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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

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The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

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Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\left[(a^2+b^2)^{1/2}\right] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\dfrac{x}{2b}}{\cos\dfrac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

CONTENTS

NEW BOOKS

Differential Geometric Control Theory. Edited by R. W. Brockett, R. S. Millman and H. J. Sussman. Birkhauser Boston, Inc., Boston, 1983. 335 pp.

This is volume 27 of the series Progress in Mathematics. It constitutes the proceedings of a conference held at Michigan Technological University, June 28–July 2, 1982. The backbone of the conference was formed by a series of lectures on exterior differential systems by Robert Gardner and one on control theory by Hector Sussmann. They form the bulk of this book. There are also ten invited addresses by control theorists and geometers. Most of these have an expository component in addition to presenting recent results.

The Physics of Vibration, Volume 2, *The Simple Vibrator in Quantum Mechanics*. By A. B. Pippard. Cambridge University Press, New York, 1983. 202 pp. $39.50.

Volume 1 of *The Physics of Vibration* dealt with simple vibrating systems in terms of classical physics. In Volume 2 the behaviour of these systems is discussed in those cases when the use of quantum mechanics is appropriate, for example in systems of atomic dimensions. Only an elementary knowledge of quantum mechanics is needed. The argument throughout is based on direct solutions of Schrödinger's equation. The interaction of atomic and molecular vibrators with light and other forms of electromagnetic radiation plays a central part in the argument, and leads to a detailed analysis of the ammonia maser, as well as to successive discussions at ever-deeper levels of the problem of incorporating dissipative effects into quantum mechanics. Throughout the book the remarkable similarity of the results of applying classical and quantum physics to these problems is stressed, although care is taken to pinpoint the limits of validity of classical thought. Chapter headings: 13. The quantized harmonic vibrator and its classical features; 14. Anharmonic vibrators; 15. Vibrations and cyclotron orbits in two dimensions; 16. Dissipation, level broadening and radiation; 17. The equivalent classical oscillator; 18. The two-level system; 19. Line broadening; 20. The ammonia maser; 21. The family of masers: from laser to travelling-wave oscillator.

Seminar on Stochastic Processes, 1981. Edited by E. Cinlar, K. L. Chung, and R. K. Getoor. Birkhäuser Boston, Inc., Boston, 1981. 242 pp. $17.50.

This is volume 1 in the series Progress in Probability and Statistics. It consists of eight of the papers presented during a three day seminar on stochastic process held at Northwestern University in April 1981. The authors are K. L. Chung, K. M. Rab, R. K. Getoor, M. J. Sharpe, F. B. Knight, J. W. Pitman, A. O. Pittenger, Z. R. Pop-Stojanovic, J. Walsh, W. Winkler, E. Cinlar, J. Jacod.

Branching Processes. By S. Asmussen and H. Hering. Birkhäuser Boston, Inc., 1983. 451 pp. $34.95.

This is volume 3 in the series Progress in Probability and Statistics. The authors' aim is to single out some of the more recent developments in the field and to present them with sufficient background material to obtain a largely self-contained treatment intended to supplement previous monographs rather than to overlap them. The body of the text is divided into four parts. Part A is a short introduction, stressing examples and applications. Part B is a self-contained and up-to-date presentation of the classical limit theory of simple branching processes, viz. the Galton-Watson process and its continuous time analogue. Part C deals with the limit theory of Markov branching processes with a general set of types under conditions tailored to (multigroup) branching diffusions on bounded domains, a setting which also covers the ordinary multitype case. The aim of Part C is to treat a large subfield to the highest degree of generality and completeness possible. Part D concerns simple branching diffusions on unbounded domains, age-dependent processes and their generalizations to models in which the individuals have completely unrestricted reproduction patterns, and finally models in which different individuals cooperate in producing offspring. For easier reference a number of auxiliary results and tools used by the authors are compiled in an Appendix. Each of the eleven chapters concludes with a section of bibliographical comments.

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Quantum Theory, Groups, Fields and Particles. Edited by A. O. Barut. D. Reidel Publishing Co., Dordrecht, Holland, 1983. 334 pp. $45.50.

This is volume 4 in the series Mathematical Physics Studies. The contributions collected here were originally delivered at two meetings in Turkey, at Blacksea University in Trabzon and at the University of the Bosphorus in Istanbul, but they have been thoroughly revised, updated and extended for this volume. The volume collects self-contained introductory contributions of some of the recent developments both in mathematical concepts and in physical applications of Symmetry and Dynamics which are becoming very important in current research. It treats, on the one hand, differential geometry, group representations, topology and algebras and on the other hand, particle equations, particle dynamics and particle interactions. Specifically, it contains a complete exposition of the theory of deformations of symplectic algebras and quantization, expository material on topology and geometry in physics, and group representations. On the more physical side, there are studies on the concept of particles, on conformal spinors of Cartan, on gauge and supersymmetric field theories, and on relativistic theory of particle interactions and the theory of magnetic resonances. The eleven contributions are grouped in three parts: 1. Quantum Theory and Deformations, 2. Group Representations and Gauge Theories; 3. Particle Dynamics.

Combinatorial Enumeration. By I. P. Goulden and D. M. Jackson. John Wiley & Sons, Inc., New York, 1983. 541 pp. $47.50.

This is a volume in the Wiley-Interscience Series in Discrete Mathematics. The theory of enumeration has developed rapidly during the past century, with the increasing awareness of the importance of discrete structures. Work on its mathematical foundations has been inspired by MacMahon's "Combinatory Analysis," published in 1915, and Rota's series entitled "On the Foundations of Combinatorial Theory," begun in 1964. The authors' objectives in writing this book are to give a unified account of a generating function approach to this area and to give a reasonably complete collection of representative results. They have illustrated the theory with a range of examples to reveal something of its generality and subtlety. The book is written not only for the combinatorial theorist but also for the mathematician, the physicist, and the computer scientist, in whose fields problems of this type occur. Hitherto, much of the material included here has been available only in the research journals. The book is divided into five parts: 1. Mathematical Preliminaries; 2. The Combinatorics of the Ordinary Generating Function; 3. The Combinatorics of the Exponential Generating Function; 4. The Combinatorics of Sequences; and 5. The Combinatorics of Paths. A feature of the book is that the exercises are organized as a compendium of supplementary results whose solutions are given in detail and form an important part (two hundred pages) of the work.

Geometric Quantization in Action: Applications of Harmonic Analysis in Quantum Statistical Mechanics and Quantum Field Theory. By Norman E. Hurt. D. Reidel Publishing Co., Dordrecht, Holland, 1983. xiv + 336 pp. $49.50.

This is volume 8 in the series Mathematics and Its Application. The major thrust of this volume is to present the single theme that geometric quantization provides the structure for the geometric realizations of the irreducible unitary representations of the groups involved in physics. It also presents some examples of the use of this representation theory in two areas of physics—quantum statistical mechanics and quantum field theory. Chapter headings: 0. Survey of Results; 1. Representation Theory; 2. Euclidean Group; 3. Geometry of Symplectic Manifolds; 4. Geometry of Contact Manifolds; 5. The Dirac Problem; 6. Geometry of Polarizations; 7. Geometry of Orbits; 8. Fock Space; 9. Borel-Weil Theory; 10. Geometry of C-Spaces and R-Spaces; 11. Geometric Quantization; 12. Principal Series Representations; 13. Geometry of DeSitter Spaces; 14. Discrete Series Representations; 15. Representations and Automorphic Forms; 16. Thermodynamics of Homogeneous Spaces; 17. Quantum Statistical Mechanics; 18. Selberg Trace Theory; 19. Quantum Field Theory; 20. Coherent States and Automatic Forms.

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Functional Integration and Semiclassical Expansions. By F. Langouche, D. Roekaerts, and E. Tirapegui. D. Reidel Publishing Co., Dordrecht, Holland, 1982. xii + 313 pp. $39.50.

This is volume 10 in the series Mathematics and Its Applications. It is intended as a fairly complete presentation of what the authors call the discretization approach to functional integrals, i.e. path integrals defined as limits of discretized expressions. In its main parts it is based on the original work of the authors. Their main concern has been to present a formalism that is practical and which can be adapted to make computation in the numerous areas where path integrals are being increasingly used. For these reasons applications, illustrative examples, and detailed calculations are included. There is also a complete and up-to-date bibliography. Chapter headings: 1. Functional integrals defined as limits of discretized expressions; 2. Correspondence rules and functional integral representations; 3. Functional integral representations of expectation values; 4. Perturbation expansions; 5. Short time propagators and the relations between them; 6. Covariant definitions of functional integrals; 7. Functional integral methods in Fokker-Planck dynamics; 8. Product integrals; 9. The semiclassical expansion in phase space; 10. The semiclassical expansion in configuration space; 11. Other approaches; 12. Computation of the propagator on the sphere s^3.

Percolation Theory for Mathematicians. By Harry Kesten. Birkhäuser Boston, Inc., Boston, 1982. 412 pp. $30.00.

This is volume 2 in the series Progress in Probability and Statistics. Percolation theory had its origin in a genuine applied problem and has, at the same time, been a source of many important mathematical problems. The author aims in this book to treat some basic results with rigorous proofs. Although being a research monograph, the only prerequisite is a standard graduate course in probability theory. Table of contents: 1. Introduction and summary; 2. Which graphs do we consider? 3. Periodic percolation problems; 4. Increasing events; 5. Bounds for the distribution of the number of edges; 6. The Russo-Seymour-Welsh Theorem; 7. Proofs of certain theorems; 8. Power estimates; 9. The nature of a singularity; 10. Inequalities for critical probabilities; 11. Resistance of random electrical networks; 12. Unsolved problems.

Numerical Treatment of Inverse Problems in Differential and Integral Equations. Edited by P. Deufhard and E. Hairer. Dirkhauser Boston, Inc., Boston, 1983. 354 pp. $27.50.

This is volume 2 in the series Progress in Scientific Computing. It presents the Proceedings of an international Workshop, held at Heidelberg, August 30–September 2, 1982. It deals with the problem: given measured data and an associated theoretical model, determine unknown parameters in that model (or unknown functions to be parametrised) in such a way that some measure of the discrepancy between data and model is minimal. Fields of applications presented include: chemistry, molecular biology, physics, geophysics, astronomy, reservoir simulation, electrocardiology, computed tomography, and control system design. The models are expressed in terms of ordinary, partial, and integral equations.

Differential Topology: An Introduction. By David B. Bauld. Marcel Dekker, Inc., New York, 1982. 256 pp. $29.75.

This is volume 72 in the series Pure and Applied Mathematics. The aim of this book is to present a classification of surfaces from the viewpoint of differential topology. Along the way the reader is introduced to topological spaces then proceeds to a study of differentiable manifolds. Morse theory and surgery are studied at a fairly general level before attention is restricted to the effect of surgery on a surface. This surgery then leads to the classification. A solution manual is available upon request.

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Foundations of Euclidean and Non-Euclidean Geometry. By Richard L. Faber. Marcel Dekker, Inc., New York, 1983. 350 pp. $49.75.

This is volume 73 in the series Pure and Applied Mathematics. It is intended for use in undergraduate courses and builds a solid foundation of important concepts in geometry and an understanding of their historical development. There are numerous computational and theoretical problems, and a solution manual is available upon request. Chapter headings: 1. The Beginnings; 2. Greek Geometry; 3. The Axiomatic Method; 4. History of the Parallel Postulate; 5. Fundamentals of Lobachevskian Geometry; 6. The Trigonometric Formulas; 7. The Weierstrass Model; 8. Lobachevskian Geometry and Physical Space.

Statistical Theory and Random Matrices. By Moshe Carmeli. Marcel Dekker, Inc., New York, 1983. 215 pp. $35.00.

This is volume 74 in the series Pure and Applied Mathematics. The question whether the highly excited states of a physical system may be understood by assuming no structure to the system, and that no quantum number other than the spin and the parity remains good leads to the statistical theory of energy levels and its relation to random matrices. Such a statistical theory is designed to describe the general appearance and the degree of irregularity of the level structure that occurs in a complex physical system. The standard type of statistical mechanics is inadequate for the discussion of energy levels since statements on the fine detail of the energy level structure cannot be made in terms of an ensemble of states. What is required is a different kind of statistical mechanics in which one renounces the exact knowledge not of the state of he system but of the nature of the system itself. The problem then is to define in a mathematically precise way an ensemble of systems in which all possible laws of interactions are equally probable. The idea of a statistical mechanics of nuclei, which is based on an ensemble of systems, is due to Wigner and to von Neumann. This book summarizes the fundamentals of this theory. Chapter headings: 1. Introduction; 2. Highly Excited Systems; 3. Symmetry Properties of Physical Systems; 4. Gaussian and Orthogonal Ensembles; 5. Unitary Ensembles; 6. Eigenvalue-Eigenvector Distributions of the Gaussian Ensemble; 7. Distribution of the Widths; 8. Symplectic Group and Quaternions; 9. More on the Gaussian Ensemble; 10. Summary.

The Theory of Topological Semigroups. By H. H. Carruth. Marcel Dekker, Inc., New York, 1983. 264 pp. $34.75.

This is volume 75 in the series Pure and Applied Mathematics. The goal of this book is to survey the field of topological semigroups, and to make this body of information accessible to the average graduate student. The spirit of the exposition follows that of A. D. Wallace, along the lines of the question "What topological spaces admit a continuous associative multiplication with unit?" The answers to these questions involve more algebra and topology than is the case for compact groups, where there is a representation theory due to the presence of Haar measure. The coverage here includes background material, internal structure, products, quotients, and semigroups with some special algebraic or topological property. In a subsequent volume, some of the aspects which rely on cohomology or category theory will be treated.

The Method of Discretization in Time and Partial Differential Equations. By Karel Rektorys. D. Reidel Publishing Co., Dordrecht, Holland, 1982. xviii + 451 pp. $69.00.

This is volume 4 in the series Mathematics and Its Applications (East European Series). It is devoted to results obtained by the author's method in his seminar during recent years. The method is a modification of the Rothe method, or method of lines, for converting the solution of a parabolic boundary value problem of second order in two variables x, t into the solution of p ordinary differential equation, with corresponding boundary conditions, by dividing the interval for the variable t into p subintervals and replacing the partial time derivative at each point of the division by a difference quotient. There are twenty-one chapters, two introductory ones, seven consisting of examples, and the others giving theoretical aspects of the method of discretisation in time.