

QUARTERLY
OF
APPLIED MATHEMATICS

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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, RI 02912, either directly or through any one of the Editors. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proof only. The author's institution will be requested to pay a publication charge of \$30 per page which, if honored, entitles the author to 100 free reprints. Detailed instructions will be sent with galley proofs.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated. Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{[a^2 + b^2]^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \frac{x}{2b}}{\cos \frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (for books—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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Introduction to Mathematical Control Theory. By S. Barnett and R. G. Cameron. Clarendon Press, Oxford, 1985, pp. xi + 404. \$29.95.

This is the second edition of a volume in the Oxford Applied Mathematics and Computing Series, first published in 1975. A substantial amount of new material has been added, including many problems and topics, such as the critical case for algebraic stability tests, as well as a new chapter on frequency-domain methods for linear multivariate systems which complements the original emphasis on state space theory and reflects recent progress in the field.

Lectures on Viscoelasticity Theory. By A. C. Pipkin. Springer-Verlag, New York, 1986. pp. viii + 188.

This is volume 7 in Applied Mathematical Sciences and is the second edition of a text first published in 1972. Chapter headings: 1. Viscoelastic response in shear. 2. Fourier and Laplace transforms. 3. Relations between modulus and compliance. 4. Some one-dimensional dynamical problems. 5. Stress analysis. 6. Thermal effects. 7. Large deformations with small strains. 8. Slow viscoelastic flow. 9. Viscometric flow.

The Theory of Topological Semigroups: Volume 2. By J. H. Carruth, J. A. Hildebrandt, and R. J. Koch, Marcel Dekker, Inc., New York, 1986. pp. x + 195. \$49.50.

This is volume 100 in the series Pure and Applied Mathematics. Discussing the application of cohomology theory to the theory of compact monoids, the book includes coverage of the concepts of middle and edge points, and a review of the conditions one can impose upon a compact monoid to make it a group. With regard to topologic semilattices, it proves theories on Lawson semilattices and continuous semilattices.

Probability and Measure. By Patrick Billingsley. John Wiley & Sons, New York, 1986. pp. xii + 622. \$44.95.

This is a newly revised and expanded edition of a volume in the Wiley Series in Probability and Mathematical Statistics. In particular, the treatment of several topics, such as asymptotic behavior of Markov chains, is expanded, and some new ones, such as the central limit theorem for martingales, are added. Chapter headings: 1. Probability. 2. Measure. 3. Integration. 4. Random variables and expected values. 5. Convergence of distributions. 6. Derivatives and conditional probability. 7. Stochastic processes.

Tables of Progressive Gravity Waves. By John M. Williams. Pitman Publishing, Inc., Marshfield, Mass. 1985. pp. 1 + 640. \$135.00.

The tables forming the major part of this book provide the most comprehensive solutions yet computed of the most fundamental wave problem in hydrodynamics. Given a frictionless liquid of uniform undisturbed depth with an infinite train of uniform, symmetrical, progressive gravity waves propagated upon it, what is the wave profile, what are the displacements, velocities, accelerations and pressures at all points in the fluid, and what are the paths of the fluid particles? To compute waves of other than very small amplitudes, means must be found of dealing with the nonlinear boundary conditions at the free surface. This volume is a compendium of results achieved by several authors (including M. S. Longuet-Higgins and the author) over the past ten years, accompanied by a concise exposition of their formulation and use. The book will be useful to practicing oceanographers and engineers, who may want to consult the tables for direct application to practical problems; to users who may wish to use the data from the volume as input to a computer program to process a large number of wave calculations; and to applied mathematicians pursuing research.

Continued from page 244

Process Modelling. By Morton M. Denn. Pitman Publishing Inc., Marshfield, Mass., 1986. pp. xi + 324. \$34.95.

This book is an attempt to present a systematic treatment of the methods that are useful in the development and application of mathematical models. Chapter headings: 1. What is a model? 2. Why model? 3. Constructing a model. 4. Modelling reactors: Conservation of mass. 5. Modelling reactors: Conservation of energy. 6. Scaling. 7. Case study: Coal gasification. 8. Stability. 9. Modal analysis. 10. Discretization. 11. From discrete to continuous. 12. Case study: Fiber spinline. 13. Model validation. 14. Case study: Activated sludge process.

Time Series: Theory and Methods. By Peter J. Brockwell and Richard A. Davis. Springer-Verlag, New York, 1987. pp. xiv + 519. \$48.00.

This is a volume in the Springer Series in Statistics. The authors attempt in this book to give a systematic account of linear time series models and their application to the modeling and prediction of data collected sequentially in time. The aim is to provide specific techniques for handling data and at the same time to provide a thorough understanding of the mathematical basis for the techniques. Both time and frequency domain methods are discussed but the book is written in such a way that either approach could be emphasized. Distinctive features of the book are the extensive use of elementary Hilbert space methods and recursive prediction techniques based on innovations, use of the exact Gaussian likelihood and AIC for inference, a thorough treatment of the asymptotic behaviour of the maximum likelihood estimators of the coefficients of univariate ARMA models, extensive illustrations of techniques by means of numerical examples, and a large number of problems for the reader. The companion diskette contains programs written for the IBM PC, which can be used to apply the methods described in the text. Chapter headings: 1. Stationary time series. 2. Hilbert spaces. 3. Stationary ARMA processes. 4. The spectral representation of a stationary process. 5. Prediction of stationary processes. 6. Asymptotic theory. 7. Estimation of the mean and the autocovariance function. 8. Estimation for ARMA models. 9. Model-building and forecasting with ARMA processes. 10. Inference for the spectrum of a stationary process. 11. Multivariate time series. 12. Further topics.

Intrinsic Geodesy. By Antonio Marussi. Springer-Verlag, New York, 1985. pp. xvii + 219. \$56.00.

The papers in this volume (translated by W. I. Reilly) cover the period from 1950 to 1981 and are organized into six chapters: 1. Fundamentals of intrinsic geodesy. 2. Structure of the gravity field and Laplace's equation. 3. Principles of intrinsic geodesy applied to the normal reference field. 4. Mapping of the actual gravity field onto the normal reference field. 5. Mapping between surfaces. 6. Propagation of light in continuous isotropic refracting media. The subjects embrace the author's two interests: (a) Intrinsic geodesy, or the description of the Earth's gravity field in terms of coordinates natural to it—a modern extension and development of the differential geometry of Gauss, carried forward by a line of Italian mathematicians, amongst others; and (b) the theory of two- and three-dimensional representations (or mapping projections), particularly as applied to the Earth's gravity field. There is also an appendix by the translator giving the notation of vector analysis à la Burali-Forti and Marcolongo, a bibliography of Antonio Marussi, and a brief personal history of Marussi by H. Moritz.

Kiyosi Itô: Selected Papers. Edited by Daniel W. Stroock, and S.R.S. Varadhan. Springer-Verlag, New York, 1987. pp. xxi + 647. \$44.00.

This volume contains an introduction by the editors and a foreword by Kiyosi Itô, a bibliography of Itô's work, and the texts of 46 of Itô's papers, from 1941 to 1983, including his famous works on the subject of stochastic integration.

Continued from page 274

Varieties of Formal Languages. By J. E. Pin. Plenum Press, New York, 1986. pp. x + 138. \$37.50.

This is a volume in the series Foundations of Computer Science, a translation of the monograph first published in French in 1984. It presents a synthesis of the latest developments in the mathematical theory of data processing. It discusses the fundamental results of the theory of finite automata and of rational languages and examines the recent concept of the variety of languages which formalizes the connections between finite automata, recognizable languages, and finite semigroups. The author presents a review of the developments within the past decade. The first four chapters of the book are devoted to the fundamental results of the theory of automata and are illustrated by examples. The final chapter contains the most recent results of the theory (but without proofs). Chapter headings: 1. Semigroups, languages and automata. 2. Varieties. 3. Structure of finite semigroups. 4. Piecewise-testable languages and star-free languages. 5. Complementary results.

Fluid Interfacial Phenomena. Edited by Clive A. Croxton. John Wiley & Sons, New York, 1986. pp. xi + 775. \$150.00.

This volume presents reviews and assessments of some of the principal areas of investigation in the field of interfacial phenomena in fluids at the molecular level by leading exponents in the field. On an inhomogeneous boundary between two or more coexistent phases an enormous diversity of thermodynamic, structural and dynamic phenomena present themselves. Aspects of wetting, the intrinsic nature of the interface, its dynamical behaviour, orientational structure, electrolytes, metallic, liquid crystal and polymeric systems have all been considered in this book, as has that most complex of familiar systems, water. There are sixteen papers, some of monograph length.

Foundations of Algebraic Analysis. By Masaki Kashiwara, Takahiro Kawai, and Tatsuo Kimura. Princeton University Press, Princeton, 1986. pp. xii + 254. \$38.00.

This is volume 37 in the Princeton Mathematical Series, and is a translation from the Japanese. The use of algebraic methods for studying analysis is an important theme in modern mathematics. The most significant development in this field is microlocal analysis, that is, the local study of differential equations on cotangent bundles. This treatise provides a thorough description of microlocal analysis starting from its foundations. The book begins with the definition of a hyperfunction. It then carefully develops the microfunction theory and its applications to differential equations and theoretical physics. It also provides a description of microdifferential equations, the microlocalization of linear differential equations. Finally, the authors present the structure theorems for systems for microdifferential equations, where the quantized contact transformations are used as a fundamental device.

Mathematical Concepts in Organic Chemistry. By Ivan Gutman and Oskar E. Polansky. Springer-Verlag, New York, 1986. pp. x + 212. \$71.00.

This book is an attempt to outline some mathematical aspects of modern organic chemistry, with attention focused on topological, graph-theoretical, and group-theoretical features. Only some knowledge of elementary algebra and calculus is required, with less well-known but still elementary mathematical facts collected in four appendices. Mathematical rigor, however, is not avoided: the author's intention was to show not only the results of mathematical chemistry but also their derivation. Chapter headings: 1. Topological aspects of chemistry. 2. Molecular topology. 3. Chemical graphs. 4. Fundamentals of graph theory. 5. Graph theory and molecular orbitals. 6. Special molecular graphs. 7. Fundamentals of group theory. 8. Symmetry groups. 9. Automorphism groups. 10. Some interrelations between symmetry and automorphism groups. 11. Topological indices. 12. Thermodynamical stability of conjugated molecules. 13. Topological effect on molecular orbitals.

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Topics in Analytic Number Theory. Edited by Sidney W. Graham and Jeffrey D. Vaaler. University of Texas Press, Austin, 1985. pp. vii + 312. \$35.00.

These twelve papers constitute the proceedings of a conference held at the University of Texas at Austin in 1982. Two of the papers, by Henryk Iwaniec on modular forms and Heni Halberston on sieves, are long expositions based on their lectures.

Statistical Image Processing and Graphics. Edited by Edward J. Wegman and Douglas J. DePriest. Marcel Dekker, Inc. New York, 1986. pp. xii + 351. \$74.95.

This is volume 72 in the series Statistics: Textbooks and Monographs. It is the proceedings of a workshop held, under ONR auspices, in Luray, Virginia, in May 1983. The 19 papers are grouped under the headings Statistical Image Processing and Statistical Graphics.

Survey Design and Analysis. By F. R. Jolliffe. Ellis Horwood Ltd., 1986, distributed by John Wiley & Sons, New York, pp. 1 + 178. \$64.95.

This is a volume in the Ellis Horwood Series "Mathematics and its Applications". It is concerned with the statistical aspects of sample surveys, and also to some extent censuses. The level of treatment is intermediate: it assumes some knowledge of statistics, but is not highly mathematical. Chapter headings: 1. Introduction. 2. Sampling from finite populations. 3. Non-sampling errors. 4. Distribution of estimators and some applications. 5. Processing of results. 6. Contingency tables and their analysis. 7. Multivariate analysis.

Ideal and Incompressible Fluid Dynamics. By Michael E. O'Neill and Frank Chorlton. Ellis Horwood Ltd., 1986, distributed by John Wiley & Sons, New York. pp. 1 + 412. \$89.95.

This is a volume in the Ellis Horwood Series "Mathematics and its Applications". It is the aim of this book to provide a readily accessible introductory text on the theory of fluid mechanics which is as far as possible self-contained with regard to mathematical methods and techniques. Chapter headings: 1. Vector and tensor methods. 2. Kinematics of fluids in motion. 3. Mechanics of fluid motion. 4. Potential flow. 5. The stream functions. 6. Two-dimensional flow. 7. Conformal transformation and its applications. 8. Waves.

An Asymptotic Theory for Empirical Reliability and Concentration Processes. By Miklós Csörgö, Sándor Csörgö, and Lajos Horváth. Springer-Verlag, New York, 1986. pp. 1 + 171. \$17.40.

This is volume 33 in Lecture Notes in Statistics. Chapter headings: 1. Introduction. 2. The basic setting for the approximations and variance preliminaries. 3. Auxiliary processes: Integrals of empirical processes. 4. Mean residual life processes. 5. Auxiliary processes: Empirical increments of Brownian bridge integrals. 6. Total time on test processes. 7. Discussion of results of total time on test processes. 8. Scaled total time on test processes. 9. Total time on tests from the first failure. 10. Unscaled empirical Lorenz processes. 11. Empirical Lorenz processes. 12. Discussion of results on empirical Lorenz processes. 13. The empirical concentration process of Goldie. 14. Discussion of results on the Goldie concentration processes. 15. Further diversity and concentration processes. 16. Indices of inequality, diversity, and concentration. 17. Bootstrapping empirical functions. 18. References.

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Mathematical and Computational Concepts in Chemistry. Edited by N. Trinajstić. Ellis Horwood Ltd., 1986, distributed by John Wiley & Sons, New York. pp. 1 + 365. \$78.00.

This is a volume in the Ellis Horwood series Mathematics and its Applications. It contains the 29 invited and special lectures at the International Symposium on the Mathematical Concepts of Chemistry, held in Dubrovnic, Croatia, in September 1985.

Counting for Something: Statistical Principles and Personalities. By William S. Peters. Springer-Verlag, New York, 1987. pp. xviii + 275. \$33.00.

This is a volume in the Springer Texts in Statistics. It provides a survey of some of the principal ideas and concepts which have shaped the subject of statistics, with many examples drawn from real life applications, and a good deal of biographical information about its creators, including accounts of the many controversies which accompanied its history. Chapter headings: 1. Statistics—the word. 2. Distributions. 3. Special averages. 4. Making comparisons. 5. Probability. 6. Craps and binomials. 7. Horse kick, deadly quarrels, flying bombs. 8. Normal distribution. 9. Political arithmetic. 10. Regression and correlation. 11. Karl Pearson. 12. Pearson to Gosset to Fisher. 13. More regression. 14. R. A. Fisher. 15. Sampling: polls and surveys. 16. Quality control. 17. Principal components and factor analysis. 18. Jerzy Neyman. 19. The Bayesian bandwagon. 20. Observational studies and program evaluation. 21. Nonparametrics and robust method. 22. Statistics—a summing up.

Assignment Methods in Combinatorial Data Analysis. By Lawrence J. Hubert. Marcel Dekker, Inc. New York, 1987. pp. x + 326. \$69.75. (\$34.95 for classroom use on orders of five or more copies).

This is volume 73 of the series Statistics: Textbooks and Monographs. It concerns the problem a statistician faces when consulted, for instance, on the appropriate analysis of a data-set having independent group structure and the client is concerned with a difference between means, when the data was generated without an initial random assignment to the groups or random sampling from two independent populations, making standard models (such as the *t*-test) inadmissible. The question discussed in this book concerns the meaning a conventional test actually has in the absence of a context providing the usual justification. It is shown how to interpret a significance test in a nonexperimental situation, by constructing a reference distribution, considering all possible mean differences to be equally likely. The concern is with evaluating some observed index in relation to a specific randomness conjecture that is defined only in terms of the observed data. Specifically, two comprehensive statistical paradigms are reviewed. The book starts with a discussion of the linear assignment model in Chapter 1 and how it may be applied in several common data analysis contexts. This introduction then serves as a basis for extensions in Chapters 2 and 3 to more complicated data analysis situations of comparing rectangular data matrices and multiple object sets. Chapter 4 introduces a second notion of assignment or matching based on pairs of objects. In turn, this idea provides the motivation for the generalizations in Chapters 5 to 7 that discuss, respectively, alternative indices of object set correspondence and higher-order assignment measures, multiple proximity matrices, and restrictions that may be imposed on the randomization scheme.

Initial Boundary Value Problems in Mathematical Physics. By R. Leis. John Wiley & Sons, New York, 1986. pp. viii + 266. \$44.95.

The lectures presented in this book were given at the University of Bonn and in part at the University of Strathclyde, Glasgow. Chapter headings: 1. Introduction. 2. Linear operators. 3. The wave equation. 4. The spectrum and boundary value problems. 5. The free space problem for the wave equation. 6. The wave equation continued: Time-asymptotic behaviour of the solutions. 7. Linear acoustics. 8. Maxwell's equations. 9. Linear acoustics and Maxwell's equations continued. 10. A Schrödinger equation. 11. Linear elasticity. 12. The plate equation. 13. Linear thermoelasticity.

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Rotations, Quaternions, and Double Groups. By Simon L. Altmann. Clarendon Press, Oxford, 1986. pp. xiv + 317. \$49.00.

Rotation operators are often obtained as by-products of the angular momentum operators used in quantum mechanics. Partly as a result of this approach, rotations are then parametrized by means of Euler angles, which suffer from three defects: they are not always unique, they are very cumbersome to determine in finite rotation groups (point groups), and they do not provide a scheme for the multiplication of rotation. An entirely different approach to rotations was introduced by Olinde Rodrigues in 1840. The rotation operators are obtained in this approach by an entirely geometric method, which not only provides a much better insight into their nature, but also leads most naturally to the parametrization of rotations by parameters that coincide with quaternions. These parameters are unique, easy to determine, and they provide an algebra that permits the multiplication of rotations in a very simple way. At the same time, these parameters determine unambiguously the phase factors that appear in the angular-momentum representations for half-integral quantum numbers. This result leads to a rigorous formulation of the representations of the rotation group, either as projective representations or by means of double groups. The main object of this book is to present a consistent description of this geometric and quaternionic treatment of rotations. Chapter headings: 1. Introduction. 2. All you need to know about symmetries, matrices, and groups. 3. A primer on rotations and rotation matrices. 4. Rotations and angular momentum. 5. Tensor basis: Introduction to spinors. 6. The bilinear transformation: Introduction to $SU(2)$, $SU'(2)$, and rotations. More about spinors. 7. Rotations and $SU(2)$. The stereographic projection. 8. Projective representations. 9. The geometry of rotations. 10. The topology of rotations. 11. The spinor representations. 12. The algebra of rotations: Quaternions. 13. Double groups. 14. The irreducible representations of $SO(3)$. 15. Examples and applications.

Nonlinear Partial Differential Equations and Free Boundaries: Volume 1—Elliptic Equations. By J. I. Díaz. Pitman Advanced Publishing Program, Boston, 1985. pp. 1 + 323. \$44.95.

This is volume 106 of Research Notes in Mathematics. It is devoted to the study of some partial differential equations whose nonlinear character gives rise to a free boundary: the boundary of an a-priori unknown and positively measured region where the solution of the equation vanishes identically. The formulation of this free boundary holds under the same adequate balance between two of the terms of the equation representing the different peculiarities of the phenomenon under consideration: diffusion, absorption, convection, evolution, etc. The characterization of these special balances for several classes of nonlinear equations, important in the applications, is one of the main goals of the work. The first volume is dedicated to elliptic equations; a second volume dealing with parabolic and hyperbolic equations is now in preparation. Chapter headings: 1. The free boundary in the Dirichlet problem for second order elliptic quasilinear equations. 2. The free boundary in other second order nonlinear problems. 3. Existence and location of the free boundary by means of energy methods. 4. The general theory for second order nonlinear elliptic equations: A particular overview.

Hamilton's Principle in Continuum Mechanics. By A. Bedford. Pitman Advanced Publishing Program, Boston, 1985. pp. 1 + 106. \$29.95.

This is volume 139 of Research Notes in Mathematics. The objective of this monograph is to give a comprehensive account of the use of Hamilton's principle to derive the equations which govern the mechanical behaviour of continuous media. The classical theories of fluid and solid mechanics are discussed as well as two generalizations of those theories for which Hamilton's formulations have also been used to develop generalizations of classical theories of fluid and solid mechanics. Chapter headings: 1. Mechanics of systems of particles. 2. Foundations of continuum mechanics. 3. Mechanics of continuous media. 4. Mechanics of mixtures. 5. Discontinuous fields.

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Random Data: Analysis and Measurement Procedures. By Julius S. Bendat and Allan G. Piersol. John Wiley & Sons, New York, 1986. pp. xvii + 566.

This book is an extensively revised and expanded edition of the authors' 1971 book of the same title which in turn was a major revision of 'Measurement and Analysis of Random Data', first published in 1966. It has been written primarily to provide a convenient reference for practicing engineers and scientists. The problem sets at the end of each chapter also make it useful as a specialized textbook for students. A working knowledge of calculus and transform methods of applied mathematics is assumed. Chapter headings: 1. Basic descriptions and properties. 2. Linear physical systems. 3. Probability fundamentals. 4. Statistical principles. 5. Stationary random processes. 6. Single-input/output relationships. 7. Multiple-input/output relationships. 8. Statistical errors in basic estimates. 9. Statistical errors in advanced estimates. 10. Data acquisition and processing. 11. Digital data analysis. 12. Nonstationary data analysis. 13. The Hilbert transform.

Numerical Analysis. Edited by D. F. Griffiths and G. A. Watson. John Wiley & Sons, New York, 1986. pp. 1 + 262.

This is volume 140 of the Pitman Research Notes in Mathematics Series. It contains full versions of the 16 invited talks held at the 11th Dundee Biennial Conference on Numerical Analysis, June 1985.

Functional Analysis and Two Point Differential Operators. By John Locker. John Wiley & Sons, New York, 1986. pp. 1 + 257. \$44.95.

This is volume 144 of the Pitman Research Notes in Mathematics Series. The monograph grew out of lectures given by the author at Colorado State University and its purpose is to study the topics of two point differential operators and linear boundary value problems in Hilbert space. Throughout the presentation emphasizes the modern viewpoint of linear operator theory, while still trying to make the appropriate connections with the classical theory.

Modeling and Analysis of Conventional Defense in Europe: Assessment of Improvement Options. Edited by Reiner K. Huber. Plenum Press, New York, 1986. pp. viii + 215. \$45.00.

This book presents a collection of contributions to a workshop on "Long-term development of NATO's conventional defense" to which a German Strategy Forum had invited some 50 systems analysts and defense experts from the United States, the United Kingdom, the Federal Republic of Germany, and the SHAPE Technical Center. The workshop was held in Bonn from 2-4 December, 1984. The contributions are in four parts: 1. Basic premises for the evaluation. 2. Conceptual approaches to the assessment of alternatives. 3. Analysis of improvement proposals. 4. Assessment group summaries.

Ergodic Theory of Random Transformations. By Yuri Kifer. Birkhäuser, Boston, 1986. pp. 1 + 210.

This is volume 10 in the series Progress in Probability and Statistics. The purpose of this book is the study of the variety of ergodic theoretical properties of the evolution processes generated by independent applications of transformations chosen at random from a certain class according to some probability distribution. The book exhibits the first systematic treatment of ergodic theory of random transformations, i.e., an analysis of composed actions of independent random maps. This setup allows a unified approach to many problems of dynamical systems, products of random matrices and stochastic flows generated by stochastic differential equations. Chapter headings: 1. General analysis of random maps. 2. Entropy characteristics of random transformations. 3. Random bundle maps. 4. Further study of invariant subbundles and characteristic exponents. 5. Smooth random transformations.

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Boundary Crossing of Brownian Motion. By Hans Rudolf Lerche. Springer-Verlag, New York, 1986. pp. iv + 142. \$15.70.

This is volume 40 of Lecture Notes in Statistics. Its subheading is "its relation to the law of the iterated logarithm and to sequential analysis", which describes the point of view of the monograph. Two themes are treated which are closely related: (i) boundary first passage distributions of Brownian motion, and (ii) optimal properties of sequential tests with parabolic and nearly parabolic boundaries, which are, also, the two chapter headings.

Parallel Array Processing. By P. G. Ducksbury. Ellis Horwood Ltd., 1986, distributed by John Wiley & Sons, New York. pp. 1 + 122. \$37.95.

This is a volume in the Ellis Horwood Series in Electrical and Electronic Engineering. It is based on research undertaken at the Hatfield Polytechnic, England, on a distribution array processor, built by ICL. Chapter headings 1. Introduction. 2. Parallel computers and programming. 3. Finite element optimization. 4. The linear problem. 5. The non-linear problem. 6. Some comments on more complex problems. 7. The Navier-Stokes equation. 8. Conclusions and future research.

Multiple Parameter Stability Theory and its Applications: Bifurcations, Catastrophes, Instabilities. By Koncay Huseyin. Clarendon Press, Oxford, 1986. pp. xiii + 283. \$75.00.

This is volume 18 in The Oxford Engineering Science Series. In a 1975 monograph, the author (inspired by W. T. Koiter's work) presented a multi-parameter nonlinear theory of stability for discrete potential systems. The main goal of this book is to present a parallel nonlinear theory for nonpotential systems which can exhibit dynamic as well as static instabilities. This is achieved within the framework of an autonomous formulation. The emphasis is on the instabilities and incipient bifurcations associated within a certain equilibrium path (or surface) of the system. Dynamic instabilities are linked to the emergence of a family of periodic motions, while static instabilities involve equilibrium states only. Chapter headings: 1. Introduction. 2. Potential systems. 3. Static instability of autonomous systems. 4. Dynamic instability of autonomous systems. 5. Applications. 6. Concluding remarks.

Chaos. Edited by Arun V. Hoden. Manchester University Press, Princeton, 1986. pp. vi + 324. \$50.00 Hardcover, \$19.95 Paperback.

This volume sets out the basic applied mathematical and numerical methods of chaotic dynamics and illustrates the wide range of phenomena that can be treated as chaotic processes. It demonstrates how chaotic solutions may be processed by very similar nonlinear difference, delay or differential equations. Among the examples of apparent chaotic activity discussed are cellular metabolism, cardiac electrophysiology, population biology, electric oscillators, and laser systems. The fifteen papers, by various authors, are divided into six groups: 1. Prologue. 2. Iterations. 3. Endogenous chaos. 4. Forces chaos. 5. Measuring chaos. 6. Epilogue.

Integral Methods in Science and Engineering. By Fred R. Payne, Constantin C. Corduneanu, A. Haji-Sheikh, and Tseng Huang. Hemisphere Publishing Corporation, New York, 1986. pp. x + 651. \$95.50.

These are the proceedings of a conference held at the University of Texas, Arlington, in March 1985. The keynote address is by Hans Mark, Chancellor of the University of Texas System and former NASA Deputy Administrator. The papers are organized under the following headings: Mathematical Physics, Mathematical Analysis, Fluid Mechanics, Solid Mechanics, Thermal Sciences, Optimization, and Population Dynamics.