

QUARTERLY  
OF  
APPLIED MATHEMATICS

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# QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, RI 02912, either directly or through any one of the Editors. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proof only. The author's institution will be requested to pay a publication charge of \$30 per page which, if honored, entitles the author to 100 free reprints. Detailed instructions will be sent with galley proofs.

The current subscription price per volume (April through January) is \$40. Single issues can be purchased, as far as they are available, at \$11 and back volumes at \$40 per volume. Subscriptions and orders for back volumes must be addressed to the American Mathematical Society, P. O. Box 1571, Providence, RI 02901. All orders must be accompanied by payment. Other subscription correspondence should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, RI 02940.

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# SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\left[(a^2 + b^2)^{1/2}\right] \text{ is preferable to } e^{(a^2 + b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \frac{x}{2b}}{\cos \frac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

**Figures:** Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b. c." even if this special abbreviation is defined somewhere in the text.

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—NEW BOOKS—

*The theory and practice of reliable system design.* By Daniel P. Siewiorek and Robert S. Swarz. Digital Press, Bedford, MA, 1982. xxiv + 774 pp. \$55.00.

This book is addressed to advanced undergraduates, graduate students and practicing engineers. It provides an introduction to the broad range of issues in reliable computer design. Enough mathematics are included to make possible first-order models and evaluations of alternative designs. A major goal of the book is to provide enough concepts to enable the practicing engineer to incorporate comprehensive reliability techniques into his or her next design. Design techniques can be understood through a series of articles in Part II, which describe actual implementations of reliable computers; this information is provided by the system designers themselves. The final chapter provides a methodology for reliable system design and illustrates how this methodology can be applied in an actual design situation (the Intel 432).

*Practical methods of optimization, volume 2: Constrained optimization.* By R. Fletcher. John Wiley & Sons, New York, 1981. ix + 224 pp. \$31.95.

This is the second part of a two-volume work which presents those aspects of optimization which are currently of foremost importance in solving practical problems. This involves a study of optimality criteria, determination of suitable algorithms, and computer experimentation both under trial conditions and from real-life experience. The main theme of the book is an emphasis on practicability, and most attention is given to methods which have proved to be reliable and efficient. Volume 2 covers most types of finite-dimensional optimization problems with constraints of a continuous nature. Linear and quadratic programming are covered as various approaches to nonlinear programming. Also included are sections on the theory of Lagrange multipliers and optimality conditions, convexity and duality, integer programming, and a chapter on non-differentiable optimization which describes algorithms and theory in a unified way.

*Transonic Shock, and Multidimensional Flows.* Edited by Richard E. Meyer. Academic Press, New York, 1982. ix + 345 pp. \$21.00.

This volume collects invited lectures presented at a symposium held in Madison on May 13–15, 1981, under the auspices of the Mathematics Research Center of the University of Wisconsin. The symposium was devoted to a review of recent and current progress in the scientific computation of high-speed aerodynamic phenomena and related fluid motions. It recorded the mastering of problems generally acknowledged as insoluble only two decades ago and sketched the trends of future development. There were fifteen speakers. They surveyed the latest successful methods in aerodynamic computation and their application to the transonic flow patterns typical of contemporary flight, as well as the theoretical foundations of these methods.

*Self-organization and dissipative structures—Applications in the physical and social sciences.*  
Edited by William C. Schieve and Peter M. Allen. University of Texas Press, Austin, TX, 1982. xii + 361 pp. \$50.00.

This volume is based upon the proceedings of a workshop held in Austin in September 1978 in honor of Ilya Prigogine. Its main purpose was to bring together different aspects of recent work in the natural and human sciences which seem to offer new possibilities of linking the two. The twenty-three invited speakers came from many different fields and disciplines. They are concerned either with the development of mathematical concepts and methods for the description of forms and patterns as they evolve in the living world or with the search to reveal the variables and mechanisms necessary to describe adequately the evolution of a system containing many factors responding to subjective, qualitative stimuli.

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*Advanced programming and problem solving with Pascal.* By G. Michael Schneider and Steven C. Bruell, John Wiley & Sons, New York, 1981. \$23.95.

This is a textbook designed for a second course in computer programming (e.g., the course CS2 in ACM Curriculum '78).

*From ENIAC to UNIVAC: An appraisal of the Eckert-Mauchly computers.* By Nancy Stern. Digital Press, Bedford, MA, 1981. ix + 286 pp.

The aim of this monograph is threefold: firstly, to focus on the process of development in the computer field, taking the work of Eckert and Mauchly on four computers as a case study; secondly, to analyze the social and institutional forces influencing the development process; and finally to place the controversial achievements of Eckert and Mauchly in the appropriate historical context, always remaining as objective as possible.

*Project Whirlwind—The history of a pioneer computer.* By Kent C. Redmond and Thomas M. Smith. Digital Press, Bedford, MA, 1980. xiv + 280 pp.

Whirlwind, developed at M.I.T. under the leadership of Forrester and Everett, was the first electronic digital computer able to operate in real time, using magnetic core memory and synchronous parallel logic. The authors examine the technological traditions, the research and development policies and practices, the funding crises, the management techniques, and the administrative philosophies that went into the engineering feat.

*Principles of dynamic programming, part II: Advanced theory and applications.* By Robert E. Larson and John L. Casti. Marcel Dekker, Inc., New York and Basel, 1982. ix + 497 pp. \$39.75.

This is volume 7 of the series Control and Systems Theory. In the first part of this two-volume work the basic theme was determinism and algorithm feasibility. In this part, many of the extensions and generalizations inherent in the dynamic programming approach to the determination of optimal policies are explored. Contact with classical variational theory is made and, in a more modern spirit, a number of questions involving optimal control processes are examined. A chapter is devoted to stochastic and adaptive processes arising in engineering and management situations. Chapter headings: 1. Dynamic programming, optimal control theory, and the calculus of variations. 2. Quadratic criteria—linear equations. 3. Stochastic and adaptive systems. 4. Advanced computational methods. 5. Dynamic programming and operations research problems. 6. Two case studies.

*Elementary computability, formal languages, and automata.* By Robert McNaughton. Prentice-Hall, Englewood Cliffs, NJ, 1982. xvi + 400 pp. \$24.95.

This book is an introduction to theoretical computer science emphasizing two interrelated areas: the theory of computability and the theory of formal languages. Automata are used as precise models of computation in studies that have actual computers as their primary application. Other areas, such as semantics and computational complexity, are treated briefly, in an attempt to bring all of theoretical computer science into view. It seeks to provide a well-rounded and elementary explanation of the issues and an adequate supply of exercises, integrated with the text. There are also some advanced expository sections, but the predominating level is elementary. The bibliography emphasizes expository books and articles rather than original monographs and research papers. Table of contents: 1. The concept of algorithm. 2. Turing machines. 3. The foundational programming languages. 4. Computable functions. 5. Godel numbering and Church's thesis. 6. Unsolvable problems. 7. Context-free grammars for formal languages. 8. Parsing. 9. Regular languages and finite automata. There are six appendices: 1. Euclidean algorithm: correctness proof. 2. Labyrinth algorithm: correctness proof. 3. Basic issues of formal languages. 4. Critique of algorithms: computational complexity. 5. Proofs of mathematical induction. 6. The diagonal method.

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*Formal methods of program verification and specification.* By H. K. Berg, W. E. Boebert, W. R. Franta, and T. G. Moher, Prentice-Hall, Englewood Cliffs, NJ, 1982. xiv + 207 pp.

The principal application of formal methods in the development of software lies in the area of verification, where mathematical techniques are used to argue that a given program is correct. Such an argument requires a rigorous definition of correctness; this definition is given by a formal specification. This book is an introduction to the fundamental techniques in both specification and verification. Chapter headings: 1. Introduction. 2. Models of Computation. 3. Verification Methods. 4. Approaches to Proofs of Partial Correctness. 5. Approaches to Proofs of Total Correctness. 6. Correctness of Parallel Programs. 7. Application of the Verification Approaches. 8. Approaches to Specification. 9. State of the Art and Summary.

*Three-dimensional constitutive relations and ductile fracture.* Edited by S. Nemat-Nasser. North-Holland Publishing Co., New York, 1981. xv + 439 pp. \$95.00.

These are the proceedings of an International Union of Theoretical and Applied Mechanics (IUTAM) Symposium held in Dourdon, France, 2-5 June 1980. It was organized in an effort to bring together various significant ingredients relevant to flow and ductile fracture of materials and to focus attention on important unresolved research problems in the area. The objectives were to address the following three basic areas: (1) Mechanisms for the formation and propagation of fissures in ductile materials, (2) Constitutive equations for the plastic behavior of materials in regions close to the ends of fissures, and (3) Global macroscopic consequences of (1) and (2). These areas were extensively examined, covering theoretical and experimental aspects of various mechanisms involved in the formation, growth, and coalescence or interconnection of voids, the stress and strain fields near the tip of a stationary or mobile crack, and finally, the possibility of formulating global criteria for the initiation and growth of fissures. This volume contains twenty contributions to the Symposium as well as discussions of their presentations.

*Fourier series and integrals of boundary value problems.* By J. Ray Hanna. John Wiley & Sons, New York, 1982. xi + 271 pp. \$31.95.

This is a volume in the series Pure and Applied Mathematics. The emphasis is placed upon basic concepts and techniques rather than the development of theory. Modeling and solutions of physical problems are considered in detail. Chapter headings: 1. Linear Differential Equations. 2. Orthogonal Sets of Functions. 3. Fourier Series. 4. Fourier Integrals. 5. Mathematical Models and Boundary Value Problems. 6. Continuation of Models and Boundary Value Problems. 7. Bessel Functions and Boundary Value Problems. 8. Legendre Polynomials and Boundary Value Problems.

*Case studies in mathematical modeling.* Edited by D. J. G. James and J. J. McDonald. John Wiley & Sons, New York, 1981. viii + 214 pp. \$17.95.

This book is the result of a workshop on the teaching of mathematical modeling. Topics include crop yield, deer harvesting, leaching of nitrates, plant growth, parking a car, hydroelectric power generation, speed-wobble in motorcycles, cost of a mortgage, selling hot dogs and others.

*Tensor analysis on manifolds.* By Richard L. Bishop and Samuel I. Goldberg. Dover Publications, Inc., New York, 1980. vii + 280 pp. \$5.50.

This is an unabridged and corrected republication of the work originally published by The Macmillan Company in 1968.

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*Elementary statistical methods.* By G. Barrie Wetherill. Chapman & Hall, London and New York, 1982. xiii + 356 pp. \$14.95.

This is the third edition of a text first published in 1967. The most substantial revision is to the appendix on calculation and computing which now contains BASIC programs. It is stated that APL programs are available from the author.

*Theory of games and statistical decisions.* By David Blackwell and M. A. Girshick. Dover Publications, Inc., New York, 1979. xi + 355 pp. \$5.50.

This is an unabridged and unaltered republication of the work originally published in 1954 by John Wiley & Sons.

*Statistical analysis of geological data.* (two volumes bound as one). By George S. Koch, Jr. and Richard F. Link. Dover Publications, Inc., New York, 1980. ix + 438 pp. \$12.50.

This is a reprint of a text first published in two volumes in 1970. There are eight chapters: 1. Introduction. 2. Distributions. 3. Sampling. 4. Inference. 5. Analysis of variance. 6. Distributions and transformations. 7. Geological sampling. 8. Variability in geological data.

*Set theory and logic.* By Robert R. Stoll. Dover Publications, Inc., New York, 1979. xiv + 474 pp. \$6.50.

This is an unabridged corrected republication of the first edition, published by W. H. Freeman & Co. in 1963.

*An introduction to information theory—Symbols, signals and noise.* By John R. Pierce. Dover Publications, Inc., New York, 1980. xii + 305 pp. \$4.50.

This is an unabridged and revised version of the work originally published in 1961 by Harper & Bros. under the title *Symbols, signals and noise: the nature and process of communication*. The author has brought the work up to date and contributes a new preface.

*An introduction to the approximation of functions.* By Theodore J. Rivlin. Dover Publications, Inc., New York, 1981. vii + 147 pp. \$3.50.

This is an unabridged and corrected republication of the work originally published by the Blaisdell Company in 1969.

*Graphs, surfaces and homology: An introduction to algebraic topology.* By P. J. Giblin. Chapman & Hall, New York, 1981. xvii + 329 pp. \$15.95.

This is the second, revised edition of a text first published 1977. The most notable addition is a discussion of the computer-proof of the four-color theorem.

*Speculations on the fourth dimension—Selected writings of Charles H. Hinton.* Edited by Rudolf v. B. Rucker. Dover Publications, Inc., New York, 1980. xix + 204 pp. \$4.00.

This is a new anthology of essays by Hinton published between 1884 and 1907.

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*Contributions to analysis and geometry.* Edited by D. N. Clark, G. Pecellis and R. Sacksteder. The Johns Hopkins University Press, Baltimore, MD, and London, 1982. ix + 357 pp. \$36.50

These are the proceedings of a conference held at The Johns Hopkins University on April 24 and 25, 1980; the volume is dedicated to Philip Hartman on the occasion of his retirement. It contains the invited addresses given by Charles C. Pugh, Richard Sacksteder, Clifford Truesdell, Jack Hale and Nelson Max, as well as 22 other papers.

*Pseudo-differential operators.* By Hitoshi Kumano-go. The MIT Press, Cambridge, MA, and London, England, 1981. xviii + 451 pp.

Pseudo-differential operators are a natural extension of linear partial differential operators and their theory developed rapidly after 1965 with the systematic studies of Kohn-Nirenberg, Hormander, and others, who expressed the symbol of a differential operator by means of its asymptotic expansion. Presently, large parts of classical applied mathematics, including potentials, Green's functions, and fundamental solutions, are being newly reconsidered from the point of view of pseudo-differential operators. Heretofore, however, a fairly deep knowledge of functional analysis has been required for the understanding of the whole theory. Examining the structure of this theory, the present author noticed that the basic part of the theory can be constructed solely by means of elementary calculus and the elementary theory of Fourier transforms. This monograph is the result of that discovery. Chapter headings: 1. Spaces of Functions and Fourier Transforms. 2. Pseudo-Differential Operators of Class  $S_{\rho,\delta}^m$ . 3. Pseudo-Differential Operators on Sobolev Spaces. 4. Second-Order Hypoelliptic Operators. 5. The Index of Fredholm Operators. 6. Boundary-Value Problems for Elliptic Operators. 7. Pseudo-Differential Operators of Class  $S_{\lambda,\rho,\delta}^m$  and Initial-Value Problems. 8. Complex Powers of Hypoelliptic Operators. 9. The Fixed-Point Theorem of Atiyah-Bott-Lefschetz. 10. Fourier Integral Operators.

*Design of database structures.* By Toby J. Teorey and James P. Fry. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1982. xv + 492 pp. \$28.95.

The purpose of this book is to establish a consistent framework for multilevel database design; to define a workable methodology; and to describe a set of general principles, tools and techniques for database design at each level. The methodology uses the top-down (and iterative) design path, providing for evaluation at many points, allowing for redesign when necessary, and using the same basic terminology throughout. The goal of the design process itself is to formulate a database structure that accurately represents the real-world environment it serves and can be efficiently implemented on an existing hardware/software system. The methodology is illustrated throughout the text using detailed examples from statements of database system requirements through specification of logical and physical database structures that satisfy those requirements. The approach is applicable to network, relational, and hierarchical database systems.

*VLSI systems design for digital signal processing, volume I: Signal processing and signal processors.* By B. A. Bowen and W. R. Brown, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1982. xvii + 304 pp.

The material in this volume is organized in two parts. The first five chapters form part A which reviews the underlying theory and basic techniques of digital signal processing. Chapter 5 provides the bridge from processing to processors. Chapters 6 and 7 make up Part B and concern the architecture and performance limitations of processors and hardware evolution.

*Elementary differential equations with linear algebra.* By Albert L. Rabenstein. Academic Press, New York and London, 1982. x + 519 pp.

This is the third edition of a text emphasizing the interplay between differential equations and matrices.