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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime ($'$), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x/2b)}{\cos (\pi a/2b)} \text{ is preferable to } \frac{\cos \dfrac{\pi x}{2b}}{\cos \dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEWS—

Mathematicians and their times: history of mathematics and mathematics of history. By Laurence Young. North-Holland, Amsterdam, New York, 1981. x + 343 pp. $36.50.

This is a lively and informal account of important events and personalities in the development of mathematics. It is not intended as a systematic and complete history of the subject. Instead, it is a collection of many things: the author's recollections of some leading figures of our times, snapshots from earlier mathematical eras, and general, philosophical statements.

The book, which seems to be based on lecture notes, is organized into a very long introduction followed by three chapters. The introduction describes in about 120 pages the early periods of mathematics up to Newton, Euler, Lagrange, and Laplace. The three chapters following the introduction describe in a somewhat rhapsodic style some of the main personalities in "modern" mathematics starting with Gauss and Riemann and continuing up to our days.

What makes this unusually enjoyable is the personal style of the author, who does not hesitate to express his opinions forcefully, if not always convincingly. He seems to have known, or at least met, most of today's leading figures in mathematics and tells some wonderful ancedotes about them. Highly recommended for leisure reading.

U. Grenander (*Providence*)

Elementary stability and bifurcation theory. By Gérard Iooss and Daniel D. Joseph. Springer-Verlag, New York, Heidelberg, Berlin, 1980. xv + 286 pp.

Bifurcation theory is an important area in evolutionary equations and applications. An elementary text which presents the basic ideas in a clear and concise way does not exist. The present authors have made a first attempt to fill this gap taking as their goals (the first two are quoted from the Introduction):

1) "To make the analysis simple enough so that it can be understood by persons whose mathematical training does not extend beyond the classical methods of analysis which were popular in the 19th century."

2) "To give a complete theory for all problems which in a sense, through projections, could be said to be set in two dimensions."

3) Assuming the projection methods (center manifold theory, etc.) are known, to give indications of how to apply the results to higher-dimensional problems, including some partial differential equations.

The two-dimensional problems referred to consist only of equilibrium solutions, time-periodic solutions of autonomous and nonautonomous equations and some special classes of quasiperiodic solutions obtained through periodically forced two-dimensional systems.

The authors assume almost no knowledge of differential equations or linear algebra, give very detailed proofs of the elementary results and give many illustrations with extensive calculations. In this sense, the goals are satisfactorily achieved. To the reviewer, it seemed that the presentation assumed too little from the reader and may have violated their quote from Poincaré, "Everything should be made as simple as possible, but not simpler." Also, the reviewer feels that there is an excessive number of definitions and the references are not selected with care.

In spite of these minor criticisms, the book will be useful as an introduction—especially for persons with little mathematical background.

Jack K. Hale (*Providence*)

—BOOKS RECEIVED—

Handbook of applicable mathematics. Volume II: probability. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1980. xix + 450 pp. $85.00.

The chief editor of this Handbook is Walter Ledermann and the editor of this volume Emlyn Lloyd. The 'Handbook' consists of two sets of books. On the one hand, there are (or will be) a number of guidebooks written by experts in various fields in which mathematics is used (e.g., medicine, sociology, management, economics). These guidebooks are by no means comprehensive treatises; each is intended to treat a small number of particular topics within the field, employing, where appropriate, mathematical formulations and mathematical reasoning. A typical guidebook will consist of a discussion of a particular problem, or related set of problems, and will show how the use of mathematical models serves to solve the problem. Wherever any mathematics is used in a guidebook, it is cross-referenced to an article (or articles) in the core volume. There are 6 core volumes devoted respectively to Algebra, Probability, Numerical Methods, Analysis, Geometry and Combinatorics and Statistics. These volumes are texts of mathematics—but they are no ordinary mathematical texts. They have been designed specifically for the needs of the professional adult and they stand or fall by their success in explaining the nature and importance of key mathematical ideas to those who need to grasp and to use those ideas. Either through their reading of a guidebook or through their own work or outside reading, professional adults will find themselves needing to understand a particular mathematical idea (e.g., linear programming, statistical robustness, vector product, probability density, round-off error); and they will then be able to turn to the appropriate article in the core volume in question and find out just what they want to know. The chapter headings in this volume are: 1. Introduction: Probability and statistics. 2. Basic concepts of probability. 3. The calculus of probability. 4. Random variables and their realizations; probability distributions. 5. A first catalogue of discrete probability distributions (integer-valued variables). 6. Independent and non-independent random variables and their joint distributions, bivariate and multivariate; marginal and conditional distributions. 7. The addition of random variables: convolution. 8. Expectation (expected value). 9. Variance, covariance, and higher moments. 10. Continuous distributions. 11. A catalogue of continuous probability distributions. 12. Generating functions. 13. Multivariate continuous distributions. 14. Miscellaneous topics on distributions. 15. Order statistics. 16. Miscellaneous topics involving conditional probability. 17. Laws of large numbers. Stable distributions. 18. Stochastic processes: introduction and examples. 19. Processes in discrete time: Markov chains. 20. Continuous-time processes. 21. Renewals. 22. Complements on stochastic processes: Second order processes.

Essays in general relativity. Edited by Frank J. Tipler. Academic Press, Inc., New York, 1980. xvii + 236 pp. $30.00.

This is a Festschrift for Abraham Taub. The contents are: 1. On Schwarzschild causality—A problem for "Lorentz covariant" general relativity, by Roger Penrose. 2. Comments on the topology of nonsingular stellar models, by Lee Lindblom and Dieter R. Brill. 3. General relativity and the eternal return, by Frank J. Tipler. 4. Energy and momentum of the gravitational field, by James W. York, Jr.. 5. The beam and stay of the Taub universe, by John Archibald Wheeler. 6. Tidal forces in a highly asymmetric Taub universe, by L. C. Shepley. 7. Symmetry breaking in general relativity, by Arthur E. Fischer, Jerrold E. Marsden, and Vincent Moncrief. 8. Gauge-invariant perturbation theory in spatially homogeneous cosmology, by Robert T. Jantzen. 9. Locally isotropic space-time nonnull homogeneous hypersurfaces, by M. A. H. MacCallum. 10. The gravitational waves that bathe the earth: upper limits based on theorists' cherished beliefs, by Mark Zimmermann and Kip S. Thorne. 11. General relativistic hydrodynamics: the comoving, Eulerian, and velocity potential formalisms, by Larry Smarr, Clifford Taubes, and James R. Wilson. 12. Lagrangian relativistic hydrodynamics with Eulerian coordinates, by Tsvi Piran. 13. Some thoughts on the origin of cosmic inhomogeneities, by E. P. T. Liang. 14. Automorphisms of formal algebras associated by deformations with a symplectic manifold, by Andre Lichnerowicz. 15. A remark on time-independent asixymmetric fields, by A. Papapetrou.

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Lecture Notes in Mathematics. Springer-Verlag, Berlin, Heidelberg, New York. 1980.

Ordinary and partial differential equations. Edited by W. N. Everitt. xvi + 271 pp. $16.80.

This is volume 827 of this series and the proceedings of a conference on ordinary and partial differential equations, held at the University of Dundee, Scotland, in March 1978, and dedicated to the memory of Arthur Erdelyi (1908–1977).

Semi-martingales et grossissement d'une filtration. By Thierry Jeulin. ix + 142 pp. $11.80.

This is volume 833 of the series.

Model theory of algebra and arithmetic. Edited by L. Pacholski, J. Wierzejewski and A. J. Wilkie. vi + 410 pp. $24.50.

This is volume 834 of this series, and the proceedings of the conference on applications of logic to algebra and arithmetic held at Karpacz, Poland, in September 1979. It was the fourth in the series Set Theory and Hierarchy Theory organized by the Institute of Mathematics of the Technical University of Wrocaw. There are 21 papers.

Introduction to data analysis and statistical inference. By Carl N. Morris and John E. Rolph. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1981. xx + 389 pp.

This text grew out of a two-quarter service course given jointly by the authors in the RAND Graduate Institute of Policy Studies since 1971. It is recommended that the text be used in conjunction with the statistical package STATLIB (by W. A. Brelsford and D. A. Relles, Prentice-Hall, 1981). The fourteen chapters are divided into three parts: 1. Data analysis, and 2. Statistical inference for linear models, 3. Robustness.

Approximation theory and applications. Edited by Zvi Ziegler. Academic Press, Inc., New York, 1981. xi + 358 pp. $26.00.

These are the proceedings of a workshop held at the Technion, Haifa, Israel, in May 1980. There are 24 papers on the subject of the workshop.

Theoretical and applied mechanics. Proceedings of the XVth International Congress of Theoretical and Applied Mechanics, University of Toronto, Canada, August 17-23, 1980. Edited by F. P. J. Rimrott and B. Tabarrok, North-Holland Publishing Co., Amsterdam and New York, 1981. xxxii + 458 pp. $78.00.

This volume contains four general lectures (by H. H. E. Leipholz on the analysis of nonconservative, nonholomic systems, J. C. R. Hunt on environmental fluid mechanics, E. Sternberg on singular problems in linearized and finite elastostatics, and E. Fichera on analytic problems of materials with memory) and 33 sectional lectures presented at the Congress. They cover the latest developments of all aspect of mechanics. The other 320 contributed papers are listed by author and title.

The origins of Cauchy's rigorous calculus. By Judith V. Grabiner. MIT Press, Cambridge, MA, 1981. xi + 252 pp. $25.00.

This book explores the background of a major intellectual revolution, the rigorous reinterpretation of the calculus undertaken by Augustin-Louis Cauchy and his contemporaries in the first part of the nineteenth century. Their generation changed the calculus from a method of solving problems to a collection of theorems, based on precise definitions, about limits, continuity, series, derivatives, and integrals. Chapter headings: 1. Cauchy and the nineteenth-century revolution in calculus. 2. The status of foundations in eighteenth-century calculus. 3. The algebraic background of Cauchy's new analysis. 4. The origins of the basic concepts of Cauchy's analysis: limit, continuity, convergence. 5. The origins of Cauchy's theory of the derivative. 6. The origins of Cauchy's theory of the definite integral.

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Quadratic form theory and differential equations. By John Gregory. Academic Press, New York, 1980. xii + 237 pp.

This is volume 152 in the series Mathematics in Science and Engineering. Historically, quadratic form theory has been treated as a rich but misunderstood uncle. It appears briefly, almost as an afterthought, when needed to solve a variety of problems. A partial list of such problems includes the Hessian matrix in n-dimensional calculus; the second variational (Jacobi or accessory) problem in the calculus of variations and optimal control theory; Rayleigh-Ritz methods for finding eigenvalues of real symmetric matrices; the Aronszajn-Weinstein methods for solving problems of vibrating rods, membranes, and plates; oscillation, conjugate point, and Sturm comparison criteria in differential equations; Sturm-Liouville boundary-value problems; spline approximation ideas for numerical approximations; Gershgorin-type ideas (and the Euler-Lagrange equations) for banded symmetric matrices; Schrödinger equations; and limit-point-limit-cycle ideas of singular differential equations in mathematical physics. A major purpose of this book is to develop a unified theory of quadratic forms to enable mathematicians to handle the mathematical and applied problems described above in a more meaningful way. The author's development is on four levels and should appeal to a variety of users of mathematics. For the theoretically inclined, he presents a new formal theory of approximations of quadratic forms/linear operators on Hilbert spaces. These ideas allow him to handle a wide range of problems. They also allow him to solve these problems in a qualitative and quantitative manner more easily than with more conventional methods. His second level of development is qualitative in nature. Using this theory, he can derive very general qualitative comparison results such as generalized Sturm separation theorems of differential equations and generalized Rayleigh-Ritz methods of eigenvalues. This theory is also quantitative in nature. He derives in level three an approximation theory that can be applied in level four to give numerical algorithms that are easy to implement and give good numerical results.

Advances in control and dynamic systems, vol. 17. Edited by C. T. Leondes. Academic Press, New York, 1981. xvii + 424 pp. $32.50.

This volume contains the following review articles: An overview of differential games (N. K. Gupta); Use of parameter optimization methods to determine the existence of game-theoretic saddle points (J. E. Rader); Solution techniques for realistic pursuit-evasion games (J. Shinar); Differential dynamic programming techniques in differential games (B. A. S. Järmark); Stochastic differential game techniques (B. Mons); Algorithms for differential games with bounded control and state (A. Chompaisal); Air combat systems analysis techniques (U. H. D. Lynch); Reachable set methods (N. K. Gupta); Zero-sum dynamic games (P. R. Kumar and T. H. Shiau); Singular differential game techniques and closed-loop strategies (K. Forouhar).

A brief course of higher mathematics. By V. A. Kudryavtsev and B. P. Demidovich. Mir Publishers, Moscow, 1981. Distributed by Imported Publications, Inc., Chicago. 693 pp. $16.00.

This book outlines those aspects of mathematics which are essential for students of the natural sciences. The twenty-six chapters range from geometry via calculus of one and several variables to probability theory and linear programming. There are many examples and exercises, with their solutions.

A book of problems in ordinary differential equations. By M. L. Krasnov, A. I. Kiselyou, and G. I. Makarenko. Translated by Vladimir Shokurov. Mir Publishers, Moscow, 1981. Distributed by Imported Publications, Inc., Chicago. 332 pp. $6.80.

The topics in this book include the method of isoclines for equations of the first and second order, problems in finding orthogonal trajectories, the use of the method of superposition in solving linear differential equations of rth order, linear dependence and linear independence of a system of functions, problems in solving linear equations with constant and variable coefficients, boundary-value problems for differential equations, integrating equations in power series, asymptotic integration, integrating systems of differential equations, Lyapunov stability, and the operator method.

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Advanced calculus: an introduction to modern analysis. Pure and Applied Mathematics: A Series of Monographs and Textbooks, Volume 63. By William L. Voxman and Roy H. Goetschel, Jr. Marcel Dekker, New York, 1981. 608 pp. $55.00.

This text provides an introduction to the theory and applications of elementary analysis and aims to maintain a careful balance between abstract concepts and applied results of significance.

Towards a formal description of Ada. Edited by D. Bjorner and O. N. Oest. Springer-Verlag, Berlin, Heidelberg, New York, 1980. xiv + 630 pp. $27.20.

This is volume 98 of Lecture Notes in Computer Science, edited by G. Goos and J. Hartmanis. Ada is a programming language developed on behalf of the U. S. Department of Defense by CII-Honeywell-Bull in Paris. It was named in honor of Ada Augusta, Lady Lovelace, a colleague and inspirer of Charles Babbage and daughter of Lord Byron. There are four chapters: 1. What is Ada? 2. The DDC Ada compiler and run-time system development method; 3. Quality assurance; 4. Software engineering support.

Graphs, networks, and algorithms. By M. N. S. Swamy and K. Thulasiraman. John Wiley & Sons, New York, 1981. xviii + 592 pp. $37.50.

This book is addressed to students in mathematics, electrical engineering, and computer science. It is organized into three parts, dealing with the theory of graphs, electrical networks, and graph algorithms. In part I (Chapters 1 to 10) the authors discuss the theory of graphs, trees, Hamilton and Euler graphs, directed graphs, matrices of a graph, planarity, connectivity, matching, coloring, and an introduction to matroid theory. Among the matroid topics presented here are Minty's self-dual axiom system which makes obvious the duality between circuits and cutsets of a graph, the arc-coloring lemma, and the greedy algorithm and its initimate relationship with matroids. Part II (chapters 11 to 13) is concerned with a discussion of those aspects of electrical network theory whose development is essentially graph-theoretic. In chapter 11 the authors discuss, among other things, the principal partition of a graph and its application in the mixed-variable method of networks. In chapter 12 they discuss several results in the theory of resistance networks and a method for realizing circuit and cutset matrices. In chapter 13 they develop topological formulas for network functions. Part III, which deals with graph algorithms, is organized into two chapters: chapter 14 on algorithms for the analysis of graphs and chapter 15 on algorithms that concern optimization problems on graphs. The authors' main concern here is the theory underlying the design, proof of correctness, and analysis of several graph algorithms. Among other things they include algorithms for flow graph reducibility, dominators, shortest paths, matchings, optimum binary search trees, network flows, and optimum branchings, as well as Hopcroft and Karp's analysis of a bipartite matching algorithm and Edmonds and Karp's analysis of Ford-Fulkerson's labeling algorithm. A major omission from this book is a discussion of NP-complete problems, which is beyond its scope.

Applied functional analysis. 2nd edition. By A. V. Balakrishnan. Springer-Verlag, New York, 1981. xiii + 373 pp. $34.00.

This is the second edition of a work first published in 1976. Errors have been corrected and new material has been added reflecting relevant recent work.

Higher geometry. By Nikolai V. Efimov. Translated by P. C. Sinha. Mir Publishers, Moscow, 1980. Imported Publications, Inc., Chicago. 560 pp. $11.00.

This is a translation of the sixth Russian edition published in 1977. It gives a systematic presentation of the fundamentals of Euclidean geometry, non-Euclidean geometry of Lobachevsky and Riemann, projective geometry and the geometrical aspects of special relativity theory. It also gives a general introduction to constant curvature geometries. The author is a professor at the Moscow State University.

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Applied probability. By Frank A. Haight. Plenum Press, New York, 1981. xi + 290 pp. \$35.00.

This book is designed for students with some experience in mathematics but no prior knowledge of probability. The author's intuitive approach emphasizes random variables and their distributions. Difficult combinatorics are omitted, allowing the student to move quickly to modern practical applications—especially queueing theory. The mathematical levels are gradually increased throughout the text to include techniques such as Laplace transforms and Riemann-Stieltjes intergration. Stochastic processes in queueing theory are covered in detail without elaborate or cumbersome notation. Chapter headings: 1. Discrete probability; 2. Conditional probability; 3. Markov chains; 4. Continuous probability distributions; 5. Continuous time processes; 6. The theory of queues.

The photic field. By Parry Moon and Domina E. Spencer. The MIT Press, Cambridge, MA, 1981. xi + 257 pp. \$25.00.

This book is devoted to that part of geometrical optics which deals with amounts of radiant energy. By utilizing the potentialities of field theory, it unifies the subject and creates a useful tool for handling practical applications such as solar heating, lighting design, photographic exposure, and color specification. Chapter headings: 1. Physical and geometrical optics; 2. Basic concepts; 3. The reflectance holor; 4. The pharosage vector; 5. Potentials and quasipotentials; 6. Contour integration; 7. The vector potential; 8. Total pharos; 9. Two-dimensional fields; 10. Weighting functions; 11. Polarization.

Mathematics tomorrow. Edited by Lynn Arthur Steen. Springer-Verlag, New York, 1981. xi + 250 pp. \$18.00.

This collection of essays by noted mathematicians is divided into four parts: What is mathematics? Teaching and learning mathematics. Issues of equality. Mathematics for tomorrow. The essays represent their author's opinions and predictions about the direction mathematics research and mathematics education should take in the immediate future.

Matrix-geometric solutions in stochastic models. By Marcel F. Neuts. Johns Hopkins University Press, Baltimore, 1981. xiii + 332 pp. \$32.50.

This book is an outgrowth of a series of lectures the author gave at The Johns Hopkins University, Baltimore, Maryland, from July 20 to July 24, 1979. It contains a variety of results on queues and other stochastic models with the unifying feature of ready algorithmic implementation. The material discussed in it comes under the broader heading of computational probability, the study of stochastic models with a genuine added concern for algorithmic feasibility over a wide, realistic range of parameter values. Careful and exhaustive computer studies are stressed. Chapter headings: 1. Matrix-geometric invariant vectors; 2. Probability distributions of phase type; 3. Quasi-birth-and-death processes; 4. The GI/PH/1 queue and related models; 5. Buffer models; 6. Queues in a random environment.

A course in descriptive geometry. By V. O. Gordon and M. A. Sementsov-Ogievskii. Edited by V. O. Gordon. Translated from the Russian by Leonid Levant. MIR Publishers, Moscow, 1980. 376 pp. \$10.00.

This is a translation of a revised 1977 Russian edition. It was written as a textbook for students of mechanical engineering. Chapter headings: 1. The method of projecting. 2. The point and the straight line. 3. The plane. 4. The relative positions of two planes; the relative positions of a straight line and a plane. 5. The method of replacing projection planes and the method of revolution. 6. Polyhedrons. 7. Curved lines. 8. Curved surfaces. 9. Intersection of curved surfaces with a plane and a straight line. 10. The intersection of curved surfaces. 11. The development of curved surfaces. 12. Axonometric projections.

Lecture Notes in Pure and Applied Mathematics. Marcel Dekker, Inc., New York and Basel.

Volume 64. *Power series over commutative rings.* By James W. Brewer. 1981. 112 pp. $17.50.

This monograph presents a unified treatment of power series over commutative rings on an introductory level. Special attention is given to important topics of ongoing research activity and particularly to power series in a single variable. The book is divided into three chapters which deal with positive results, negative results, and the study of R-homomorphisms of power series rings.

Volume 65. *Sequence spaces and series.* By P. K. Kamthan and Manjul Gupta, 1981. 384 pp. $45.00.

This monograph begins with an introduction that presents major results from the theory of topological vector spaces. The discussion then proceeds to the basic material in the theory of sequence spaces and lays the foundation for the natural structures and topologies as well as the duality theory of sequence spaces. The final chapter deals with advanced topics in sequence spaces, reflecting a wide spectrum of recent investigations.

Volume 67. *Integral and functional differential equations.* Edited by Terry L. Herdman, Samuel M. Rankin, III, and Harlan W. Stech, 1981. 296 pp. §32.50.

This volume presents nineteen original papers in the areas of ordinary, partial, and functional differential equations, and integral equations. The book contains a survey of recent developments in the theory of abstract Volterra equations which arise, for example, as models of heat flow in homogeneous materials with memory. The book also contains papers on topics ranging from questions involving the fundamental theory of differential equations, to approximation techniques, to the analysis of particular equations that arise from industrial applications, mathematical epidemiology, and theoretical population biology.

The handbook of artificial intelligence. Edited by A. Barr and E. A. Feigenbaum. William Kaufmann, Inc., Los Altos, 1981. xiv + 409 pp. $30.00.

This is the first of three projected volumes. It consists of five parts: introduction, search, knowledge representation, understanding natural language, and understanding spoken language. The editors present a survey of AI research that is motivated historically and scientifically. One can view this volume as an encyclopedia of AI programming techniques, their successful applications, some of their limitations, and the computational concepts that have been used to describe them.

Lectures on geometric methods in mathematical physics. By Jerrold E. Marsden. SIAM Publications, Philadelphia, 1981, iii + 97 pp. $12.50.

The topics selected for these lectures, given at a CBMS-NSF Regional Conference in 1979, aim to illustrate some of the ways geometry and analysis can be used in mathematical problems of physical interest. A recurring theme is the role of symmetry, bifurcation and Hamiltonian systems in diverse applications. Very little is known about dynamical systems that are close to a completely integrable Hamiltonian system. The simplest classical examples, such as the harmonic oscillator, Duffing's equation, the spherical pendulum, the rigid body and the equations of a perfect fluid, show that this program is both interesting and complex. Symmetry is relevant because many examples are, or are close to, a Hamiltonian system which is invariant under some Lie group. Such dynamical systems are very sensitive to perturbations in their equations. This means that they are bifurcation points in the set of all dynamical systems. This theme predominates in the lectures, whose titles are: 1. Infinite-dimensional Hamiltonian systems. 2. Elasticity as a Hamiltonian system. 3. Symmetry and reduction. 4. Applications of reduction. 5. Two completely integrable systems. 6. Bifurcations of a forced beam. 7. The traction problem in elastostatics. 8. Bifurcations of momentum mappings. 9. The space of solutions of Einstein's equations: regular points. 10. The space of solutions of Einstein's equations: singular points.

Optimization. By D. M. Greig. Longman Inc., New York, 1980. xi + 179 pp. \$14.50.

This is a volume in Longman Mathematical Texts. The problem considered in it is the optimization of the numerical value of a function of one or more variables which may be subject to constraints. In Chapter 1 the classical theory of optimization with equality constraints is presented using Lagrange multipliers. The corresponding results in the inequality-constrained case, the Kuhn-Tucker conditions, are then described with a discussion of convexity and of the theory of duality. Chapter 2 describes methods for finding a minimum in the unconstrained case. These are all iterative, proceeding from an initial point x by successive improvements, but differ in the way these improvements are made. Chapter 3 deals with the special case of linear programming, the simplex method, and the revised simplex method. Chapter 4 deals with applications and special cases of linear programming. Game theory and sensitivity are shown to be closely linked with the dual problem, and special algorithms are described for transportation, trans-shipment and assignment problems. Chapter 5 describes methods for nonlinear programming. Particular cases are quadratic programming and convex programming. Methods used to solve constrained problems are of two general types, depending on their treatment of the constraints. In penalty-function methods the constraints are incorporated into a modified objective function which is then treated as unconstrained. Augmented Lagrangian methods are of this type. The other approach is to use the constraints to guide the sequence of values of x, and this gives rise to projective and Newton-type methods.

Boundary-value problems governed by second-order elliptic systems. By D. L. Clements. Pitman Publishing Co., London, 1981. viii + 162 pp. £21.00.

This book presents the derivation of the solutions to various boundary-value problems governed by second-order elliptic partial differential equations that have been found useful in applications, particularly to problems in elasticity and heat transfer. It provides: various boundary integral equations for the numerical solution of boundary-value problems governed by the elliptic system; the analytical solution to various important boundary-value problems in anisotropic thermostatics and elastostatics; the derivation of the analytical solution of a number of boundary-value problems for this elliptic system; and an examination of the constants occurring in the general solution of the system in terms of arbitrary analytic functions. Chapter headings: 1. The second-order system of equations. 2. Physical problems governed by the system. 3. Boundary-value problems. 4. Some problems in elasticity and thermostatics. 5. Singular solutions. 6. Boundary integral equations. 7. Applications of the boundary integral equations.

Notes on Banach spaces. Edited by H. Elton Lacey. The University of Texas Press, Austin, Texas, 1980. viii + 439 pp. \$30.00

The nine articles contained in this book are derived from various seminars and couses given in the Mathematics Department of the University of Texas at Austin from 1975 to 1979. For example, there is one on integration in Banach spaces by H. -H. Kuo (39 pages) and one on the geometry of finite-dimensional Banach spaces and operator ideals by A. Pelczynski (101 pages).

Queueing tables and graphs. By Frederick S. Hillier and Oliver S. Yu with David M. Avis, Lawrence D. Fossett, Frederick D. Lo and Martin I. Reiman. Elsevier North-Holland, New York, 1981. 231 pp. \$32.95.

This book is the product of a ten-year study at Stanford University. It focused on integrating recent theory with algorithmic development and computer programming; queueing systems involving new combinations of exponential, Erlang, and degenerate distributions for interarrival times and service times were successfully solved. The Stanford team generated comprehensive numerical results for six major types of queueing systems. These results are presented in tabular form for precise analysis and in graphical form for quick analysis. For each type of queuing system considered, essentially every tractable and computationally feasible case was exhaustively investigated. Chapter headings: 1. Introduction, 2. Random arrivals and exponential service times ($M/M/c$), 3. Random arrivals and Erlang service times ($M/E_k/c$), 4. Random arrivals and constant service times ($M/D/c$), 5. Constant interarrival times and exponential service times ($D/M/c$), 6. The effect of Erlang interarrival times ($E_m/M/c, E_m/E_k/c$).

Linear and combinatiorial optimization in ordered algebraic structures. By U. Zimmermann. North-Holland Publishing Co., Amsterdam and New York, 1981. x + 380 pp. $61.00.

The object of this book is to provide an account of results and methods for linear and combinatorial optimization problems over ordered algebraic structures. In linear optimization the set of feasible solutions is described by a system of linear constraints; to a large extent such linear characterizations are known for the set of feasible solutions in combinatorial optimization, too. Minimization of linear objective functions subject to linear constraints is a classical example which belongs to the class of problems considered. In the last thirty years several optimization problems have been discussed which appear to be quite similar. The difference between these problems and classical linear or combinatorial optimization problems lies in a replacement of linear functions over real (integer) numbers by functions which are linear over certain ordered algebraic structures. This work contains a description of algebraic formulations of such problems which make the relationship of similar problems more transparent. Then results and methods in different algebraic settings appear to be instances of general results and methods. Further specializing of general results and methods often leads to new results and methods for a particular problem. The author develops the foundations of the theory of ordered algebraic structures, covering all results which are used in the algebraic treatment of linear and combinatorial optimization problems. The first part of the book is self-contained; the representation theorems presented provide a general view of the scope of the underlying algebraic structure. Theoretical concepts and solution methods for linear and combinatorial optimization over such ordered algebraic structures are developed in the second part of the work. Results from Part I are explicitly and implicitly used throughout the discussion of these problems. In addition, the volume features an improved presentation of many results from the widespread literature (more than 250 references) as well as several new results. The thirteen chapters are divided into two parts: 1. Ordered algebraic structures, 2. Linear algebraic optimization.

An end-user's guide to data base. By James Martin. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1981. xiii + 144 pp. $21.95.

This is an elementary account of what databases are all about: no technical words and—of course—no mathematics; but useful and perceptive.

Projection method in constrained optimization and applications to optimal policy decision. By Berc Rustem. Springer-Verlag, Berlin, Heidelberg, New York, 1981. xv + 315 pp. $17.40

This is volume 31 of Lecture Notes in Control and Information Sciences. It consists of two parts. The first part, Chapters 1-4, is concerned with the development of projection techniques for different aspects of constrained optimisation. Chapter 1 provides a unified approach to the derivation of projection techniques and reviews existing methods. The application of projection techniques to the computation of a feasible point of linearly constrained regions is discussed in Chapter 2. In Chapter 3 a projection method is discussed for the quadratic programming problem of minimizing a quadratic objective function subject to inequality constraints. This method requires an initial feasible point. In Chapter 4 a method is discussed for solving the nonlinear programming problem. This method requires solutions to quadratic minimization subproblems. The second part of the work is concerned with the application of projection techniques to computational problems in policy optimization. A fundamental problem in the optimization of policy decisions is the specification of a suitable objective function. In Chapter 5 an iterative method is given for specifying objective functions. In Chapter 6 policy optimization algorithms, based on extensions of projection methods in Chapter 1, are discussed. These algorithms minimize quadratic objective functions subject to large nonlinear models treated as equality constraints.

Computer arithmetic: logic and design. By Otto Spaniol. John Wiley & Sons, Inc., New York, 1981. 280 pp. $39.50.

This is a volume in the Wiley Series in Computing. The contents of the book are represented by the chapter headings: 1. Notation, 2. Adders, 3. Multiplication, 4. Division, 5. Redundant notation, 6. Calculation of special functions, 7. Time complexity of arithmetic operations.

Hydrodynamic stability. By P. Drazin and W. Reid. Cambridge University Press, New York, 1981. xiv + 525 pp. $77.00.

The aim of the authors in writing this treatise on a subject which for nearly a century has been recognized as one of the central problems in fluid dynamics has been twofold: first, in Chapters 1-4, to describe the fundamental ideas, methods, and results in three major areas of the subject: thermal convection, rotating and curved flows, and parallel shear flows; secondly, to provide an introduction to some aspects of the subject of current research interest. These include some of the more recent developments in the asymptotic theory of the Orr-Sommerfeld equation in Chapter 5, some applications of the linear stability theory in Chapter 6 and finally, in Chapter 7, a discussion of some of the fundamental ideas involved in current work on the nonlinear theory of hydrodynamic stability. Each chapter ends with a number of problems which often extend or supplement the main text as well as provide exercises to help the reader understand the topics. The subject is concerned with the problem of when and how laminar flows break down, their subsequent development, and their eventual transition to turbulence. It has many applications in engineering, in meteorology and oceanography, and in astrophysics and geophysics. Some of these applications are mentioned, but the book is written from the point of view intrinsic to fluid mechanics and applied mathematics. Thus, although the authors emphasize the analytical aspects of the theory, they have also tried, wherever possible, to relate the theory to experimental and numerical results. Chapter headings: 1. Introduction, 2. Thermal instability, 3. Centrifugal instability, 4. Parallel shear flows, 5. Uniform asymptotic approximations, 6. Additional topics in linear stability theory, 7. Nonlinear stability. There is an appendix on a class of generalized Airy functions.

Stationary random processes associated with point processes. By Tomasz Rolanski. Springer-Verlag, New York, Heidelberg, Berlin, 1981. vi + 139 pp. $12.00.

This is volume 5 of "Lecture Notes in Statistics". It studies a notion of a random process associated with a point process. In Chapter 1 the preliminaries are given, including basic facts from ergodic theory, an important mathematical tool in these notes, and from queueing theory. Chapter 2 deals with discrete, and chapter 3 with continuous time theory. In chapter 4 three applications of the latter are given and in chapter 5 the results are applied to nonstandard single-server queues.

Applications of number theory to numerical analysis. By Hua Loo Keng and Wang Yuan. Springer-Verlag, Berlin, Heidelberg, New York, Science Press, Beijing, 1980. 241 pp. $39.00.

This volume is the first in a collection of joint publications of Springer-Verlag and Science Press (Beijing, The People's Republic of China) aimed at making contributions of Chinese mathematicians more readily available to the international mathematical community. It is a revised edition of the original Chinese edition published by Science Press in 1978 as the first volume in the Academia Sinica's Series in Pure and Applied Mathematics. The authors describe the number-theoretic method as follows: "We use number theory to construct a sequence of uniformly distributed sets in the s-dimensional unit cube G_s, where $s \geq 2$. Then we use the sequence to reduce a difficult analytic problem to an arithmetic problem which may be calculated by computer. For example, we may use the arithmetic mean of the values of the integrand in a given uniformly distributed set ofG_s to approximate the definite integral over G_s such that the principal order of the error term is shown to be of the best possible kind, if the integrand satisfies certain conditions. It is worth mentioning that the principal order of the error term of the Cartesian product formula for a classical single quadrature formula depends on and increases very rapidly with the dimension s. And though the error term in the Monte Carlo method is independent of s, it is in the sense of probability there, not in the usual sense of error. The number theoretic method may also be used to construct an approximate polynominal for the periodic function of s variables and to treat the problems of the approximate solutions to integral equations and partial differential equations of certain types." Chapter headings: 1. Algebraic number fields and rational approximation. 2. Recurrence relations and rational approximation. 3. Uniform distribution. 4. Estimation of discrepancy. 5. Uniform distribution and numerical integration. 6. Periodic functions. 7. Numerical integration of periodic functions. 8. Numerical error for quadrature formula. 9. Interpolation. 10. Approximate solution of integral equations and differential equations.

Seminar on micro-local analysis. By Victor W. Guillemin, Masaki Kashiware, and Takahiro Kawai. Princeton University Press, New Jersey, 1979. vii + 137 pp.

This is number 93 of Annals of Mathematics Studies. By micro-local analysis the authors mean the study of generalized functions as local objects on the cotangent bundle. It has its roots in the work of Cauchy, Riemann and Hadamard on the relationship between singularities of solutions of partial differential equations and the geometry of their characteristics.

Petri net theory and the modeling of systems. By James L. Peterson. Prentice-Hall, Inc. Englewood Cliffs, N.J., 1981. x + 290 pp. $22.95.

This book brings together the major parts of Petri net theory, presenting them in a coherent and consistent manner. Petri net theory can be applied to a large number of areas and knowledge of its fundamentals is becoming necessary for computer scientists, system analysts and engineering professionals. Table of contents: 1. Introduction, 2. Basic definitions, 3. Modeling with Petri nets, 4. Analysis of Petri nets, 5. Complexity and decidability, 6. Petri net languages, 7. Extended and restricted Petri net models, 8. Related models of parallel computation.

A Lie group: rotations in quantum mechanics. By Jean-Marie Normand. North-Holland Publishing Co., Amsterdam and New York, 1980. xvi + 486 pp. $61.00.

The three-dimensional rotation group and the quantum theory of angular momenta are widely studied subjects, but this volume differs from other works on the subject in that it is not only devoted to the rotation group in quantum mechanics. The author has aimed to bring out a set of general methods and results concerning the representations of compact Lie groups (mainly the linear ones) while deriving many specific properties and computational methods for the representations of the three-dimensional rotation group in quantum mechanics. The book is divided into roughly four parts. The first five chapters are devoted to general results concerning the rotation group and its representations. The unitary representations of the two-dimensional special unitary group are studied in Chapters 6 and 7. The important mathematical tool of tensor algebra is developed in detail in Chapters 8 and 9 and the last two chapters are devoted to the representations in commonly used functional vector spaces and to the applications to quantum mechanics. Mathematical definitions and theorems about vector spaces algebras and groups are given in appendices, as are specific results, tables and examples in connection with the rotation group.

Markov random fields and their applications. By Ross Kindermann and J. Laurie Snell. American Mathematical Society, Providence, 1980. 142 pp. $8.80.

This is volume 1 of the series Contemporary Mathematics. Markov random fields is a new branch of probability theory that promises to be important both in the theory and application of probability. The existing literature on the subject is quite technical and often understandable only to the expert. This paper is an attempt to present to a wider audience the basic ideas of the subject and its application. The authors have relied on examples and computer graphics to convey the meaning of the results when they are too technical to prove. Chapter headings: 1. The Ising model, 2. Markov fields on graphs, 3. Finite lattices, 4. Dynamic models, 5. The tree models, 6. Additional applications.

General topology and modern analysis. Edited by L. F. McAuley and M. M. Rao. Academic Press, New York, 1981. xv + 514 pp. $36.00.

This volume contains the proceedings of the Conference on General Topology and Modern Analysis held in May 1980 at the University of California, Riverside, in honor of the retirement of Professor F. Burton Jones. There is an inaugural lecture by R. H. Bing entitled Metrization Problems. The contributed papers are divided into three sections: Aposyndetic Continua (12 lectures); Algebraic, Differential and General Topology (24 papers); Modern Analysis and Set Theory (15 papers). There is also an annotated bibliography on aposyndesis by E. E. Grace.