectivity, trees, planarity, coloring maps, decomposition problems.

Common Principal Components and Related Multivariate Models. By Bernhard Flury. John Wiley & Sons, 1988. 258 pp., $39.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is its purpose to provide a comprehensive review of some multivariate statistical models that have evolved over the last few years, such as, for instance, generalizations of principal component analysis to several groups. The book is aimed at both the researcher in multivariate statistics and the advanced practitioner who would like to use such methods. Chapter headings: 1. Preliminaries; 2. Principal Component Analysis; 3. Relationship between Covariance Matrices; 4. Common Principal Components; 5. Proportional Covariance Matrices; 6. Partial Common Principal Component and Common Space Analysis; 7. How Different are Several Covariance Matrices? 8. Miscellanea; 9. Numerical Methods. There are also three appendicies, two on matrix analysis and one on a Fortran program for the FG algorithm.

Introduction to the Theory of Coverage Processes. By Peter Hall. John Wiley & Sons, 1988. viii + 408 pp., $39.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It develops mathematical theories for describing random patterns in one or more dimensions. A coverage process, which can be generated in many ways, may be characterized by a countable sequence of sets in Euclidean space, the centers of the sets comprising a stochastic point process. The most interesting examples occur when the sets have positive measure, or are at least uncountable. The subject is developed primarily in the continuum, with emphasis on the cases of two or three dimensions, the one-dimensional case (e.g., segments, queues, and counters) being addressed in the context of classical stochastic processes. Early chapters investigate covering problems such as lattice processes, and parking and packing problems. Middle chapters address Poisson-centered line segments, the concept of vacancy, random sets, and Boolean models. Closing chapters examine problems of counting and clumping in Euclidean space (with applications to the theory of estimation), and elements of inference. Examples treated include queueing theory, renewal processes, point processes, shot noise processes, branching processes, percolation, random set theory, integral and differential geometry, and all types of stochastic limit theory. Actual applications range from image processing to industrial safety.

Geometric Reasoning. Edited by John Woodwark. Oxford University Press, 1990. xiii+282 pp., $49.95.

These are proceedings of a conference on problems having to do with information about the shape of objects, image analysis, computer graphics, computational geometry, and similar fields which can come under the umbrella of Geometric Reasoning. There are fifteen papers by various authors.

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Mathematical Bioeconomics: The Optimal Management of Renewable Resources. By Colin W. Clark. John Wiley & Sons, 1990. xii+386 pp., $49.95.

This is the second, updated, edition of a volume in the series Pure and Applied Mathematics. There are new chapters on the theory of resource regulation, and stochastic resource models show how to map out an economically effective resource management system. It also includes new sections on the issue of irreversible investment, game theoretic models, and dynamic programming, as well as an expanded bibliography.

Statistical Methods for Quality Control. By Thomas P. Ryan. John Wiley & Sons, 1989. xviii + 446 pp., $49.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It presents an in-depth treatment of techniques useful in modern methods of quality improvement. This includes the traditional topics of control charts and acceptance sampling, but also tools such as an analysis of means and statistical design of experiments which are becoming recognized as relevant to the subject. It provides a critical review of Taguchi methods, a variety of control charts with emphasis on cumulative sum procedures, and a bibliography of control charts for both manufacturing and nonmanufacturing applications. The sixteen chapters are divided into three parts: 1. Fundamental Quality Control and Statistical Concepts; 2. Control Charts and Process Capability; 3. Beyond Control Charts: Graphical and Statistical Methods.

The Evolving Role of Statistical Assessment as Evidence in the Courts. Stephen E. Fienberg (editor). Springer-Verlag, 1989. vii + 357 pp., $34.00.

This book is the product of the Panel on Statistical Assessments as Evidence in the Courts, which was convened by the Committee on National Statistics of the National Research Council to help clarify the issues raised by the increasing frequency with which the proof of facts in legal proceedings has in recent times involved the use of quantitative information. This has brought about the need for judges, lawyers, statisticians, social scientists, and others, to address issues such as the evaluation of quantitative evidence, the ethical and professional obligations of expert witnesses, and the roles of court-appointed witnesses. The report—the result of more than three years of research and deliberation—addresses a variety of issues that arise in federal and state court proceedings when statistical assessments such as quantitative descriptions, causal inferences, and predictions of events based on earlier occurrences are presented as evidence. Its recommendations include several innovations to improve the comprehension of statistical evidence by judges and juries.

Probabilistic Reasoning in Expert Systems: Theory and Algorithms. By Richard E. Neapolitan. John Wiley & Sons, 1990. xiii+433 pp., $44.95.

This text is written from the perspective of a mathematician, with the emphasis being on the development of theorems and algorithms. Since every effort is made to keep the material selfcontained, the necessary background in probability and graph theory, as well as the philosophical foundations of probability theory, are reviewed. Chapter headings: 1. Introduction. 2. Probabilistic considerations; 3. Graph theoretic considerations; 4. Rule-based systems versus causal (belief) networks; 5. Causal (belief) networks; 6. Probability propagation in singly connected causal networks; 7. Probability propagation in trees of cliques; 8. Abductive inference; 9. Applications to decision theory; 10. Variability and uncertainty in probabilities; obtaining probabilities.

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Probability Approximations via the Poisson Clumping Heuristic. By D. Aldous. Springer-Verlag, 1989. xv + 269 pp., $44.00.

This monograph addresses problems such as the following. What is the distribution of the radius of the largest circle containing none of the points placed randomly in a unit square, or of the smallest circle containing four? Why do Brownian sample paths have local maxima but no points of increase? How long does it take a particle performing a random walk on the vertices of a high-dimensional cube to visit every vertex, or a particle moving under the influence of a potential field and random perturbations of velocity to escape from a deep potential well? What is the longest stretch of empty road seen from a car moving with constant speed (random from car to car) on a freeway? The common theme in all these problems, and the others (over 100), discussed in this book is that they all deal with maxima or minima, and it is the purpose of the book to explain a simple idea which enables one to write down solutions to such problems. The idea is that such problems can often be translated into problems about sparse random sets resembling i.i.d. random clumps centered at points of a Poisson process, and that the problem reduces to estimating mean clump size which can be estimated by approximating the underlying random process by a simpler process for which explicit calculations are possible. The author contends that this idea, though rarely explicitly stated, is known in several specific settings, but that its power and range seems not to be appreciated. He claims that it provides the correct way to look at extrema and rare events in a wide range of probabilistic settings, and sets out to demonstrate this assertion. The arguments are informal. Chapter headings: 1. The heuristics; 2. Markov chain hitting times; 3. Extremes of stationary processes; 4. Extremes of locally Brownian motion; 5. Simple combinatorics; 6. Combinatorics for processes; 7. Exponential combinatorial extrema; 8. Stochastic geometry; 9; Multi-dimensional diffusions; 10. Random fields; 11. Brownian motion: local distributions; 12. Miscellaneous examples; 13. The eigenvalue problem.

A Journey into Gravity and Spacetime. By John Archibald Wheeler. Scientific American Library, W. H. Freeman and Company, 1990. 299 pp. with 150 illustrations, $32.95.

In this beautifully produced, printed, and illustrated book, the author, one of the giants of modern physics, explores Einstein's geometric theory of gravity. He begins by tracing the development of this doctrine, chronicling the ideas and contributions of Newton, Leibniz, Gauss, Riemann, and Mach and ends by applying the theory to both familiar and exotic phenomena, all without using mathematics. The chapter headings are: 1. Great men, great ideas; 2. From fall to flat; 3. Interval: revelation that all of space is ours; 4. Boomeranging through the earth; 5. Tides: the grip of mass on spacetime; 6. Momenergy: the grip of spacetime on mass; 7. The boundary of a boundary: where the action is; 8. From potter's wheel to space geometry; 9. Picturing space and spacetime around a center of mass; 10. Stones in flight and planets in orbit; 11. Gravity waves; 12. Black holes; 13. A farewell look at gravity.

Pooled Cross-sectional and Time Series Data Analysis. By Terry E. Dielman. Marcel Decker, Inc., 1989. vii + 249 pp.

This is Volume 97 in the series Statistics: Textbooks and Monographs. In econometrics, a data base that provides a multivariate statistical history for each of a number of individual entities is called a pooled cross-sectional and time series data base; the terms panel data or longitudinal data are also used. This book reviews methods for estimating multivariate relationships particular to each individual entity in the data base and for summarizing these relationships for a number of individuals. Methodologies examined include classical pooling, error components, analysis of covariance, seemingly unrelated regression, and random coefficient regression. In each case the model is presented with assumptions necessary for estimation and inference, examples are given, the limitation of the model discussed, and available computer software is mentioned. The orientation of the book is applied, with emphasis on the analysis of data. Properties of various estimators are discussed, but proofs are limited to those not readily available in the literature.