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A RESPONSE BOUND FOR HYSTERETIC SECOND ORDER SYSTEMS*

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The behavior of many engineering systems is governed by the second order differential equation

$$\ddot{u}(t) + f[u(t)] = g(t), \tag{1}$$

where $g(t)$ is a specified oscillatory function of time, dot denotes differentiation with respect to t, $f(u)$ is a nonlinear restoring function representing the system hysteresis, as shown in the figure, and $u(0) = \dot{u}(0) = 0$. To obtain the bound, both sides of equation (1) are multiplied by $\dot{u}$. Integration of the resulting expression over the time interval t_i to t_{i+1} yields

$$\left.\frac{\dot{u}^2(t)}{2}\right]_{t_i}^{t_{i+1}} + \int_{t_1}^{t_{i+1}} f(u)\dot{u}(\tau)\,d\tau = \int_{t_i}^{t_{i+1}} g(\tau)\dot{u}\,d\tau. \tag{2}$$

If t_i and t_{i+1} are two consecutive times of zero crossing of $\dot{u}(t)$, then the first term on the left hand side of the above equation vanishes. Therefore,

$$\int_{t_i}^{t_{i+1}} f(u)\dot{u}(\tau)\,d\tau = \int_{t_i}^{t_{i+1}} g(\tau)\dot{u}(\tau)\,d\tau. \tag{3}$$

Since $u(t)$ attains extremum values at times t_i and t_{i+1}, then one may assume $\bar{u} = \sup|u(t)|$ to occur either at $t = t_i$ or $t = t_{i+1}$. Therefore, the right hand side of equation (3) may be represented by

$$\int_{t_i}^{t_{i+1}} g(\tau)\dot{u}(\tau)\,d\tau = \int_{u(t_i)}^{u(t_{i+1})} g(\tau)\,du \leqslant 2\bar{g}\bar{u}, \tag{4}$$

where $\bar{g} = \sup|g(t)|$. The left hand side of equation (3) may also be represented as

$$\int_{t_i}^{t_{i+1}} f(u)\dot{u}(\tau)\,d\tau = \int_{u(t_i)}^{u(t_{i+1})} f(u)\,du = \alpha(2\bar{f}\bar{u}), \tag{5}$$

where $\bar{f} = \sup|f(u)|$ and $0 < \alpha \leqslant 1$ is the reduction factor which makes the equality satisfied. The value of α may be calculated from the ratio $A/4\bar{f}\bar{u}$, where A is the area of the hysteresis loop. Comparisons of equalities (3) and (5) and inequality (4) yield

$$\bar{f} \leqslant \bar{g}/\alpha. \tag{6}$$

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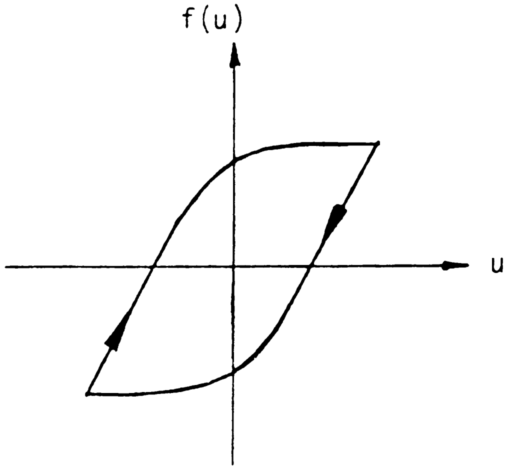

A representation of $f(u)$

If it is assumed that $\bar{u}$ and $\bar{f}$ occur simultaneously, i.e. $\bar{f} = |f(\bar{u})|$, it is concluded that

$$f(\bar{u}) \leqslant \bar{g}/\alpha. \tag{7}$$

This inequality yields an upper bound on $\bar{u}$ with α as a parameter.