# QUARTERLY

OF

# APPLIED MATHEMATICS



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# QUARTERLY OF APPLIED MATHEMATICS

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# SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter $O$, between the numeral one (1), the letter $l$ and the prime ($'$), between alpha and $a$, kappa and $k$, mu and $u$, nu and $v$, eta and $n$.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2+b^2)^{1/2}] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\frac{x}{2b}}{\cos\frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

**Figures:** Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

## CONTENTS

**Vol. XLVII, No. 1** **March 1989**

*Mixture Models: Inference and Applications to Clustering.* By Geoffrey J. McLachlan and Kaye E. Basford. Marcel Dekker, New York, 1988. pp. x+253.

This is volume 84 of the series Statistics: Textbooks and Monographs. The purpose of this book is to highlight the important role of finite mixture distributions in modelling heterogeneous data, with the focus on applications in the field of cluster analysis. The practical applications of mixture models are presented against the background of the existing literature for inference undertaken in a finite mixture framework. Attention is concentrated on the fitting of mixture models by a likelihood-based approach, using maximum likelihood where appropriate. The EM algorithm provides a convenient way for the iterative computation of solutions of the likelihood equation. The emphasis is on mixtures of normal distributions. There is also an account of items such as the detection of atypical observations, the assessment of model fit, and robust estimation for finite mixture models. Chapter headings: 1. General introduction. 2. Mixture models with normal components. 3. Applications of mixture models to two-way data sets. 4. Estimation of mixing proportions. 5. Assessing the performance of the mixture likelihood approach to clustering. 6. Partitioning of treatment means in ANOVA. 7. Mixture likelihood approach to the clustering of three-way data.

*Limit Theorems for Stochastic Processes.* By J. Jacob and A. N. Shiryaev. Springer-Verlag, New York, 1987. pp. xvii+600. $98.00.

This is volume 288 of Grundlehren der mathematischen Wissenschaften, A Series of Comprehensive Studies in Mathematics. The limit theorems in this book belong to the theory of weak convergence of probability measures on metric spaces. More precisely, the authors' main aim is to give a systematic exposition of the theory of convergence in law for those stochastic processes that are semimartingales. The choice of the class of semimartingales as the authors' chief object of study has two reasons. One is that this class is broad enough to accommodate most common processes: discrete-time processes, diffusions, many Markov processes, point processes, solutions of stochastic differential equations, etc. The second reason is that it represents a very powerful tool for studying these processes, namely the stochastic calculus. The first two chapters constitute a rather complete account of the theory of semimartingales and related topics as random measures. The scope of this and other chapters is represented by the table of contents: 1. The general theory of stochastic processes, semimartingales and stochastic integrals. 2. Characteristics of semimartingales and the process with independent increments. 3. Martingale problems and changes of measures. 4. Hellinger processes, absolute continuity and singularity of measures. 5. Contiguity, entire separation, convergence in variation. 6. Skorokhod topology and convergence of processes. 7. Convergence of processes with independent increments. 8. Convergence to a process with independent increments. 9. Convergence to a semimartingale. 10. Limit theorems, density processes and contiguity.

*Multiple Comparison Procedures.* By Yosef Hochberg and Ajit C. Tamhane. John Wiley & Sons, New York, 1987. pp. xxii+450. $44.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. The authors focus on problems involving multiplicity and selection ("data-snooping") of inferences when comparing treatments based on univariate responses. Roy's union-intersection method forms the unifying theme used to derive the various classical multiple comparison procedures for these problems. In Chapter 1, the authors elaborate their philosophy and approach to multiple comparison problems, and discuss the basic notions of families, error rates, and control of error rates. The remainder of the book is divided into two parts. Part I, consisting of Chapters 2–6, deals with MCPs based on a classical error rate control approach for fixed-effects of linear models under the usual normal theory assumptions. Part II, consisting of Chapters 7–11, deals with MCPs for other models and problems, and MCPs based on alternative approaches (e.g., decision-theoretic and Bayesian). Chapter headings: 1. Introduction. 2. Some theory of multiple comparison procedures for fixed-effects linear models. 3. Single-step procedures for pairwise and more general comparisons among all treatments. 4. Stepwise procedures for pairwise and more general comparisons among all treatments. 5. Procedures for some nonhierarchical finite families of comparisons. 6. Designing experiments for multiple comparisons. 7. Procedures for one-way layouts with unequal variances. 8. Procedures for mixed two-way layouts and designs with random covariates. 9. Distribution-free robust procedures. 10. Some miscellaneous multiple comparison problems. 11. Optimal procedures based on decision-theoretic, Bayesian, and other approaches.

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*Lectures on Stochastic Analysis: Diffusion Theory.* By Daniel W. Stroock. Cambridge University Press, New York, 1987. pp. ix+128. $34.50 Hardcover, $11.95 Paperback.

This is volume 6 of the series London Mathematical Society Student Texts. It grew out of lectures given at M.I.T. It was the author's purpose to provide a reasonably self-contained introduction to some stochastic analytic techniques which can be used in the study of certain analytic problems, and his method was to concentrate on a particularly rich example rather than to attempt a general overview. The example which he chose was the study of second-order partial differential operators of parabolic type. The material covered is derived from the book *Multidimensional Diffusion Processes*, by the author and S. R. S. Varadhan, Springer-Verlag, 1979. However, the presentation is different in that here, the emphasis is on conceptual clarity rather than generality and detail. It also eases the presentation by using the modern theory of martingales and stochastic integration. Chapter headings: 1. Stochastic processes and measures on function space. 2. Diffusions and martingales. 3. The martingale problem formulation of diffusion theory.

*Irregularities of Distribution.* By Józesf Beck and William Chen. Cambridge University Press, New York, 1987. pp. xiv+294. $54.50.

This is volume 89 of Cambridge Texts in Mathematics. It is an up-to-date description of the various approaches to and methods in the theory of irregularities of distribution. The subject is primarily concerned with number theory, but also borders on combinatorics and probability theory. The work is in three parts. The first (Chapters 1–4) is concerned with the classical problem, complemented with more recent results. In the second part (Chapters 5–8), the authors study generalizations of the classical problem, pioneered by Schmidt. Here, they include chapters on the integral equation method of Schmidt and the more recent Fourier transform technique. The final part (Chapter 9) is devoted to Roth's "$\frac{1}{4}$-theorem"; some recent advances are also discussed. Chapter headings: 1. Van der Corput's conjecture. 2. Lower bounds—Roth's method. 3. Upper bounds. 4. Lower bounds—a combinatorial method of Schmidt. 5. Schmidt's work. 6. A Fourier approach. 7. Further applications of the Fourier transform methods. 8. More upper bounds. 9. Miscellaneous questions.

*Principles of Thermodynamics and Statistical Mechanics.* By D. F. Lawden. John Wiley & Sons, New York, 1987. pp. xi+154. $34.95.

This text is based on a course of lectures given by the author to third-year mathematics students at the University of Aston in Birmingham. The sequence of chapters is planned along chronological lines, starting with the classical theory of thermodynamics and ending with the derivation of the statistics for bosons and fermions by use of the grand canonical ensemble. Chapter headings: 1. Classical thermodynamics. 2. Applications of classical theory. 3. Classical statistics. Maxwell's distribution. 4. Method of mean value. 5. Quantum statistics. 6. Crystals and magnets. 7. Bose–Einstein and Fermi–Dirac statistics.

*Extreme Values , Regular Variation, and Point Processes.* By Sidney I. Resnick. Springer-Verlag, New York, 1987. pp. xii+320. $71.00.

This is volume 4 of Applied Probability, a Series of the Applied Probability Trust. Extreme value theory is an elegant theory as well as a subject with applications to situations where the features of most interest depend on the largest and smallest values of the data collected (air pollution monitoring, design strength of buildings due to wind stress, heights of dams, the drilling of core samples to determine ore concentrations, etc.). The book is primarily concerned with the behavior of extreme values of independent, identically distributed observations. The treatment is organized around two themes. The first is that the central analytic tool of extreme value theory is the theory of regularly varying functions, and the second is that the central probabilistic tool is point process theory and, in particular, the Poisson process. Chapter headings: 0. Preliminaries. 1. Domains of attraction and norming constants. 2. Quality of convergence. 3. Point processes. 4. Records and extremal processes. 5. Multivariate extremes.

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## *Linear Models for Unbalanced Data.* By Shayle R. Searle. John Wiley & Sons, New York, 1987. pp. xxiv+536.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is a textbook for students and a reference book for working statisticians who want to learn the fundamentals of linear statistical models. It is also designed for trained statisticians who want to broaden their understanding of linear models in order to better interpret their data from analyses that come from statistical computing packages. The three salient features of the book are (i) its total devotion to unbalanced data—i.e., data having unequal numbers of observations in the subclasses, (ii) its emphatic dependence on the up-to-date cell means model approach to linear models for unbalanced data, and (iii) its organization, in that the first half of the book requires no knowledge of matrices, whereas the second half does. Unbalanced data are the focal point of the book because (a) they are harder to analyze and understand than balanced data; (b) unbalanced data are becoming increasingly prevalent through computer storage of data; and (c) since the staggering amount of arithmetic often required for analyzing unbalanced data can be done so quickly and cheaply by today's statistical computing packages, analysis of data are being increasingly done by people who are not fully knowledgeable about the statistical implications. Chapter headings: 1. An up-dated viewpoint: Cell means models. 2. Basic results for cell means models: The 1-way classification. 3. Nested classifications. 4. The 2-way crossed classification with all-cells-filled data: Cell means models. 5. The 2-way classification with some-cells-empty data: Cell means models. 6. Models with covariables (analysis of covariance): The 1-way classification. 7. Matrix algebra and quadratic forms (a prelude to Chapter 8). 8. A general linear model. 9. The 2-way crossed classification: overparametrized models. 10. Extended cell means models. 11. Models with covariables: The general case and some applications. 12. Comments of computing packages. 13. Mixed models: A thumbnail survey.

## *Teichmüller Theory and Quadratic Differentials.* By Frederick P. Gardiner. John Wiley & Sons, New York, 1987. pp. xvii+236. $46.95.

This is a volume in the series Pure and Applied Mathematics, founded by Richard Courant. It is its purpose to give a self-contained development of Teichmüller theory and related results, with the most important background material presented in the first chapter. A first-year graduate course in complex analysis is assumed in the background of the reader. Chapter headings: 1. Results for Riemann surface theory and quasiconformal mapping. 2. Minimal norm properties for holomorphic quadratic differentials. 3. The Reich–Strebel inequality for Fuchsian groups. 4. Density theorems for quadratic differentials. 5. Teichmüller's theory. 6. Teichmüller's theorem. 7. Teichmüller's and Kobayashi's metrics. 8. Discontinuity of the modular group. 9. Holomorphic self-mappings of Teichmüller space. 10. Quadratic differentials with closed trajectories. 11. Measured foliations.

## 300 *Years of Gravitation.* Edited by S. W. Hawking and W. Israel. Cambridge University Press, New York, 1987. pp. xiii+684. $69.50.

To commemorate the 300th anniversary of the publication of Isaac Newton's *Philosophiae Naturalis Principia Mathematica*, Stephen Hawking and Werner Israel have assembled a series of unique review papers by many of the world's foremost researchers in cosmology, relativity, and particle physics. The resulting volume reflects the significant and exciting advances that have been made since the editors' acclaimed volume published in 1979: *General Relativity: An Einstein Centenary Survey.* Newton's immense contribution to the physical sciences is assessed, and its relevance to today's physics made clear. The international contributors then chart the major developments in the study of gravitation, from Newtonian gravity to black hole physics. In the fields of galaxy formation, inflationary and quantum cosmology, and superstring unification, the book provides important overviews written by workers involved in many of the advances described. In addition to the editors, the following authors are represented: S. Weinberg, R. Penrose, A. H. Cook, C. M. Will, T. Darmour, R. D. Blanford, K. S. Thorne, M. J. Rees, A. Vilenkin, S. K. Blau, A. H. Guth, A. Linde, J. H. Schwarz, C. Crnkovic, and E. Witten.

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*Measurement Error Models.* By Wayne A. Fuller. John Wiley & Sons, New York, 1987. pp. xxiii+440. $44.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is an outgrowth of research on the measurement error—also called response error—in data collected from human respondents. It was written with the objective of increasing the use of statistical techniques explicitly recognizing the presence of measurement error. There is an introduction to techniques for simple models, such as known measurement variance, instrumental variable estimation, factor analysis, and others. Subsequent chapters examine, in detail, vector explanatory variables, extensions of the single relation model, and multivariate models. Chapter headings: 1. A single explanatory variable. 2. Vector explanatory variables. 3. Extensions of the single relation model. 4. Multivariate models.

*Regular variation.* By N. H. Bingham, C. M. Goldie, and J. L. Teugels. Cambridge University Press, New York, 1987. pp. xix+491. $75.00.

This is a volume in the distinguished Encyclopedia of Mathematics and its Applications, edited by Gian-Carlo Rota, and now published by the Cambridge University Press. It is a comprehensive account of the theory and applications of regular variation, and concerned with the asymptotic behaviour of a real function of a real variable $x$ which is "close" to a power of $x$. Such functions are much more than a convenient extension of powers. In many limit theorems regular variation is intrinsic to the result, and exactly characterises the limit behaviour. The book emphasises such characterisations, and gives a comprehensive treatment of those applications where regular variation plays an essential (rather than merely convenient) role. The authors rigorously develop the basic ideas of Karamata theory and de Haan theory including many new results and "second-order" theorems. They go on to discuss the role of regular variation in Abelian, Tauberian, and Mercerian theorems. These results are then applied in analytic number theory, complex analysis, and probability, with the aim above all of setting the theory in context. A widely scattered literature is thus brought together in a unified approach. Chapter headings: 1. Karamata theory. 2. Further Karamata theory. 3. de Haan theory. 4. Abelian and Tauberian theorems. 5. Mercerian theorems. 6. Applications to analytic number theory. 7. Applications to complex analysis. 8. Applications to probability theory. There are also six appendices: 1. Regular variation on more general settings. 2. Differential equations. 3. Functional equations. 4. Subexponentiality. 5. Calculating the de Bruijn conjugate. 6. Bounded variation.

*The Theory of Fourier Series and Integrals.* By P. L. Walker. John Wiley & Sons, New York, 1986. pp. viii+192. $29.95.

This text is addressed to two groups of readers: students who are approaching the subject in a first course in analysis, and those who are meeting it in the context of science or engineering. The historically vital area of the conduction of heat is emphasized. Chapter headings: 1. Fourier coefficients and Fourier series. 2. Convergence theory. 3. The Dirichlet problem and the Poisson integral. 4. Conjugate functions and conjugate series. 5. The Fourier integral. 6. Multiple Fourier series and integrals.

*The Non-Euclidean Revolution.* By Richard J. Trudeau. Birkhäuser, Boston, 1987. pp. xii+269. $39.00.

This book, which features an introduction by H. S. M. Coxeter, provides an account of the historical processes by which mathematicians came up with alternatives to Euclid's traditional classical geometry. The book proceeds on three levels: it is a treatise on non-Euclidean geometry with extra material on history and philosophy. On a second level, it stresses the events depicted as another scientific revolution. And on a third level, it propounds the view that absolutely certain knowledge about the world is impossible. The book is written so as to be readable by nonspecialists. Chapter headings: 1. First things. 2. Euclidean geometry. 3. Geometry and the diamond theory of truth. 4. The problem with postulate 5. 5. The possibility of non-Euclidean geometry. 6. Hyperbolic geometry. 7. Consistency. 8. Geometry and the story of truth.

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*Gauge Field Studies.* By Stefan Pokorski. Cambridge University Press, New York, 1987. pp. xii+394. $89.50.

This book has its origin in a long series of lectures given at the Institute for Theoretical Physics, Warsaw University. It is addressed to graduate students and research workers in theoretical physics who have some knowledge of quantum field theory in its canonical formulation. The book is designed to be a concise reference to some of the field-theoretical tools used in contemporary research in the theory of fundamental interactions. Physical problems are discussed only as illustrations of certain theoretical ideas and of computational methods. Chapter headings: 1. Introduction. 2. Path integral formulation of quantum field theory. 3. Feynman rules for Yang–Mills theories. 4. Introduction to the theory of renormalization. 5. Quantum electrodynamics. 6. Renormalization groups. 7. Scale invariance and operator product expansion. 8. Quantum chromodynamics. 9. Chiral symmetry; spontaneous symmetry breaking. 10. Spontaneous and explicit global symmetry breaking. 11. Spontaneous breaking of gauge symmetry. 12. Chiral anomalies. 13. Effective Lagrangians. 14. Introduction to supersymmetry.

*Measure Theory and Integration.* By M. M. Rao. John Wiley & Sons, New York, 1987. pp. xii+540. $59.95.

This is a volume in the series Pure and Applied Mathematics, founded by Richard Courant. It presents a detailed exposition of the general theory of measure and integration, and is meant to be a text for a first-year graduate course. The material is unified from various sets of notes, and of experience gained, from the author's frequent teaching of such a class since 1960. Each topic is introduced with ample motivation. The author starts with an abstraction of lengths, areas, volumes and other measurements of known geometric figures and develops the basic ideas of Lebesgue in $R^n$. This is then used as a model and a reference for the general study leading to the Carathéodory process. The measure approach as a basic step is natural in such areas as functional analysis, probability and statistics, and ergodic theory, whereas reference to Lebesgue's method keeps in view the applications to differential equations and mathematical physics among others. In this book, the Carathéodory process takes center stage and helps in an efficient presentation. Chapter headings: 1. Introduction and preliminaries. 2. Measurability and measures. 3. Measurable functions. 4. Integration. 5. Differentiation and duality. 6. Product measures and integrals. 7. Capacity theory on integration. 8. The lifting theorem. 9. Topological measures. 10. Some complements and applications.

*Plots, Transformations, and Regression: An Introduction to Graphical Methods of Diagnostic Regression Analysis.* By A. C. Atkinson. Clarendon Press, Oxford, 1985. pp. xi+282. $45.00.

This is a volume in the Oxford Statistical Science Series. It is about methods for checking statistical models and the data to which they are fitted. The main interest is in multiple regression models and least squares. The essential feature of the procedures which are described is that they are suitable for routine use. Thus they require little extra computation above that required to fit the model. The results are often best represented as plots which have a more immediate impact than lists of numbers and so are more suitable for routine checking. A selection of the methods is available in several statistical packages under the name regression diagnostics. The book provides a clear introduction to these methods and suggests useful extensions to them. The material of the book falls into three main parts. Chapters 1–5 describe methods of checking regression models for inadequacies both of the model and of the data. Inadequacies of the data include the presence of outliers and of individual observations which strongly influence the conclusions drawn from the data. Chapters 6–9 are concerned with the transformation of variables in an equation, especially the response. The remaining three chapters are concerned with more advanced and specialized topics including generalized linear models and robust estimation. The theoretical discussion is accompanied by the analysis of 14 sets of data, many of which make several appearances during the course of the book. The analyses are illustrated by over 150 plots.

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*Sphere Packings, Lattices and Groups.* By J. H. Conway and N. J. A. Sloan (with additional contributions by E. Bannai, R. E. Bocherds, J. Leech, S. P. Norton, A. M. Odlyzko, R. A. Parker, L. Queen, and B. B. Venkov). Springer-Verlag, New York, 1988. pp. xxvii+663. $87.00.

This is volume 290 of Grundlehren der mathematischen Wissenschaften, A Series of Comprehensive Studies in Mathematics. The book is mainly concerned with the problem of packing spheres in Euclidean space of dimensions 1,2,3,4,5,.... Given a large number of equal spheres, what is the most efficient (or densest) way to pack them together? The authors also study several closely related problems: the kissing number problem, which asks how many spheres can be arranged so that they all touch one central sphere of the same size; the covering problem, which asks for the least dense way to cover $n$-dimensional space with equal overlapping spheres; and the quantizing problem, important for applications to analog-to-digital conversion (or data compression), which asks how to place points in space so that the average second moment of their Voronoi cells is as small as possible. Attacks on these problems usually arrange the spheres so their centers form a lattice. Lattices are described by quadratic forms, and the authors study the classification of quadratic forms. Most of this book is devoted to these five problems. Chapter headings: 1. Sphere packing and kissing numbers. 2. Coverings, Lattices and Quantizers. 3. Codes, designs and groups. 4. Certain important lattices and their properties. 5. Sphere packing and error-correcting codes. 6. Laminated lattices. 7. Further connections between codes and lattices. 8. Algebraic constructions for lattices. 9. Bounds for codes and sphere packings. 10. Three lectures on exceptional groups. 11. The Golay codes and the Mathieu groups. 12. A characterization of the leech line. 13. Bounds on kissing numbers. 14. Uniqueness of certain spherical codes. 15. On the classification of integral quadratic forms. 16. Enumeration of unimodular lattices. 17. The 24-dimensional odd unimodular lattices. 18. Even unimodular 24-dimensional lattices. 19. Enumeration of extremal self-dual lattices. 20. Finding the closest lattice point. 21. Voronoi cells of lattices and quantization errors. 22. A bound for the covering radius of the Leech lattice. 23. The covering radius of the Leech lattice. 24. Twenty-three constructions for the Leech lattice. 25. The cellular structure of the Leech lattice. 26. Lorentzian forms for the Leech lattice. 27. The automorphism group of the 26-dimensional even unimodular Lorentzian lattice. 28. Leech roots and Vinberg groups. 29. The monster group and its 196884-dimensional space. 30. A monster Lie algebra?

*Signal Detection in Non-Gaussian Noise.* By Saleem A. Kassam. Springer-Verlag, New York, 1988. pp. ix+234. $48.00.

This is a volume in the series Springer Texts in Electrical Engineering. It contains a unified treatment of the problem of the detection of signals in additive noise which is not required to be Gaussian. The probability density function of the observations is assumed to be known to within a finite number of unknown parameters in a known functional form. The three canonical problems covered are: detecting a completely known deterministic signal in noise, a random nondeterministic signal in noise, and a phase-incoherent narrowband signal in narrowband noise. Chapter headings: 1. Elements of statistical hypothesis testing. 2. Detection of known signals in additive noise. 3. Some univariate noise probability density function models. 4. Optimization data quantization in known-signal detection. 5. Detection of known narrowband signals in narrowband noise. 6. Detection of narrowband signals with random phase angles. 7. Detection of random signals in additive noise.

*Transportation and Traffic Theory.* Edited by Nathan H. Gartner and Nigel H. M. Wilson. Elsevier Science Publishing Co., New York, 1987. pp. xviii+506. $89.00.

This volume represents the Proceedings of the Tenth International Symposium on Transportation and Traffic Theory, held July 8–10, 1987, at M.I.T. The symposium was organized in honor of Robert Herman. The papers contain studies on how traffic flows in urban areas, the optimal scheme for controlling and routing urban traffic, and the analysis of the performance of transportation systems. There are 26 papers from Europe, North America, and Japan, with the foci of transportation research tending to be rather different in these three regions of the world, reflecting the diversity in the social and economic systems of which transportation is an integral part.

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*Modelling, Robustness and Sensitivity Reduction in Control Systems.* Edited by Ruth F. Curtain. Springer-Verlag, New York, 1987. pp. ix+492. $89.00.

This is volume 34 of Series F: Computer and Systems Sciences of the NATO Advanced Science Institute Series. The NATO Advanced Research Workshop, of which these are the Proceedings, was held in Groningen, The Netherlands, December 1–5, 1986. The 27 papers are divided into six groups, entitled: 1. $H^\infty$-mathematical foundations. 2. Model approximation and robustness in control design. 3. Modelling and signal processing. 4. Modelling and system identification. 5. $H^\infty$-techniques. 6. Optimal sensitivity reduction. 7. List of participants.

*General Relativity and Gravitation.* Edited by M. A. H. MacCallum. Cambridge University Press, New York, 1987. pp. xvii+407. $49.50.

These are the Proceedings of the 11th International Conference on General Relativity and Gravitation, held in Stockholm, July 6–12, 1986. It contains the texts of thirteen plenary lectures as well as the 18 reports of the symposium chairmen. The authors of the plenary lectures are M. A. Abromowicz, M. J. Duff, J. Ehlers, M. B. Green, L. P. Grishchuk, C. J. Isham, P. O. Mazur, R. Penrose, T. Piran, J. J. Stachel, J. H. Tayler, M. S. Turner, and S.-T. Yau.

*Probability and Bayesian Statistics.* Edited by R. Viertl. Plenum Press, New York, 1987. pp. xii+510. $89.50.

This book contains selected and refereed contributions to the "International Symposium on Probability and Bayesian Statistics", which was organized to celebrate the 80th birthday of Professor Bruno de Finetti at his birthplace, Innsbruck, Austria. Since Professor de Finetti died in 1985 the symposium was dedicated to the memory of Bruno de Finetti and took place at Igls near Innsbruck from 23 to 26 September, 1986. The contributions cover a broad spectrum ranging from foundations of probability via psychological aspects of formulating subjective probability statements, abstract measure-theoretical considerations, contributions to theoretical statistics and stochastic processes, to applications in economics, reliability, and hydrology. There are 52 papers in all.

*Mathematical Topics in Population Biology, Morphogenetics and Neurosciences.* Edited by E. Teramoto and M. Yamaguti. Springer-Verlag, New York, 1987. pp. ix+348.

This is volume 17 of Lecture Notes in Biomathematics. It constitutes the Proceedings of an International Symposium held in Kyoto, Japan, November 10–15, 1985. There are two introductory lectures: Chaos and fractals, by M. Yamaguti, and Recurrent themes in mathematical biology, by S. A. Levin. The other 31 papers are grouped under the headings: Mathematical ecology and population biology, Mathematical theories of patterns and morphogenesis, and Theoretical neurosciences and related problems in physiology.

*Combinatorics of Finite Sets.* By Ian Anderson. Oxford University Press, New York, 1987. pp. xv+250. $45.00.

The purpose of this work is to be a readable introduction to an area of combinatorics which, the author believes, is now gradually being moulded into an elegant unity: a theory whose origin can be traced back to Sperner's 1928 theorem, that if one wants to find as many subsets of an $n$-element set as possible, subject to the condition that no subset is contained in another, then one cannot do better than to choose all the subsets of size $n/2$. The resulting theory of posets is called Sperner theory; the author, however, only uses Sperner's theorem as a springboard. Chapter headings: 1. Introduction and Sperner's theorem. 2. Normalized matching and rank numbers. 3. Symmetric chains. 4. Rank and number of multisets. 5. Intersecting systems and the Erdös–Ko–Rado theorem. 6. Ideals and a lemma of Kleitman. 7. The Kruskal–Katona theorem. 8. Antichains. 9. The generalized Macaulay theorem for multisets. 10. Theorems for multisets. 11. The Littlewood–Offord problem. 12. Miscellaneous methods. 13. Lattices of antichains and saturated chain partitions.