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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

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The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

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Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\left[(a^2+b^2)^{1/2}\right] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\dfrac{x}{2b}}{\cos\dfrac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

CONTENTS

David C. Barnes: Buckling of columns and rearrangements of functions 169

Gabriele Anzellotti: On the existence of the rates of stress and displacement for Prandtl-Reuss plasticity 181

Mark H. Holmes: A nonlinear diffusion equation arising in the study of soft tissue 209

Z. U. A. Warsi: A note on the mathematical formulation of the problem of numerical coordinate generation 221

A. D. Solomon, D. G. Wilson and V. Alexiades: Explicit solutions to phase change problems 237

Nima Geffen: Alternative variational formulations for first order partial differential systems 245

L. M. de Socio and G. Gualtieri: A hyperbolic Stefan problem 253

A. E. Green: The solitary wave with surface tension 261

New Books 244, 260, 263, 264

— NEW BOOKS —

The theory and applications of graphs. Edited by Gary Chartrand, Yousef Alavi, Donald L. Goldsmith, Linda Lesniak-Foster and Don R. Lick. John Wiley & Sons, New York, 1981. xvi + 611 pp. $29.95.

This book contains the nearly 50 refereed papers presented at the fourth international conference on the theory and applications of graphs, held at Western Michigan University, May 6–9, 1980. A number of open problems suggested by the conference participants are posed. Research topics include tournaments, Ramsey theory, topological graph theory, Hamiltonian graphs, colorings, enumeration, connectivity and trees. Applications include problems in traffic phasing, mission and computer networks, linguistics, chemistry and biochemistry.

Catastrophe theory and applications. Edited by D. K. Sinha. John Wiley & Sons, New York, 1981. xi + 131 pp. $15.95.

These are the proceedings of a seminar on applicable catastrophe theory, held at Jadavpur University in 1979. There are texts of fourteen lectures, six of these by E. C. Zeeman, and an exhaustive bibliography of 22 pages.

An introduction to applied optimal control. By Greg Knowles. Academic Press, New York, 1981. x + 180 pp. $19.50.

This is a text for an introductory-level course to students of varied background, who are mainly interested in applications. Chapter I introduces the concept of optimal control by means of examples. In Chapter II necessary conditions for optimality for the linear-time optimal control problem are derived geometrically, and illustrations are given. Chapters III and IV discuss the Pontryagin maximum principle, its relation to the calculus of variations, and its application to various problems in science, engineering, and business. Since the optimality conditions arising from the maximum principle can often be solved only numerically, numerical techniques are discussed in Chapter V. In Chapter VI the dynamic programming approach to the solution of optimal control problems and differential games is considered; in Chapter VII the controllability and observability of linear control systems are discussed, and in Chapter VIII the extension of the maximum principle to state-constrained control problems is given. Finally, for more advanced students with a background in functional analysis, Chapter IX considers several problems in the control of systems governed by partial differential equations.

Mathematical basis of statistics. By Jean-Rene Barra; translation edited by Leon Herbach. Academic Press, New York, 1981. xvi + 249 pp. $39.50

This is a volume in the series Probability and Mathematical Statistics. As Yu. V. Linnik says in his foreword, the author's work is "an excellent mathematical introduction to modern mathematical statistics." The book is based on the Bourbaki school. As such, it is intended for advanced students. Fundamental to the presentation is the notion of a 'statistical space'. There is an appendix dealing with the modern notion of conditional probability.

Advances in applied mechanics. Edited by Chia-Shun Yih. Academic Press, New York, 1981.

This volume contains three papers: Variational and Related Methods for the Overall Properties of Composites by J. R. Willis, Elastic Wave Propagation in Stratified Media by B. L. N. Kennett, and Elastic Behavior of Composite Materials: Theoretical Foundations by L. J. Walpole.

Continued on Page 260

Continued from Page 244

Multivariable calculus.By Lawrence J. Corwin and Robert H. Szczarba. Marcel Dekker, Inc., New York, 1979. xi + 525 pp.

This is volume 69 in the series Monographs and Textbooks in Pure and Applied Mathematics. It is an introduction to the calculus of functions of several variables, suitable for students who have had two years of calculus.

Modern probability theory: an introductory textbook. By B. Ramdas Bhat. John Wiley & Sons, New York, 1981. xi + 256 pp. $14.95.

This book covers the syllabus of a one-year graduate course, emphasizing rigorous derivations using the axiomatic approach.

An introduction to statistical physics. By W. G. V. Rosser. Ellis Horwood Limited, Chichester, and John Wiley & Sons, New York, 1982. xiv + 382 pp. $55.00.

This introductory book develops statistical mechanics in a novel, logical and unified manner, in a more simplistic manner than is usual. Numerical examples are used as a clear and straightforward method of presenting new material and formulae. The subject is then developed from the three main postulates: (i) the existence of quantum states, (ii) the law of conservation of energy, and (iii) the principle of equal a priori probabilities. The book aims at three main ends. Firstly, it interprets classical equilibrium thermodynamics in terms of statistical mechanics. A review of the basic principles of classical equilibrium thermodynamics is given early in the book in a form suitable for comparison later with the approach to thermodynamics based on statistical mechanics. Secondly, it develops the Boltzmann distribution law and the approach to thermodynamics based on the partition function. The properties of the Helmholtz free energy F and the Gibbs free energy G are developed and interpreted in terms of both classical equilibrium thermodynamics and statistical mechanics. Thirdly, it applies statistical mechanics to quantum phenomena. A full discussion of Planck's radiation law is given. The Einstein and Debye theories of heat capacities are discussed, leading to the concept of a phonon. The grand canonical distribution is developed and used to derive the Fermi-Dirac and Bose-Einstein distribution functions, which are then applied to a wide range of interesting phenomena.

Current topics in survey sampling. Edited by D. Krewski. Academic Press, New York, 1981. xv + 509 pp. $29.50.

This volume contains all the invited papers as well as abstracts of a symposium on survey sampling held at Carleton University in Ottawa, May 7–9, 1980. The papers are divided into six groups: 1. William G. Cochran: In Memoriam, 2. Nonsampling Errors, 3. Current Survey Research Activity, 4. Superpopulation Models, 5. Variance Estimation, 6. Imputation Techniques.

Introduction to multivariate analysis. By Christopher Chatfield and Alexander J. Collins. Chapman and Hall, New York, 1980. viii + 246 pp. $25.00.

The eleven chapters of this introduction to the analysis of multivariate data are divided into four parts: 1. Introduction; 2. Finding new underlying variables (principal component and factor analysis); 3. Procedures based on the multivariate analysis of variance and covariance; 4. Multidimensional scaling and cluster analysis. The book provides a blend of theory and practice—enough theory is given to introduce the concepts and to make the topics mathematically interesting, but the use and misuse of the techniques in practice are also discussed and appropriate real-life examples presented.

Continued on Page 263

Continued from Page 260

The logic of quantum mechanics. By Enrico G. Beltrametti and Gianni Cassinelli. With a foreward by Peter A. Carruthers. Encyclopedia of Mathematics and its Applications, edited by Gian-Carlo Rota, volume 15. Addison-Wesley Publishing Co., Reading, MA, 1981. xxvi + 305 pp. $31.50.

This is a volume in the section Mathematics of Physics (Peter A. Carruthers, section editor). It is the first in a series on quantum mechanics and its applications. It deals with the foundations as well as the fascinating logic of quantum mechanics and is more general and at places less factual than other volumes of the encyclopedia. Nevertheless, the thorough presentation will guide the reader to a unified view of a theory that, together with relativity, is regarded as the greatest achievement of physics during this century. Table of contents: Part I, Hilbert-Space Quantum Mechanics: 1. Static Description of Quantum Systems. 2. States. 3. Physical Quantities. 4. Spin and Motion. 5. Superselection Rules. 6. Dynamical Evolution. 7. Compound Systems. 8. Elementary Analysis of the Measurement Process. 9. Mathematical Structures Emerging from the Hilbert-Space Formulation of Quantum Mechanics. Part II, Basic Structures in the Description of Quantum Systems: 10. The Typical Mathematical Structure of Propositions: Orthomodular AC Lattices. 11. Probability Measures on Orthomodular Posets and Lattices. 12. Characterization of Commutativity. 13. States and Propositions of a Physical System. 14. Quantum-Mechanical Features in Terms of the Logic of the Physical System. 15. On the Hidden-Variables Issue. 16. Proposition-State Structure and Idealized Measurements. 17. Superpositions of States and Closure Spaces. 18. Transition-Probability Spaces and Quantum Systems. 19. On the Convex-Set Approach. 20. Introduction to a Quantum Logic. Part III, Reconstruction of Hilbert-Space Quantum Mechanics: 21. The Coordinatization Problem. 22. Use of Real and Quaternionic Hilbert Spaces: A Simple Example. 23. Dynamics. 24. Composition of Physical Systems. 25. Hidden-Variable Theories and Gleason's Theorem. 26. Introduction to Quantum Probability Theory.

The representation theory of the symmetric group. By Gordon James and Adalbert Kerber. With a foreward by P. M. Cohn and an introduction by G. de B. Robinson. Encyclopedia of Mathematics and its Applications, edited by Gian-Carlo Rota, volume 16. Addison-Wesley Publishing Co., Reading, MA, 1981. xiv + 510 pp. $44.50.

This is a volume in the section Algebra (P. M. Cohn and Roger Lyndon, section editors). It provides an account of both the ordinary and modular representation theory of the symmetric groups. The range of applications of this theory is vast, varying from theoretical physics through combinatorics to the study of polynomial identity algebras; and new uses are still being found. Table of contents: 1. Symmetric Groups and Their Young Subgroups. 2. Ordinary Irreducible Representations and Characters of Symmetric and Alternating Groups. 3. Ordinary Irreducible Matrix Representations of Symmetric Groups. 4. Representations of Wreath Products. 5. Applications to Combinatorics and Representation Theory. 6. Modular Representations. 7. Representation Theory of S_n over an Arbitrary Field. 8. Representations of General Linear Groups.

Boolean matrix theory and applications. By Ki Hang Kim. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, volume 70.) Marcel Dekker, Inc., New York, 1982. xiv + 287 pp. $38.50.

This study examines topics such as combinatorics, inverses, and the sequence of powers, with applications to switching circuit synthesis, medical diagnosis, symbolic logic, finite automata, and information networks. Exercises, examples, and open research problems are included. Chapter headings: 1. Fundamental Concepts. 2. Algebraic Properties. 3. Inverses. 4. Combinatorial Properties of Order Relations. 5. Asymptotic Forms.

The mathematical theory of chromatic plane ornaments. By Thomas W. Wieting. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, volume 71.) Marcel Dekker, Inc., New York, 1982. vii + 369 pp. $55.00.

In recognition of the seminal role of the coloring problem of classifying plane ornaments within the theory of tilings, this volume develops the coloring theory for periodic planar tilings—from a concise discussion of Euclidean plane geometry to stating the coloring problem, with systematic descriptions of procedures for reducing it to machine computation. There are three chapters: 1. The Euclidean Plane. 2. Plane Ornaments. 3. Chromatic Plane Ornaments, and a catalogue of tiling of the plane.

The structure of locally compact abelian groups. By David L. Armacost. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, volume 68.) Marcel Dekker, Inc., New York, 1981. vii + 154 pp. $23.75.

This text presents a unified treatment of the algebraic and topological aspects of the structure theory of locally compact abelian groups. After a summary of background concepts, the text goes on to in-depth coverage of important topics such as topological torsion groups, splitting problems, pure subgroups, connectedness properties, and homological methods. There is a section at the end of each chapter highlighting results ranging from simple exercises to summaries of research papers; open questions are presented to stimulate further exploration by the reader.

Ordinary differential equations. By Philip Hartman. Birkhauser, Boston, Inc., Cambridge, MA, 1982. xv + 612 pp. $29.95.

This is a reprint of the second, 1973 edition of a text first published in 1964 by John Wiley and Sons. The principal changes from the first edition are in the chapter on invariant manifolds and linearizations.

Padé approximation and its applications. Edited by M. G. de Bruin and H. van Rossum. Lecture Notes in Mathematics, volume 888, Springer-Verlag, Berlin, 1981. vi + 383 pp.

These are the Proceedings of a conference held in Amsterdam, October 29–31, 1980. There were four invited addresses, by C. Brezinski, P. R. Graves-Morris, H. Werner and L. Wuytack, and 26 short communications.

Measure theory and integration. By G. de Barra. John Wiley & Sons, New York, 1981. 239 pp. $64.95.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. It approaches integration via measure rather than measure via integration. Chapter headings: 1. Preliminaries. 2. Measure on the Real Line. 3. Integration of Functions of a Real Variable. 4. Differentiation. 5. Abstract Measure Spaces. 6. Inequalities and the L^p Spaces. 7. Convergence. 8. Signed Measures and their Derivatives. 9. Lebesgue-Stieltjes Integration. 10. Measure and Integration in a Product Space. There are hints and answers to the many exercises.

Advances in applied mechanics, volume 22. Edited by Chia-Shum Yih. Academic Press, London and New York, 1982. ix + 327 pp.

This volume contains the following articles: Aspects of suspension shear flows, by S. L. Lee; Nonlinear dynamics of deep-water gravity waves, by Henry C. Yuen and Bruce M. Lake; Instability and transition in buoyancy-induced flows, by B. Gebhart and R. L. Mahajan.