# **QUARTERLY**

OF

# APPLIED MATHEMATICS

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# QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R.I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$30.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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### SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE OUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition

charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between cápital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter I and the prime ('), between alpha and a, kappa

and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated. Dots, bars, and other markings to be set above letters should be strictly avoided because they require costly handcomposition; in their stead markings (such as primes or indices) which follow the letter should be used.

Square roots should be written with the exponent  $\frac{1}{2}$  rather than with the sign  $\sqrt{\ }$ . Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

exp  $[(a^2 + b^2)^{1/2}]$  is preferable to  $e^{(a^2+b^2)^{1/2}}$ 

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos\frac{\pi x}{2b}}{\cos\frac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t$$
 is preferable to  $\cos t(a + bx)$ .

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a+(b+cx)^n]\cos ky\}^2$$
 is preferable to  $((a+(b+cx)^n)\cos ky)^2$ .

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets

The following examples show the desired arrangements: (for books—S. Timoshenko, Strength of materials, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, On the flow of viscous liquids, especially in three dimensions, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, On the flow of viscous fluids is preferable to On the Flow of Viscous Fluids, but the corresponding German title would have to be rendered as Über die Strömung zäher Flüssigkeiten.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not accept-

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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## **—BOOKS RECEIVED—**

Handbook of statistics, vol. 1. Analysis of variance. Edited by P. R. Krishnaiah. North-Holland Publishing Co., Amsterdam and New York, 1980. xvii + 1002 pp. \$134.25.

Each volume in the projected series *Handbook of Statistics* is devoted to a particular topic in statistics. The material in these volumes is essentially expository in nature, and in general the proofs of the results are not included. This series is addressed to the entire community of statisticians and scientists in various disciplines who use statistical methodology in their work. At the same time, special emphasis will be given to applications-oriented techniques, with the applied statistician in mind.

This first volume in the series is devoted to the area of analysis of variance (ANOVA). An attempt is made to cover most of the useful techniques in univariate and multivariate ANOVA in this volume. The chapters are written by prominent workers in the field for persons who are not specialists on the topic. This volume is dedicated to the memory of the late Henry Scheffe. There are 25 chapters: 1. Estimation of variance components. 2. Multivariate analysis of variance of repeated measurements. 3. Growth curve analysis. 4. Bayesian inference in MANOVA. 5. Graphical methods for internal comparisons in ANOVA and MANOVA. 6. Monotonicity and unbiasedness properties of ANOVA and MANOVA tests. 7. Robustness of ANOVA and MANOVA test procedures. 8. Analysis of variance and problems under time series models. 9. Tests of univariate and multivariate normality. 10. Transformations to normality. 11. ANOVA and MANOVA: models for categorical data. 12. Inference and structural models for ANOVA and MANOVA. 13. Inference based on conditionally specified ANOVA models incorporating preliminary testing. 14. Quadratic forms in normal variables. 15. Generalized inverse of matrices and applications to linear models. 16. Likelihood ratio tests for mean vectors and covariance matrices, 17. Assessing dimensionality in multivariate regression, 18. Parameter estimation in nonlinear regression models. 19. Early history of multiple comparison tests. 20. Representations of simultaneous pairwise comparisons. 21. Simultaneous test procedures for mean vector and covariance matrices. 22. Nonparametric simultaneous inference for some MANOVA models. 23. Comparison of computer programs for univariate and multivariate analysis of variance. 24. Computations of some multivariate distributions. 25. Inference on the structure of interaction in two-way classification model.

Spatial processes: models and applications. By A. D. Cliff and J. K. Ord. Pion Ltd., London; distributed by Methuen, New York, 1981. x + 266 pp. \$30.00.

About one-third of this book is comprised of (heavily revised) material taken from the authors' 1973 book Spatial auto-correlation, in which hypothesis testing about spatial processes was stressed. Since then, the emphasis in the field has shifted to models of such processes, a shift which is reflected in the current work. Fairly substantial applications of the methods are described at intervals throughout the book to enable those whose chief concern is practical rather than theoretical to follow the main lines of argument. Chapter 2 to 6 employ more advanced statistical methods than the rest of the book. The chapter headings are: 1. Measures of autocorrelation in the plane. 2. Distribution theory for the join-count, I, and c statistics. 3. Map comparison with application to diffusion processes. 4. The analysis of spatial point patterns. 5. Spatial correlograms and related inferential statistics. 8. The analysis of regression residuals. 9. Models containing components both regressive and autoregressive.

The mathematical theory of wave motion. By G. R. Baldock and T. Bridgeman. Ellis Horwood Ltd., Chichester, England. 261 pp. \$64.95.

This is an elementary introduction to the mathematics of wave motion, suitable for an undergraduate course. The chapter headings are: 1. Occurrence and nature of waves. 2. Solutions of the wave equation by integration. 3. The Fourier method of solution. 4. The generalized Fourier method. 5. Dispersion. 6. Discrete systems. 7. The Fourier method in two dimensions. 8. Sound waves in fluids. 9. Electromagnetic waves. 10. Water waves. 11. Theory of hyperbolic equations. 12. The Cauchy problem in two and three dimensions. 13. Appendix: special functions.

Mathematical programming in statistics. By T. S. Arthanari and Yadolah Dodge. John Wiley & Sons, New York, 1981. xviii + 413 pp. \$28.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. The techniques for solving the optimization problems occuring in development of statistical theory can be classified as classical, numerical, variational methods, and mathematical programming. The emphasis of this book is on the latter. In chapter 2 the problem of regression analysis is discussed and the different optimizing criteria are first introduced, in the simple context of a single independent variable. Multiple regression is then considered. The theory of the simplex method in LP is developed and a special algorithm for the MINMAD problem is then explained. The bounded-variable method is developed for the dual formulation. Unbiased estimation of the parameters is then considered. The MINMAXAD criterion and the geometrical properties of the problem are discussed, with the use of the various LP formulations. Other new estimators that can be obtained with LP are discussed. Convex-programming theory is developed for solving restricted least-squares problems. Also, as a compromise between least squares and MINMAD, a convex combination of the two is considered. Obtaining best sets of independent variables is first approached through the branch-and-bound procedure; then the problem is posed as a cardinality-constrained linear-programming (CCLP) problem. Computations of generalized inverses and their connection with the simplex method are brought out in chapter 3. In chapter 4 the Neyman-Pearson problem in the theory of testing statistical hypotheses is posed as a bounded-variable LP problem. Some problems in the area of sampling that are mathematical-programming problems are considered in chapter 5. Optimal allocation of sample size in stratified sampling is shown as a dynamic programming problem. In chapter 6 MINMAD estimates for a two-way classification model are considered and the problem is shown as one of transportation. The problem of clustering N elements into m clusters according to a given criterion is basically a combinatorial-optimization problem. In chapter 7 different criteria are considered, and the different mathematical-programming problems arising therefrom.

Computer solution of large sparse definite systems. By Alan George and Joseph W. Lui. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1981. xii + 324 pp. \$24.95.

To limit the size of the book and yet consider the problems in detail, the authors restrict themselves to symmetric positive definite systems of equations, which are those arising in numerous fields of science and engineering, and to one specific method for each general approach. They provide listings of Fortran subroutines and discuss them in detail, since the implementation of an algorithm profoundly affects the performance. Chapter headings: 1. Introduction. 2. Fundamentals. 3. Some graph theory notation and its use in the study of sparse symmetric matrices. 4. Band and envelope methods. 5. General sparse methods. 6. Quotient tree methods for finite element and finite difference problems. 7. One-way dissection methods for finite element problems. 8. Nested dissection methods. 9. Numerical experiments.

Mathematical theory of entropy. By Nathaniel F. G. Martin and James W. England. Addison-Wesley, Reading, MA, 1981. xxii + 257 pp. \$29.50.

This is volume 12 of the Encyclopedia of Mathematics and its Applications, edited by Gian-Carlo Rota, in the section Real Variables, edited by James K. Brooks who contributes a foreword to the book. It presents a self-contained exposition of the mathematical theory of entropy, a subject which has played a central role in a number of areas such as statistical mechanics and information theory. The connection between the various applications of entropy have become clearer in recent years by the introduction of probability theory into its foundations, and those parts of this theory which are necessary for an understanding of entropy have been included in the monograph. The last four chapters may be read independently of each other. Chapter headings: 1. Topics from probability theory. 2. Entropy and information. 3. Information theory. 4. Ergodic theory. 5. Topological dynamics. 6. Statistical mechanics. As in all volumes of this excellent Encyclopedia, there is an extensive bibliography (this one of 164 items).

Robust statistics. By Peter J. Huber. John Wiley and Sons, Inc., New York, 1981. ix + 308 pp. \$28.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is the first systematic, book-length exposition of robust statistics. From the preface: "The technical term 'robust' was coined only in 1953 (by G. E. P. Box), and the subject matter acquired recognition as a legitimate topic for investigation only in the mid-sixties, but it certainly never was a revolutionary new concept. Among the leading scientists of the late nineteenth and early twentieth century, there were several practicing statisticians (to name but a few: the astronomer S. Newcomb, the astrophysicist A. Eddington, and the geophysicist H. Jeffreys), who had a perfectly clear, operational understanding of the idea; they knew the dangers of long-tailed error distributions, they proposed probability models for gross errors, and they even invented excellent robust alternatives to the standard estimates, which were rediscovered only recently. But for a long time theoretical statisticians tended to shun the subject as being inexact and 'dirty.' The author's 1964 paper may have helped to dispel such prejudices. Amusingly (and disturbingly), it seems that lately a kind of bandwagon effect has evolved, that the pendulum has swung to the other extreme, and that 'robust' has now become a magic word, which is invoked in order to add respectability." This book gives a solid foundation in robustness to both the theoretical and the applied statistician. The treatment is theoretical, but the stress is on concepts rather than on mathematical completeness. The level of presentation is deliberately uneven: in some chapters simple cases are treated with mathematical rigor; in others the results obtained in the simple cases are transferred by analogy to more complicated situations (like multiparameter regression and covariance matrix estimation) where proofs are not always available (or are available only under unrealistically severe assumptions). Also selected numerical algorithms for computing robust estimates are described and, where possible, convergence proofs are given. Chapter headings: 1. Generalities. 2. The weak topology and its metrization. 3. The basic types of estimates. 4. Asymptotic minimax theory for estimating a location parameter. 5. Scale estimates. 6. Multiparameter problems, in particular joint estimation of location and scale. 7. Regression. 8. Robust covariance and correlation matrices. 9. Robustness of design. 10. Exact finite sample results. 11. Miscellaneous topics.

Perturbation methods in applied mathematics. By J. Kevorkian and J. D. Cole. Springer-Verlag, New York, 1981. x + 558 pp. \$42.00.

This is volume 34 of Applied Mathematical Sciences. It is a revised and updated version, including a substantial portion of new material, of J. D. Cole's text *Perturbation methods in applied mathematics*, Ginn-Blaisdell, 1968. The authors present the material at a level which assumes some familiarity with the basics of ordinary and partial differential equations. Some of the more advanced ideas are reviewed as needed; therefore this book can serve as a text in either an advanced undergraduate course or a graduate-level course on the subject.

Spectral theory of ordinary differential operators. By Erich Muller-Pfeiffer. Ellis Horwood Ltd., Chichester, England, 1981. 247 pp.

In this monograph three aspects of the spectrum of ordinary self-adjoint singular differential operators of even order are investigated: the essential spectrum, the discrete spectrum, and the non-existence of eigenvalues. Chapter headings: 1. Fundamental concepts. 2. The essential spectrum. 3. Discrete spectra. 4. Continuous spectra. 5. Sturm-Liouville operators. 6. Oscillation criteria.

A first course in differential geometry. By Chauan-Chih Hsiung. John Wiley & Sons, New York, 1981. xvi + 343 pp. \$29.95.

This is a volume in the Wiley-Interscience Series Pure and Applied Mathematics. It deals with curves and surfaces in a three-deminsional Euclidean space, with particular attention devoted to the relationships between local and global properties. Geometric interpretations are given along with analytic expressions. Chapter headings: 1. Euclidean spaces. 2. Curves. 3. Local theory of surfaces. 4. Global theory of surfaces.

The inverse scattering transformation and the theory of solitons. By Wiktor Eckaus and Aart van Harten. North-Holland Publishing Co., Amsterdam and New York, 1981. xi + 224 pp. \$31.75.

The method of inverse scattering transformation and the theory of solitons are among the most recent and fascinating achievements in the domain of applied mathematics. The development of the theory, which took place mainly in the last decade, has been explosive and far-reaching. This book provides an entirely self-contained exposition on the subject. Its primary aim is to provide the non-initiated student and practitioner of mathematical analysis with an introduction to a new field. The volume should, however, also be of great interest to the experts in soliton theory. The theory is presented in a mathematically satisfactory way and the treatment is fully rigorous, filling in many gaps in the existing literature and presenting new insights into various aspects of the theory.

Foundations of decision support systems. By Robert H. Bronczek, Clyde W. Holsapple and Andrew B. Whinston. Academic Press, New York, 1981. xvii + 393 pp. \$29.50.

The theme of this book is the assertion that the computer has a potential for playing a different kind of role in organizations from the routine one it has in the past been called upon to play, namely the support of decision-making activities. A basic objective of the book is to provide the foundation necessary for understanding and developing decision support systems. To this end, specific frameworks, strategies and techniques of data handling, model handling, language handling and problem handling are presented, and developed primarily at conceptual and operational levels. Implementation issues are briefly described. The integration of concepts and tools from various fields, including data base management, linguistics and artificial intelligence, is considered by the authors' approach. The emphasis is on an interdisciplinary and generalized approach to decision support systems. The fifteen chapters are divided into five parts: 1. Information processing, decision making, and decision support—some perspectives. 2. Representative systems for decision support. 3. Decision support systems from the data base angle. 4. Formal logic approach to decision support. 5. Integrating the data base and formal logic approaches to decision support.

Introduction to linear operator theory. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Volume 65.) By Vasile I. Istratescu. Marcel Dekker, Inc., New York, 1981. xi + 579 pp. \$32.50.

This book is an introduction to linear operator theory on Hilbert and Banach spaces aiming for a clear understanding of linear operator theory and its applications to a wide variety of mathematical problems. It treats such topics as analytic, quasi-analytic, and related classes of vectors associated with an operator, as well as Weyl's spectrum.

Mathematics in archaeology. By Clive Orton. Humanities Press, Inc., Atlantic Highlands, N.J., 1981. 248 pp. \$33.75.

The author approaches the subject via the questions archaeologists ask about their finds. Chapter 2 looks at the question "what is it?" and how artifacts can be sorted and classified; chapter 3 turns to the question "how old is it?"—ways of looking at stratigraphy and the interpretation of dating evidence. The geographical aspects are dealt with in chapters 4 and 7, and the question of function in chapter 5. Chapter 6 deals with the special difficulties of working with broken objects like pots and bones. More general questions of interpretation—what one can or cannot say about a body of data—are met in chapter 8. The final chapter is partly speculative, and tries to pick out some trends for the future. The aim throughout is to demonstrate the mathematical ideas that sit at the heart of many archaeological ideas and concepts, and to work them out through practical examples. The book is addressed to the general reader and presupposes no knowledge of mathematics at all—mathematical ideas are expressed in ordinary language.

Case studies in mathematical modeling. Edited by W. E. Boyce. Pitman Advanced Publishing Program, Boston, London, Melbourne, 1981. xiii + 386 pp. \$22.00.

This volume contains the following seven papers, which were given at R.P.I. as part of a new course on mathematical modeling: 1. A mathematical modeling relating to herbicide resistance, by Lee A. Segel. 2. Mathematical modeling of elevator systems, by Bruce A. Powell. 3. Models of traffic flow, by Donald A. Drew. 4. Semiconductor crystal growth, by Lynn O. Wilson. 5. Shortest paths in networks, by Christoph Witzgall, Judith F. Gilsinn and Douglas R. Shier. 6. Mathematical models for computer data communication, by Alan G. Konheim. 7. Operating system security verification, by Jonathan K. Millen.

The history of the calculus of variations from the 17th through the 19th century. By Herman H. Goldstine. Springer-Verlag, New York, 1980. xviii + 410 pp.

This is volume 5 of Studies in the History of Mathematics and Physical Sciences. The author limits the scope of this work to problems solved analytically and thus takes Fermat as his starting point. Chapter headings: 1. Fermat, Newton, Leibniz, and the Bernoullis. 2. Euler. 3. Lagrange and Legendre. 4. Jacobi and his school. 5. Weierstrass. 6. Clebsch, Mayer, and others. 7. Hilbert, Kneser, and others. There is also an extensive bibliography.

Approximation theory and methods. By M. J. D. Powell. Cambridge University Press, New York, 1981. x + 339 pp. \$57.50 (hardcover) \$19.95 (paperback).

This book is based on lectures to third-year undergraduates at Cambridge University. The prerequisites are an introduction to linear spaces and operators and an intermediate course on analysis. Seven of the 24 chapters are devoted to spline approximations, because they are convenient and suitable for computer calculations as well as providing optimal theoretical solutions to the estimation of functions from limited data. The classical theory of best approximations from linear spaces with respect to the minimax, least squares and L<sub>1</sub>-norms is also studied, and algorithms are described and analyzed for the calculation of these approximations. Interpolation is considered also, and the accuracy of interpolation and other linear operators is related to the accuracy of optimal algorithms. Special attention is given to polynomial functions, and there is one chapter on rational functions. Exercises are included with each chapter which support and extend the text. All references to related work are given in an appendix.

Functions of a complex variable: theory and technique. By George F. Carrier, Max Krook, and Carl E. Pearson. Hod Books, 113 Cobb Street, Ithaca, New York, 1983, ix + 438 pp. \$34.95.

Several annoying misprints have been removed from this applications-oriented complex variable text which will be printed, henceforth, by the new publisher listed above.

A second course in stochastic processes. By Samuel Karlin and Howard M. Taylor. Academic Press, Inc., New York, 1981. xviii + 542 pp. \$35.00.

This book continues the development of the theory and applications of stochastic processes from where the authors' book A first course in stochastic processes ended, and its chapters are, indeed, numbered 10 to 18, following the nine chapters of the First Course. It presents an extensive introductory account of the fundamental concepts and methodology of diffusion processes and the closely allied theory of stochastic differential equations and stochastic integrals. Chapter headings: 10. Algebraic methods in Markov chains. 11. Ratio theorems of transition probabilities and applications. 12. Sums of independent random variables as a Markov chain. 13. Order statistics, Poisson processes, and applications. 14. Continuous time Markov chains. 15. Diffusion processes. 16. Compounding stochastic processes. 17. Fluctuation theory of partial sums of independent identically distributed random variables. 18. Queueing processes.

Statistics for biologists. By D. J. Finney. Methuen, Inc., New York, 1980. vii + 162 pp. \$9.95 paperback.

This is an elementary introduction, addressed to first-year students of biology. The emphasis is on the conveying of understanding rather than on the presentation of methods and rules. Chapter headings: 1. Problems, data, questions. 2, Probability and other definitions. 3. Combining probabilities. 4. Significance, binomials, and  $X^2$ . 5. Continuous variates. 6. Inference on means; the normal distribution. 7. Unknown variance; the t-distribution. 8. Design of experiments. 9. Comparisions between means. 10. Additional topics.

Search theory and applications. Edited by K. Brian Haley and Lawrence D. Stone. Plenum Press, New York, 1980. ix + 277 pp.

These are the proceedings of the NATO Advanced Research Institute on Search Theory and Applications, held in Portugal in March 1979. There are three introductory papers, seventeen on applications (to search and rescue, surveillance, exploration, medicine, industry and clearance) and seven on theory (four on search for moving and three for stationary targets).

Techniques and applications of path integration. By Lawrence S. Schulman. John Wiley & Sons, Inc., New York, 1981. xv + 359 pp. \$31.95.

Path integrals provide a way of looking at quantum mechanics. They also serve as a tool for calculations in many areas, including statistical physics, field theory and optics. This book emphasizes many of the useful, elegant applications that have been found for path integrals. There are 32 chapters: 1. Introducing and defining the path integral, 2. Probabilities and probability amplitudes for paths, 3. Correspondence limit for the path integral (heuristic). 4. Vector potentials and another proof of the path integral formula. 5. The Ito integral and gauge transformations. 6. Doing the integral: free particle and quadratic Lagrangians. 7. Properties of Green's functions; the Feynman-Kac formula. 8. Functional derivatives and commutation relations. 9. Brownian motion and the Wiener integral; Kac's proof. 10. Perturbation theory and Feynman diagrams. 11. Asymptotic analysis. 12. The calculus of variations. 13. The WKB approximation and its application to the anharmonic oscillator. 14. Detailed presentation of the WKB approximations. 15. WKB near caustics. 16. Caustics and uniform asymptotic approximations, 17. The phase of the semiclassical amplitude, 18. The semiclassical propagator as a function of energy. 19. Scattering theory. 20. Geometrical optics. 21. The polaron. 22. Spin and related matters. 23. Path integrals for multiply connected spaces. 24. Quantum mechanics on curved spaces. 25. Relativistic propagators and black holes. 26. Applications to statistical mechanics. 27. Coherent state representation. 28. Systems with random impurities. 29. Critical droplets, alias instantons, and metastability. 30. Renormalization and scaling for critical phenomena. 31. Phase space path integral. 32. Omissions, miscellany, and prejudices.

In the labyrinths of language: a mathematician's journey. By V. V. Nalimov. Ed. by R. G. Colodny. ISI Press, Philadelphia, 1981. xix + 246 pp. \$22.50.

"The mysteries and paradoxes of human language have been explored by various scholars since the time of the Old Testament scribes and the beginning of systematic philosophy with the ancient Greeks. Now the noted Soviet mathematician V. V. Nalimov brings the new techniques of probability theory to this ancient quest. In a wide-ranging exploration of the many previous attempts to solve these riddles, he opens new vistas by creating a semantic scale of languages in which the qualities of ordinary speech find their place between common gesture at one end and mathematics and logic at the other. The probabilistic model is shown to reflect the irreducible random element in the world of both human experience and the laws of the physical cosmos. Creative thinking is shown to arise precisely from the ambiguity of the world and our human effort to think, write, and speak about it." Chapter headings: 1. What language is. 2. Probabilistic semantics. 3. The language of science. 4. Mathematics as a language. 5. Soft languages. 6. A hard language of biological codes. 7. The theory of names. 8. Language and thinking: continuity vs. discontinuity. 9. Epilogue.