

# QUARTERLY

OF

## APPLIED MATHEMATICS

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# QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, RI 02912, either directly or through any one of the Editors. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proof only. The author's institution will be requested to pay a publication charge of \$30 per page which, if honored, entitles the author to 100 free reprints. Detailed instructions will be sent with galley proofs.

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# SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{[a^2 + b^2]^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \frac{x}{2b}}{\cos \frac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

**Figures:** Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: ( for books—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.



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————— NEW BOOKS —————

*Total Curvature in Riemannian Geometry.* By T. J. Willmore. John Wiley & Sons, Inc., Chichester, England, 1982. 158 pp. \$39.95.

This is a volume in the Ellis Horwood Series in Mathematics and Its Applications. Its purpose is to introduce the reader to a branch of geometry which has developed over the last thirty years, namely a study of invariants which arise from integrating various curvature measures over manifolds. In his treatment the author has confined his attention to smooth manifolds where he has paid particular attention to those curvature measures associated with the Lipschitz-Killing curvature and the mean curvature. This has implied omitting the integral representation of such important invariants as the Pontrjagin classes, the Hizebruch genus, etc., but these lie outside the scope of an introductory book. Chapter headings: 1. Riemannian Geometry; 2. The Gauss-Bonnet Formula; 3. Total Absolute Curvature; 4. Tight Immersions; 5. Total Mean Curvature; 6. Conformal Volume.

*Surface Topology.* By P. A. Firby and C. F. Cardiner. John Wiley & Sons, Inc., Chichester, England, 1982. 211 pp. \$54.95.

This is a volume in the Ellis Horwood Series in Mathematics and Its Applications. It is the intention of the authors to provide a simple, intuitively based, and readable introduction to geometric topology which, nevertheless, achieves significant and interesting results in both the topology of surfaces and several areas of application. Chapter headings: 1. Intuitive Ideas; 2. Plane Models of Surfaces; 3. Surfaces as Plane Diagrams; 4. Distinguishing Surfaces; 5. Patterns on Surfaces; 6. Maps and Graphs; 7. Vector Fields on Surfaces; 8. Plane Tessellation Representations of Compact Surfaces; 9. Some Applications of Tessellation Representations; 10. Introducing the Fundamental Group.

*The Convolution Product and Some Applications.* By Wilhelm Kecs. D. Reidel Publishing Co., Dordrecht, Holland, 1982. xvii + 332 pp. \$69.50.

This is volume 2 of the series Mathematics and Its Applications (East European Series). It is a translation of a monograph first published in Romanian. The aim of this work is to present in a systematic manner the fundamental properties of the convolution product for functions and distributions. Additionally, it is shown how the convolution product is used in the study of mathematical physics, deformable solids, mechanical systems, electrical circuits, etc. Chapter headings: 1. Topological vector spaces; 2. The convolution product; 3. Integral transforms and periodic distributions; 4. Convolution Equations; 5. Application of the Convolution Product.

*Analytic Functional Calculus and Spectral Decompositions.* By Florian-Horia Vasilescu. D. Reidel Publishing Co., Dordrecht, Holland, 1982. xiv + 378 pp. \$78.50.

This is volume 1 in the series Mathematics and Its Applications (East European Series). It is an English version of a monograph published in Romanian, modified and brought up to date. The author defines the mathematical notion of the analytic functional calculus as "a continuous unital algebra homomorphism, associated with a finite commuting system of elements, defined on the algebra of germs of analytic functions in neighborhoods of the joint spectrum of the system, with values in a commutative algebra; moreover, the images of the germs of the coordinate functions are the elements of the given system." Chapter headings: 1. Introduction; 2. Analytic Vector-valued Functions; 3. Fredholm Theory, Joint Spectrum and Analytic Functional Calculus; 4. Spectral Decompositions; 5. Miscellaneous Applications and Examples.



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*The Lore of Prime Numbers.* By George P. Loweke. Vantage Press, New York, 1982. 248 pp. \$17.95.

This book relates many fascinating developments of the unscientific numbering systems that emerged during the early centuries. In particular, it explores the properties of prime numbers that were noted in the distant past, and the later results obtained from the more sophisticated studies of today. All composite numbers are representable by a product of nonreducible prime numbers, so that "prime numbers are to mathematics what elementary particles are to physics."

*Problem Complexity and Method Efficiency in Optimization.* By A. S. Nemirovsky and D. B. Yudin. John Wiley & Sons, Inc., New York, 1983. 379 pp. \$49.95.

This is a volume in the Wiley-Interscience Series in Discrete Mathematics. The subject of investigation in this monograph is the question of the potentially attainable effectiveness of numerical methods applied to problems of a given type. A typical question the authors consider is of this form: given a family of optimization problems together with a source of information; accessible to the methods, about each solvable problem of this family, what are the potential lower bounds of laboriousness of all possible methods which solve all problems of the family with a given accuracy? Which method realizes this potential lower limit and is therefore the best one? After proper formalisation of concepts such as 'method,' 'laboriousness of a method,' etc., the above question is applied to a number of standard non-linear programming problems. Chapter headings: 1. Introduction; 2. Convex Programming Linearly Convergent Methods for Classes of General Convex Problems; 3. Methods of Mirror Descent; 4. Complexity of Classes of General Convex Problems (Exact First-Order Oracle); 5. Problems of Convex Stochastic Programming; 6. Solution of Convex-Concave Games and Constrained Stochastic Programming Problems; 7. Strongly Convex Problems; 8. Efficiency of Standard Methods of Strongly Convex Programming; 9. Convex Programming Methods of Zeroth Order.

*Asymptotic Treatment of Chemically Reacting Systems.* By A. K. Kapila. Pitman Advanced Publishing Program, London, 1983. 109 pp. \$32.95.

In the area of chemically reacting systems, nonlinear problems are the rule rather than the exception, and exact analytical solutions are rare. Recourse to the computer is an obvious approach, but the numerical analysis becomes difficult and delicate when the problem is stiff, i.e. when zones of rapid variation occur in the solution due to the presence of several widely disparate spatial and/or temporal scales. This monograph illustrates that under these circumstances, asymptotics can provide a fruitful approach. Chapter headings: 1. Introduction; 2. Activation-energy asymptotics; 3. Isothermal systems; 4. Oscillations.

*Combined Loadings in the Theory of Plasticity.* By M. Zyczkowski. Martinus Nijhoff Publishers BV, the Netherlands, 1982. 734 pp. \$142.00.

This comprehensive book aims at a systematisation of the basic concepts in the analysis of combined loadings in the plastic range which, very important both from the theoretical and the engineering viewpoint, is one of the more difficult branches of the theory of plasticity. It also provides a description of methods of solution of typical problems and a presentation of the more important solutions; above all, however, it furnishes research workers and engineers with a guide to a very extensive bibliography, which contains more than 3,000 entries and is estimated by the author to constitute about 90 per cent of world literature in the field up to 1977. A classification of combined loadings is introduced—successively at the levels of a point of the body  $P$ , of a cross-section  $S$ , and of the body as a whole  $B$  and is used consistently throughout the book. Chapter headings: 1. Basic concepts of the theory of combined loadings; 2. Basic equations of the theory of plasticity; 3. Combined loadings at the level  $P$  of a point of the body; 4. Transition from the local level  $P$  to the integral levels  $S$  and  $B$ ; 5 and 6. Combined loadings at the level  $S$  of a cross-section of a bar; 7. Combined loadings at the level of a cross-section of a surface structure; 8. Combined loadings at the level  $B$  of the body as a whole.

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*Table of Definite and Infinite Integrals.* By Alexander Apelblat. Elsevier Scientific Publishing Co., Amsterdam, The Netherlands, 1983. 457 pp. \$106.50.

The aim of this book is to provide mathematicians, scientists and engineers with a collection of finite and infinite integrals of elementary and special functions. There exist other compilations of integrals in the literature, including tables of indefinite and definite integrals, collections of integral transforms, monographs devoted to integrals of special functions, and so forth. These, however, generally date from at least one decade or more and are in need of updating. Therefore, in this collection, special attention is directed to integrals that are not included in previous publications. Furthermore, for convenience, equivalent reducible forms obtained by substitutions, recurrence relations, and particular cases of existing integrals of special interest, are also presented. This volume should be considered supplementary to the existing literature. The integrals presented here were compiled over many years from numerous scientific and technical journals and books or were obtained by repeated use of integral transforms. The tabulation is presented in the following order: Part 1. Integrals of elementary functions; 1. Rational and irrational algebraic functions, 2. Exponential functions, 3. Logarithmic functions, 4. Trigonometric functions, 5. Inverse trigonometric functions, 6. Hyperbolic functions. Part 2. Integrals of special functions and of elementary and special functions; 7. Exponential integral and related functions, 8. Gamma function and related functions, 9. Error functions and Fresnel integrals, 10. Legendre functions, 11. Orthogonal polynomials, 12. Bessel functions and related functions, 13. Elliptic integrals, 14. Hypergeometric functions, 15. Confluent hypergeometric functions, 16. Generalized hypergeometric functions, 17. Whittaker functions, 18. Parabolic cylinder functions, 19. Meijer's  $G$ -function and MacRobert's  $E$ -function, 20. Miscellaneous functions.

*Fourier Analysis of Numerical Approximations of Hyperbolic Equations.* By Robert Vichnevetsky and John B. Bowles. SIAM, Philadelphia, 1982. 129 pp. \$21.50.

This is a volume in the series of monographs SIAM Studies in Applied Mathematics. It brings together recent results on the use of Fourier analysis to examine questions of accuracy and stability in numerical methods for hyperbolic equations. These methods introduce errors which distort the physical nature of the phenomena under study. Describing those errors by invoking concepts which originated in mathematical physics, such as energy propagation, dispersion and diffusion thus proves to be a most enlightening approach. In this respect, Fourier analysis provides an indispensable tool. Chapter headings: 1. Introduction, 2. Fourier Analysis of the Accuracy of Semi-Discretizations, 3. Higher Order Semi-Discretizations, 4. Full Discretizations, 5. Damping, Diffusion and Filtering, 6. Group Velocity, 7. Time-Fourier Transforms, 8. Fourier Analysis and  $L_2$ -Norm of the Global Error, 9. Spectral Methods, 10. Equations in Two Dimensions: Anisotropy.

*Weak Convergence of Measures.* By Harald Bergstrom. Academic Press, New York, 1982. 242 pp. \$39.50.

This is a volume in the series Probability and Mathematical Statistics. Weak convergence of measures means convergence of functionals and thus reduces to the convergence of sequences of real numbers. It concerns elementary and fundamental concepts in mathematics, such as sets, topology, mappings, measurability, continuity, integrals, functionals, measures, etc. These concepts are closely related to each other and are to a great extent unified in the concept of weak convergence. The material in this book has been arranged with an aim towards such unification. Chapter headings: 1. Spaces, Mappings, and Measures; 2. Integrals, Bounded Linear Functionals, and Measures; 3. Weak Convergence in Normal Spaces; 4. Weak Convergence on  $R^{(k)}$ ; 5. Weak Convergence on the  $C$ - and  $D$ -Spaces; 6. Weak Convergence in Separable Hilbert Spaces.

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*Reliability Evaluation of Engineering Systems: Concepts and Techniques.* By Roy Billinton and Ronald N. Allan. Plenum Press, New York, 1983. 344 pp. \$42.50.

The scope of this textbook is not limited to any one engineering discipline but reliability evaluation is treated as an important and integral feature of the planning, design and operation of all engineering systems. Its primary objective is to provide engineers who have little or no background in probability theory or statistics with the concepts and basic techniques of the field.

*Elementary Applied Partial Differential Equations.* By Richard Haberman. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1983. 517 pp. \$34.95.

This text discusses elementary partial differential equations in the engineering and physical sciences. It is suited for courses whose titles include Fourier series, orthogonal functions, or boundary value problems. It may also be used in courses on Green's functions or transform methods. Simple models (heat flow, vibrating strings and membranes) are emphasized. Equations are formulated carefully from physical principles, motivating most mathematical topics. Solution techniques are developed patiently. Mathematical results frequently are given physical interpretations. Proofs of theorems (if given at all) are presented after explanations based on illustrative examples. These are supplemented by numerous exercises of various levels of difficulty.

*Basic Mathematics and Computer Techniques for Coal Preparation and Mining.* Edited by Kenneth K. Humphreys and Joseph W. Leonard. Marcel Dekker, Inc., New York, 1983. 248 pp. \$45.00.

This is volume 17 in the Energy, Power and Environment Series. It provides many of the decision-making procedures which have been developed by the Coal Research Bureau of the West Virginia University College of Mineral and Energy Resources. Among the techniques discussed which require computers are methods for determining optimum sites for mining and preparation facilities, and techniques for improving property evaluations and projecting drill core data.

*Introduction to Dynamics.* By Ian Percival and Derek Richards. Cambridge University Press, Cambridge, 1982. 221 pp.

Modern dynamics owes as much to Poincaré and Liapounov as to Lagrange and Hamilton, to the authors introduce Hamiltonian dynamics through the qualitative theory of differential equations and highlight the geometry of phase curves and the theory of stability. Each subject, from the elementary theory of first-order systems, up to the discoveries on chaotic motion in recent decades, is introduced through simple examples. The authors restrict themselves to first- and second-order systems and to Hamiltonian systems with one degree of freedom. This enables them to introduce to undergraduates many important ideas that have previously been confined to graduate teaching or research. The only mathematical background required is  $2 \times 2$  matrices, ordinary differential equations and calculus of two variables. Chapter headings: 1. First-order autonomous systems; 2. Linear transformations of the plane; 3. Second-order autonomous systems; 4. Conservative Hamiltonian systems of one degree of freedom; 5. Lagrangians; 6. Transformation theory; 7. Angle-action variables; 8. Perturbation theory; 9. Adiabatic and rapidly oscillating conditions; 10. Linear Systems; 11. Chaotic motion and non-linear maps.

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*Linear Algebra and Group Representations.* By Ronald Shaw. *Volume I: Linear Algebra and Introduction to Group Representations.* 261 pp. \$32.00. *Volume II: Multilinear Algebra and Group Representations.* 566 pp. \$35.00. Academic Press, London, 1982.

This two volume work consists of an account of the twin topics of group representations and of linear and multilinear algebra. It is chiefly concerned with finite-dimensional vector spaces and group representations, and hence chiefly with algebraic, rather than topological and analytical, ideas. Chapter headings, Volume 1: 1. Non-metrical linear algebra I: basic notions; 2. Non-metrical linear algebra II: canonical forms for linear operators; 3. Group representations I: basic notions (non-metrical); 4. Metrical linear algebra I: O-, Sp-, and U-geometries; 5. Metrical linear algebra II: linear operators on scalar product spaces; 6. Metrical linear algebra III: canonical forms; 7. Group representations II: metrical aspects. Chapter headings, Volume 2: 8. Multilinear algebra I: tensor algebra; 9. Multilinear algebra II: exterior algebra; 10. Multilinear algebra III: symmetric tensor algebra; 11. Multilinear algebra IV: metrical aspects; 12. Group representations III: tensor product representations; 13. Group representations IV: orthogonality relations, characters, group algebra; 14. Group representations V: multiplier representations; 15. Group representations VI: induced representations.

*Queues and Point Processes.* By P. Franken, D. König, U. Arndt, and V. Schmidt. John Wiley & Sons, Inc., New York, 1982. 189 pp. \$29.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It gives a unified treatment of some basic problems in queueing theory using the theory of stationary random marked point processes. No independence assumptions concerning the input are generally made, but its authors confine themselves to the consideration of queueing systems in steady state. Chapter headings: 1. Random marked point processes and processes with an embedded marked point process; 2. Time- and customer-stationary processes for queueing systems. Existence and ergodic theorems; 3. Continuity theorems for time- and customer-stationary quantities; 4. Relationships between time- and customer-stationary quantities, I: Basic systems; 5. Relationships between time- and customer-stationary quantities, II: Further systems; 6. Insensitivity of stationary state probabilities for a class of queueing systems.

*Function Theory on Planar Domains: A Second Course in Complex Analysis.* By Stephen D. Fisher, John Wiley & Sons, Inc., New York, 1983. 261 pp. \$34.95.

This is a volume in the Wiley Series Pure and Applied Mathematics. It is a high level treatment of complex analysis which concentrates on function theory on a finitely-connected planar domain. It is a complete presentation of the basic facts in the theory of functions—in particular, the Hardy  $H^p$  spaces on multiply connected domains in the complex plane. Special emphasis is placed on domains bounded by a finite number of disjoint analytic simple closed curves. The opening chapter surveys the fundamental ideas of potential theory, including the solution of the Dirichlet problem by means of subharmonic functions, harmonic measure, the Green's function of a domain, and the logarithmic capacity of a planar set. The second chapter is devoted to a proof of the uniformization theorem. The third, fourth, and fifth chapters develop the structure of the Hardy  $H^p$  spaces on planar domains and examine a number of classical linear extremal problems on such domains. Recent work on the structure of the set of maximal ideals of the space of bounded analytic functions on an arbitrary domain is presented in the sixth chapter. The final chapter contains work on linear operators on  $H^p$  spaces and includes a discussion of the optimal estimation of holomorphic functions, a new and rapidly developing area of research.

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*The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors.* By Colin Sparrow. Springer-Verlag, 1982. 261 pp. \$21.50.

This is volume 41 in the series "Applied Mathematical Sciences." The equations studied in this monograph were first presented in 1963 by E. N. Lorenz. They define a three-dimensional system of ordinary differential equations that depends on three real positive parameters. As the parameters are varied, the behaviour of the flow determined by the equations is changed. For some parameter values, numerically computed solutions of the equations oscillate, apparently forever, in the pseudo-random way called "chaotic"; this is the main reason for the immense amount of interest generated by the equations in the eighteen years since Lorenz first presented them. In addition, there are some parameter values for which one sees "preturbulence", a phenomenon in which trajectories oscillate chaotically for long periods of time before finally settling down to stable stationary or stable periodic behaviour, others in which one sees "intermittent chaos", where trajectories alternate between chaotic and apparently stable periodic behaviours, and yet others in which one sees "noisy periodicity", where trajectories appear chaotic though they stay very close to a non-stable periodic orbit. The monograph is divided into nine chapters and eleven appendices. The first four chapters review what is known about the Lorenz equations in the most widely studied parameter ranges. Chapter 1 contains some general remarks and a description of those simple properties of the equations that can be deduced mathematically. In Chapter 2, the author studies the bifurcation associated with a homoclinic orbit. In Chapter 3, he describes the parameter range where it is believed that one has a well understood strange attractor in a whole interval of parameter values. Chapter 4 contains a description of the results of some simple numerical experiments in a parameter range where period doubling is observed. Chapter 5 is an attempt to reconcile Chapters 3 and 4. Using a combination of different numerical techniques and a careful theoretical analysis of the changes in the behaviour of the unstable manifold of the origin the author shows how the behaviour changes from strange attractor to period doublings. Chapter 6 is an attempt to justify the methods used earlier in the text to describe periodic orbits and other trajectories using sequences of symbols. Chapter 7 contains an outline of an analysis of the behaviour when one of the parameters becomes large. Chapter 8 contains a study of the Lorenz equations for parameter values suggested by Chapter 7, and Chapter 9 contains a brief summary, a quick examination of some of the approaches used by other authors on the Lorenz equations, and a brief discussion of some Lorenz-like equations at present under investigation.

*Stochastic Modeling of Elementary Psychological Processes.* By J. T. Townsend and F. G. Ashby. Cambridge University Press, 1984. 482 pp. \$69.50 HB, \$27.95 PB.

This book addresses the problem of designing mathematical models for the simple types of human cognitive and perceptual processes studied by experimental psychologists. After introducing the problem and reviewing the elements of probability theory that their readers will need, the authors discuss situations in which psychological mechanisms must operate on or process some number of items (such as memory search). They explore the issue of models oriented around opposing psychological concepts that nevertheless make similar or equivalent predictions for real data. They go on to develop mathematical concepts and experimental techniques whereby large opposing classes of models may be tested against one another. Numerous examples from the authors' own research and the experimental literature are included. Chapter headings: 1. Modeling elementary processes; reaction time and a little history; 2. Some basic issues and deterministic models of processing; 3. Mathematical tools for stochastic modeling; 4. Stochastic models and cognitive processing issues; 5. Compound processing models; 6. Memory and visual search theory; 7. Self-terminating vs. exhaustive search strategies; 8. Nonparametric RT predictions; distribution-ordering approaches; 9. Reaction time models and accuracy losses: varied state and counting models; 10. Random walk models of reaction time and accuracy; 11. Investigating the processing characteristics of visual whole report behaviour; 12. Additivity of processing times from separate subsystems and related issues; 13. The parallel-serial testing paradigm; 14. Stochastic equivalence and general parallel-serial equivalence relations when system differences are minimal or ignored; 15. A general discussion of equivalent and nonequivalent properties of serial and parallel systems and their models.

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*Statistical Decision Theory and Related Topics III, Volume 1.* Edited by Shanti S. Gupta and James O. Berger. Academic Press, Inc., New York, 1982. 526 pp. \$36.00.

This is volume 1 of the Proceedings of the Third Purdue Symposium, held June 1–5, 1981. It contains twenty-two of the invited papers presented at the symposium and includes works on general decision theory, multiple decision theory, optimum experimental design, sequential and adaptive inference, Bayesian analysis, robustness, and large sample theory. Volume 2 will contain the remaining twenty-eight invited papers.

*Structural Models in Anthropology.* By P. Hage and F. Harary. Cambridge University Press, 1984. 170. pp. \$39.50 Hardcover, \$12.95 Paperback.

This is volume 46 of the "Cambridge Studies in Social Anthropology." The authors present a comprehensive introduction to the use of graph theory in social and cultural anthropology. Using a wide range of empirical examples, they illustrate how graph theory can provide a language for expressing in a more exact fashion concepts and notions that can only be imperfectly rendered verbally. They show how graphs, digraphs, and networks, together with their associated matrices and duality laws, facilitate the study of such diverse topics as mediation and power in exchange systems, reachability in social networks, efficiency in cognitive schemata, logic in kinship relations, and productivity in subsistence modes. The interaction between graphs and groups provides further means for the analysis of transformations in myths and permutations in symbolic systems. The totality of these structural models aids in the collection as well as the interpretation of field data. The presentation is readily accessible to the nonmathematical reader. It emphasizes the implicit presence of graph theory in much of anthropological thinking.

*Inviscid Fluid Flows.* By H. Ockendon and A. B. Tayler. Springer-Verlag, 1983. 137 pp. \$16.00.

This is volume 45 in the series "Applied Mathematical Sciences." It is based on notes used at Oxford University in association with a course given to final year undergraduate and first year graduate students in mathematics. Chapter headings: 1. Mathematical models of fluid flows; 2. Free boundary problems; 3. Nonlinear surface waves; 4. Compressible flow; 5. Shock waves; 6. Approximate solutions for compressible flow; 7. Complex variable methods.

*Linear Optimization and Approximation.* By K. Glashoff and S. Gustafson. Springer-Verlag, 1978. 192 pp. \$19.80.

This is volume 45 in the series "Applied Mathematical Sciences." It is an introduction to the theoretical analysis and numerical treatment of semi-infinite programs, by which is meant the task of minimizing a linear real-valued function of finitely many variables subject to infinitely many linear constraints. The mathematical properties of such problems are investigated and algorithms for their computational solution are presented. The classical linear optimization problems, linear programs, occur naturally as a special case. Applications are discussed in detail. The central concept of the book is that of duality, investigated not for its own sake but as an effective tool, in particular for the numerical treatment of linear optimization problems. Chapter headings: 1. Introduction and Preliminaries; 2. Weak Duality; 3. Applications of Weak Duality in Uniform Approximation; 4. Duality Theory; 5. The Simplex Algorithm; 6. Numerical Realization of the Simplex Algorithm; 7. A General Three-phase Algorithm; 8. Approximation Problems by Chebyshev Systems; 9. Examples and Applications of Semi-Infinite Programming.

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*Regular and Stochastic Motion.* By A. J. Lichtenberg and M. A. Lieberman. Springer-Verlag, 1983. 470 pp. \$38.00.

This is volume 38 in the series "Applied Mathematical Sciences." It treats stochastic motion in nonlinear oscillatory systems, with applications in science and engineering, including astronomy, plasma physics, statistical mechanics and hydrodynamics. The main emphasis is on intrinsic stochasticity in Hamiltonian systems, where the stochastic motion is generated by the dynamics itself and not by external noise. However, the effects of noise in modifying the intrinsic motion are also considered. A thorough introduction to chaotic motion in dissipative systems is given in the final chapter. Physical insight rather than mathematical rigor is emphasized throughout. Practical methods are presented for describing the motion, for determining the transition from regular to stochastic behavior, and for characterizing the stochasticity. The authors rely heavily on numerical computations to illustrate the methods and to validate them. The book is intended to be a self-contained text for physical scientists and engineers who wish to enter the fields and a reference for those researchers already familiar with the methods. Some familiarity with Hamiltonian mechanics at the graduate level is desirable but not necessary; an extensive review of the required background material is given. Chapter headings: 1. Overview and Basic Concepts; 2. Canonical Perturbation Theory; 3. Mappings and Linear Stability; 4. Transition to Global Stochasticity; 5. Stochastic Motion and Diffusion; 6. Three or More Degrees of Freedom; 7. Dissipative Systems. Appendix A: Applications to planetary motion, accelerators and beams, charged particle confinement, charged particle heating, chemical dynamics, and quantum systems. Appendix B: Hamiltonian Bifurcation Theory.

*Linear Operator Theory in Engineering and Science.* By A. Naylor and G. Sell. Springer-Verlag, 1982. 607 pp. \$29.95.

This is volume 40 in the series "Applied Mathematical Sciences." It is the second, corrected, edition of the text first published in 1971. Its goal is to present the basic facts of functional analysis in a form suitable for engineers, scientists and applied mathematicians. Careful attention is given to motivation of the material covered and many illustrative examples are presented. Chapter headings: 1. Introduction; 2. Set-Theoretic Structure; 3. Topological Structure; 4. Algebraic Structure; 5. Combined Topological and Algebraic Structure; 6. Analysis of Linear Operators (Compact Case); 7. Analysis of Unbounded Operators.

*The Classification of Finite Simple Groups. Volume 1: Groups of Noncharacteristic 2 Type.* By Daniel Gorenstein. Plenum Press, 1983. 474 pp. \$59.50.

This is a volume in the "University Series in Mathematics." One of the milestones in the history of mathematics is the classification of finite simple groups. The result is a theorem whose proof was developed over a 30-year period by about 100 group theorists and is the union of some 500 journal articles covering approximately 10,000 printed pages. This volume, along with its companion volumes "Finite Simple Groups" and the forthcoming Volume 2 of "The Classification of Finite Simple Groups," offers a historical record of the original classification of simple groups against which any subsequent improvements and revisions can be measured. Drawing upon the results described in "Finite Simple Groups," this volume provides a comprehensive outline of the classification of simple groups of noncharacteristic 2 type. The book is divided into five chapters corresponding to the major divisions of the proof: 1. simple groups of 2-rank  $\leq 2$ ; 2. simple groups of low 2-rank; 3. centralizers of involutions in simple groups; 4. the  $B$ -theorem and locally unbalanced groups; 5. the classification of groups of component type. This volume makes the classification of groups of noncharacteristic 2 type accessible to any mathematician.

*Probability Methods in Applied Mathematics.* A. T. Bharucha-Reid, editor. Academic Press, 1973. 336 pp. \$77.00.

This volume contains the following articles: "Statistically Well-Set Cauchy Problems" by G. Birkhoff, J. Bona, and J. Kampe de Fériet; "Wave Propagation in Random Anisotropic Media" by D. Dence and J. E. Spence; "Stochastic Processes in Heat and Mass Transport" by V. A. Lo Dato; "Potential Theory for Markov Chains" by R. Syski; "On Some Stochastic Differential Games" by C. R. Tsokos and W. G. Nichols.