

QUARTERLY

OF

APPLIED MATHEMATICS

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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R.I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$30.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and n, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{a^2 + b^2^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$([a + (b + cx)^n] \cos ky)^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOKS RECEIVED—

Developments in statistics, vol. 3. Edited by Paruchuri R. Krishnaiah. Academic Press, New York, 1980. xiv + 254 pp. \$35.00.

This volume consists of four invited papers which give authoritative reviews of the present state of their topics: 1. Asymptotic expansions in parametric statistical theory, by J. Pfanagl; 2. Orthogonal models for contingency tables, by H. O. Lancaster; 3. The increased use of statistical concepts in economic analysis, by Henri Theil; 4. Path analysis: an exposition, by Kathy M. Kang and E. Seneta.

Nonlinear waves in one-dimensional dispersive systems. By P. L. Bhatnager. Oxford University Press, New York, 1980. xii + 142 pp. \$22.50.

This introductory book on the mathematical theory of nonlinear dispersive waves gives a self-contained account of the important recent developments in the subject. Chapters include: linear waves, group velocity, and energy propagation; the effect of nonlinearity and its role in balancing diffusion and dispersion in steady solutions (Berger's and Korteweg-de-Vries' equation); soliton interaction by the inverse scattering method; Lax's results on general equations of evolution, and Whitman's modulation equations and group velocity in nonlinear waves.

Introduction to pseudodifferential and Fourier integral operators. By Francois Treves. Vol. I: *Pseudodifferential operators*, \$29.95; Vol. II: *Fourier integral operators*, \$35.00. 649 pp. (both volumes). Plenum Publishing Corp., New York and London, 1980.

This text provides an exposition of the handling of the techniques of pseudodifferential and Fourier operators and mathematical analysis of many examples of their use. The orientation is pragmatic, but theory is addressed whenever it serves to buttress applications. Theoretical chapters usually begin with the construction of special solutions to linear partial differential equations. The first volume covers such topics as standard pseudodifferential operators, applications of elliptic equations to boundary problems, and analytic pseudodifferential operators. The second volume addresses the naive theory of Fourier integral operators, Fourier distributions and global Fourier integral operators, standard micro-local forms of pseudodifferential operators, and applications to Riemannian geometry.

Fundamental principles of general relativity theories. By Hans-Jurgen Treder, Horst-Heino van Borzeszkowski, Alwyn van der Merwe and Wolfgang Yourgrau. Plenum Press, New York, 1980. 216 pp. \$27.50.

While the microscopic features of gravitation are emphasized by all relativistic field theories, the telescopic character of this phenomenon is underlined by a theory constructed around the Mach-Einstein doctrine. This theory rests on global formulations of the fundamental gravitational principles—i.e., the principles of relativity and inertia—and bases its considerations on the formulations of gravitational theory established by Newton and Einstein.

This volume examines the microscopic and telescopic aspects of gravitation in detail, and discusses the basic structures of field and cosmological versions of gravitational theory. These alternative approaches are compared, and experimental predictions are assessed for their significance in broadening the experimental basis for the theory of gravitation.

The first of two appendices presents a brief review of the basic concepts of four-dimensional spaces. Tetrad formalism is covered in the second appendix, which also provides a mathematical formulation of the relativity principle for quantum fields.

Continued from Page 128

Mathematical programming: structures and algorithms. By J. F. Shapiro. John Wiley & Sons, New York, 1979. xvi + 388 pp. \$23.95.

This book incorporates and integrates most of the central constructs and main results of mathematical programming and presents statements of many of the important algorithms for solving mathematical programming problems. The topics covered include linear programming, simplex methods and duality, network and graph optimization, discrete dynamic programming and Markovian decision theory, Lagrangean duality and the Kuhn-Tucker theorem, nondifferentiable optimization and methods for large-scale linear programming, complementary pivot theory and fixed-point algorithms, generalized duality theory and penalty function methods for nonlinear programming, group-theoretic and branch and bound methods for integer programming, and the traveling-salesman problem. Presented as the single most important integrating factor in this treatment is the role played by Lagrangean duality theory and related concepts of convex analysis—examined so as to motivate and discuss the global optimality conditions for all mathematical programming problems, relating them to constructive dual methods, convex analysis, and the Kuhn-Tucker theorem.

Green's functions and boundary value problems. By Ivar Stakgold. John Wiley & Sons, 1979. xv + 638 pp. \$29.50.

This is a volume in the series Pure and Applied Mathematics founded by Richard Courant. It is a new work based on the author's previous two-volume book *Boundary value problems of mathematical physics*, consolidating and reorganizing it into a single volume that can be covered in a one-year course. It is addressed to students who have completed the traditional "methods" course consisting of vector analysis, complex variables, and an introduction to Fourier series and boundary-value problems. After presenting a systematic development of linear boundary-value problems for ordinary and partial differential equations, the author concludes with a substantial chapter on nonlinear problems including bifurcation theory and monotone iteration. In the linear theory, a major role is played by related methods based on Green's functions, eigenfunction expansions, and integral equations. The necessary background in functional analysis is provided through the study of distributions, contraction mappings, Hilbert spaces, and operator theory. The linear theory culminates in the chapters on spectral theory and partial differential equations. Other features are the careful derivations of some of the equations that arise in application; the treatment of variational methods and perturbation theory; the use of maximum principles; the constructive approach to the continuous spectrum and transforms.

Mathematical statistical mechanics. By Colin J. Thompson. Princeton University Press, Princeton, 1979. ix + 278 pp. \$5.95.

This is a paperback edition of the book first published in 1972. Chapter headings: 1. Kinetic theory; 2. Thermodynamics; 3. The Gibbs ensembles and the thermodynamic limit; 4. Phase transitions and critical points; 5. The Ising model: algebraic approach; 6. The Ising model: combinatorial approach; 7. Some applications of the Ising model to biology; Appendixes.

Dynamic regression: theory and algorithms. By M. H. Pesaran and L. J. Slater. John Wiley & Sons, New York, 1980. 363 pp. \$74.95.

This is volume 5 of the series Computers and their Applications. It consists of five chapters entitled: 1. The analysis of economic time series; 2. Regression models with simple dynamic specification of the stochastic part; 3. Dynamic specifications for the stochastic part-autoregressions of general order; 4. First-order MA and mixed ARMA error specifications; 5. Distributed lag models with autocorrelated errors; and a second part devoted to four algorithms and their associated computer programs in Fortran IV (AR: regression model with autoregressive disturbances; ARMA: regression models with moving-average and mixed autoregressive moving average error specifications; DL1, DL2: autocorrelated error models with one and two distributed lag variables respectively). The first part discusses the purpose of the computer programs and the rationale behind their development. Mathematical derivations of the exact maximum likelihood estimators and their corresponding computational algorithms are described, and a brief review of the literature of regression models with autocorrelated errors is given.

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Interval mathematics 1980. Edited by Karl L. E. Nickel. Academic Press, Inc., New York, 1981. xv + 554 pp. \$29.50.

The present volume contains the proceedings of an international symposium held in Freiburg i. Br., Germany in March 1980. There are eleven invited and thirty contributed lectures.

Interval mathematics began with the goal of automating computational error analysis and has grown to include a much broader range of topics. This volume is an introduction to some of the most important work on the past decade. The application of interval mathematics to computing has several objectives: to provide computer algorithms for finding sets containing unknown solutions; to make these sets as small as possible; and to do all this as efficiently as possible. Toward these objectives, set-to-set mappings replace point-to-point mappings, and set inclusions replace approximate equalities. For the practical realization of this approach to computing one needs sets that are convenient and easy to describe. These sets are usually intervals in partially ordered spaces, but circles (disks) in the complex plane, spheres (balls) or ellipsoids in n -dimensional metric spaces, polyhedra and, sometimes, finite unions and intersections of such sets are also used. The interests of the authors of this volume range from purely theoretical topics to computational methods and even computer architecture. Their theoretical investigations concern set-valued mappings in partially ordered spaces, approximation theory, integration theory, Hausdorff distance, 3-valued relations, and non-Archimedean number systems.

Introduction to functional analysis : Banach spaces and differential calculus. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Volume 60.) By Leopoldo Nachbin. Translated by Richard M. Aron. Marcel Dekker, Inc., New York, 1981. 182 pp. \$19.75.

This text presents an introduction to Banach spaces and differential calculus on them. It shows aspects of the increasingly sophisticated uses of functional analytical techniques in mathematics, physics, engineering, economics, statistics, and information theory. The book is divided into two parts of twelve sections each. The first part is a self-contained introduction to Banach spaces. The second part presents differential calculus in normed spaces, independent of coordinate systems.

Enzyme mathematics. By P.-P. Kernevez. North-Holland, Amsterdam and New York, 1980. x + 262 pp. \$41.50.

This book deals with the mathematical aspects of simultaneous biochemical reaction and transport processes. It presents a new perspective for the biologist to consider in trying to explain the basis of biological organization, where these processes clearly play a central role. It also provides a clear presentation of the analytical techniques appropriate to the study of the equations describing these processes. Each chapter is devoted to the description of a precise system, together with the numerical and mathematical methods employed for its analysis. The mathematical technicalities are postponed until the last sections of each chapter, so that non-mathematicians may read the first sections dealing with the description of the phenomena, the numerical methods employed for their simulation, and the results of these numerical simulations. At the end of each chapter references are given to literature on the topics discussed.

A scrapbook of complex curve theory. By C. Herbert Clemens. Plenum Publishing Co., New York, 1980. ix + 186 pp. \$22.50.

This book contains musings on aspects of the theory of complex algebraic curves. Utilizing an informal approach, the volume offers provocative ramblings, with an emphasis on motivating future research as opposed to presenting a finished product. Topics run from elementary to the most sophisticated questions in algebra. Among the various topics discussed are the old-fashioned theory of conics, the elliptic integral, the number theory of cubics, meromorphic functions on cubics, the Jacobi inversion theorem, quartics, quintics, and the Schottky relation.

Continued from Page 150

Approximation theorems of mathematical statistics. By Robert J. Serfline. John Wiley & Sons, New York, 1980. xiv + 371 pp. \$34.95.

This book covers a broad range of limit theorems useful in mathematical statistics, along with methods of proof and techniques of application. The emphasis is on the manipulation of "probability" theorems to obtain "statistical" theorems.

Chapter 1 presents a variety of tools and foundation basic to asymptotic theory in statistics, including modes of convergence of a sequence of random variables and probability limit laws. Chapter 2 deals with the usual statistics computed from a sample. Properties such as asymptotic normality and almost sure convergence are derived. Deeper insights, such as Bahadur's representations, are pursued. Chapter 3 treats the asymptotics of statistics concocted as transformations of vectors of more basic statistics. The next six chapters deal with important special classes of statistics: statistics arising in classical parametric inference and contingency table analysis (Chapter 4); Hoeffding's U -statistics (Chapter 5); von Mises' differentiable statistical functions (Chapter 6); statistics obtained as solutions of equations (Chapter 7); linear functions of order statistics (Chapter 8); and rank statistics (Chapter 9). Chapter 10 surveys approaches to asymptotic relative efficiency of statistical test procedures, with special emphasis on the contributions of Pitman, Chernoff, Bahadur and Hoeffding.

The genetics of altruism. By Scott A. Boorman and Paul R. Levitt. Academic Press, New York, 1980 xx + 459 pp. \$29.50.

This book is the outcome of combining two historically separate areas of population biology. The formal models in *The Genetics of Altruism* continue the tradition of mathematical population genetics stemming from Wright, Hadane, and Fisher. The substantive area to which these models are applied is the comparative evolutionary biology of social behavior, taking cases and data both from the social insects (the social hymenoptera and the termites) and from social vertebrates (chiefly mammals). Connecting these two fields is a specific new approach to unifying a broad set of evolutionary insights into social behavior through the use of social network models to describe the impacts of altruism and cooperation on individual fitness. Use of network models gives the modeling developments a quite combinatorial character. The models developed are all evolutionary models directed to explanation on the time scales of organic evolution. A natural three-way division of alternative genetic models gives this subject its basic organization. The division is among reciprocity selection, kin selection, and group selection, each of which operates through mechanisms quite different from the other two. The first and last chapters are nonmathematical syntheses of the main findings, including the relationships between the present field and allied areas of population biology and social science (general evolutionary theory, ecology, demography, animal behavior studies, and the abstract analysis of social structure). The other ten chapters are divided into the above-mentioned three parts: the theories of reciprocity, kin and group selection.

Singular perturbations and asymptotics. Edited by Richard E. Meyer and Seymour V. Parter. Academic Press, New York, 1980. ix + 409 pp. \$22.00.

There are the proceedings of an advanced seminar conducted by the Mathematics Research Center at the University of Wisconsin, in May 1980. The papers delivered at that seminar are divided into four parts: I. Theory of singular layer problems (four papers); II. Resonance in singular perturbations and applications to physical chemistry (three papers); III. Multivariate methods and applications (three papers); IV. Turning-point theory and applications (five papers).

Plane algebraic curves. (Pure and Applied Mathematics: A series of monographs and textbooks, Volume 61.) By Grace Orzech and Morris Orzech. Marcel Dekker, Inc., New York, 1981. viii + 225 pp. \$29.50.

This is a textbook for advanced undergraduate and beginning graduate students in mathematics. The book introduces contemporary notions of algebraic varieties, morphisms of varieties, and adeles to the classical subject of plane curves over algebraically closed fields.

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Handbook of applicable mathematics. Volume II: probability. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1980. xix + 450 pp. \$85.00.

The chief editor of this Handbook is Walter Ledermann and the editor of this volume Emlyn Lloyd. The 'Handbook' consists of two sets of books. On the one hand, there are (or will be) a number of guidebooks written by experts in various fields in which mathematics is used (e.g., medicine, sociology, management, economics). These guidebooks are by no means comprehensive treatises; each is intended to treat a small number of particular topics within the field, employing, where appropriate, mathematical formulations and mathematical reasoning. A typical guidebook will consist of a discussion of a particular problem, or related set of problems, and will show how the use of mathematical models serves to solve the problem. Wherever any mathematics is used in a guidebook, it is cross-referenced to an article (or articles) in the core volume. There are 6 core volumes devoted respectively to Algebra, Probability, Numerical Methods, Analysis, Geometry and Combinatorics and Statistics. These volumes are texts of mathematics—but they are no ordinary mathematical texts. They have been designed specifically for the needs of the professional adult and they stand or fall by their success in explaining the nature and importance of key mathematical ideas to those who need to grasp and to use those ideas. Either through their reading of a guidebook or through their own work or outside reading, professional adults will find themselves needing to understand a particular mathematical idea (e.g., linear programming, statistical robustness, vector product, probability density, round-off error); and they will then be able to turn to the appropriate article in the core volume in question and find out just what they want to know. The chapter headings in this volume are: 1. Introduction: Probability and statistics. 2. Basic concepts of probability. 3. The calculus of probability. 4. Random variables and their realizations; probability distributions. 5. A first catalogue of discrete probability distributions (integer-valued variables). 6. Independent and non-independent random variables and their joint distributions, bivariate and multivariate; marginal and conditional distributions. 7. The addition of random variables: convolution. 8. Expectation (expected value). 9. Variance, covariance, and higher moments. 10. Continuous distributions. 11. A catalogue of continuous probability distributions. 12. Generating functions. 13. Multivariate continuous distributions. 14. Miscellaneous topics on distributions. 15. Order statistics. 16. Miscellaneous topics involving conditional probability. 17. Laws of large numbers. Stable distributions. 18. Stochastic processes: introduction and examples. 19. Processes in discrete time: Markov chains. 20. Continuous-time processes. 21. Renewals. 22. Complements on stochastic processes: Second order processes.

Essays in general relativity. Edited by Frank J. Tipler. Academic Press, Inc., New York, 1980. xvii + 236 pp. \$30.00.

This is a Festschrift for Abraham Taub. The contents are: 1. On Schwarzschild causality—A problem for "Lorentz covariant" general relativity, by Roger Penrose. 2. Comments on the topology of nonsingular stellar models, by Lee Lindblom and Dieter R. Brill. 3. General relativity and the eternal return, by Frank J. Tipler. 4. Energy and momentum of the gravitational field, by James W. York, Jr.. 5. The beam and stay of the Taub universe, by John Archibald Wheeler. 6. Tidal forces in a highly asymmetric Taub universe, by L. C. Shepley. 7. Symmetry breaking in general relativity, by Arthur E. Fischer, Jerrold E. Marsden, and Vincent Moncrief. 8. Gauge-invariant perturbation theory in spatially homogeneous cosmology, by Robert T. Jantzen. 9. Locally isotropic space-time nonnull homogeneous hypersurfaces, by M. A. H. MacCallum. 10. The gravitational waves that bathe the earth: upper limits based on theorists' cherished beliefs, by Mark Zimmermann and Kip S. Thorne. 11. General relativistic hydrodynamics: the comoving, Eulerian, and velocity potential formalisms, by Larry Smarr, Clifford Taubes, and James R. Wilson. 12. Lagrangian relativistic hydrodynamics with Eulerian coordinates, by Tsvi Piran. 13. Some thoughts on the origin of cosmic inhomogeneities, by E. P. T. Liang. 14. Automorphisms of formal algebras associated by deformations with a symplectic manifold, by Andre Lichnerowicz. 15. A remark on time-independent axisymmetric fields, by A. Papapetrou.

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Lecture Notes in Mathematics. Springer-Verlag, Berlin, Heidelberg, New York. 1980.

Ordinary and partial differential equations. Edited by W. N. Everitt. xvi + 271 pp. \$16.80.

This is volume 827 of this series and the proceedings of a conference on ordinary and partial differential equations, held at the University of Dundee, Scotland, in March 1978, and dedicated to the memory of Arthur Erdelyi (1908–1977).

Semi-martingales et grossissement d'une filtration. By Thierry Jeulin. ix + 142 pp. \$11.80.

This is volume 833 of the series.

Model theory of algebra and arithmetic. Edited by L. Pacholski, J. Wierzejewski and A. J. Wilkie. vi + 410 pp. \$24.50.

This is volume 834 of this series, and the proceedings of the conference on applications of logic to algebra and arithmetic held at Karpacz, Poland, in September 1979. It was the fourth in the series Set Theory and Hierarchy Theory organized by the Institute of Mathematics of the Technical University of Wrocław. There are 21 papers.

Introduction to data analysis and statistical inference. By Carl N. Morris and John E. Rolph. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1981. xx + 389 pp.

This text grew out of a two-quarter service course given jointly by the authors in the RAND Graduate Institute of Policy Studies since 1971. It is recommended that the text be used in conjunction with the statistical package STATLIB (by W. A. Brelsford and D. A. Relles, Prentice-Hall, 1981). The fourteen chapters are divided into three parts: 1. Data analysis, and 2. Statistical inference for linear models, 3. Robustness.

Approximation theory and applications. Edited by Zvi Ziegler. Academic Press, Inc., New York, 1981. xi + 358 pp. \$26.00.

These are the proceedings of a workshop held at the Technion, Haifa, Israel, in May 1980. There are 24 papers on the subject of the workshop.

Theoretical and applied mechanics. Proceedings of the XVth International Congress of Theoretical and Applied Mechanics, University of Toronto, Canada, August 17-23, 1980. Edited by F. P. J. Rimrott and B. Tabarrok, North-Holland Publishing Co., Amsterdam and New York, 1981. xxxii + 458 pp. \$78.00.

This volume contains four general lectures (by H. H. E. Leipholz on the analysis of nonconservative, nonholonomic systems, J. C. R. Hunt on environmental fluid mechanics, E. Sternberg on singular problems in linearized and finite elastostatics, and E. Fichera on analytic problems of materials with memory) and 33 sectional lectures presented at the Congress. They cover the latest developments of all aspect of mechanics. The other 320 contributed papers are listed by author and title.

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The origins of Cauchy's rigorous calculus. By Judith V. Grabiner. MIT Press, Cambridge, MA, 1981. xi + 252 pp. \$25.00.

This book explores the background of a major intellectual revolution, the rigorous reinterpretation of the calculus undertaken by Augustin-Louis Cauchy and his contemporaries in the first part of the nineteenth century. Their generation changed the calculus from a method of solving problems to a collection of theorems, based on precise definitions, about limits, continuity, series, derivatives, and integrals. Chapter headings: 1. Cauchy and the nineteenth-century revolution in calculus. 2. The status of foundations in eighteenth-century calculus. 3. The algebraic background of Cauchy's new analysis. 4. The origins of the basic concepts of Cauchy's analysis: limit, continuity, convergence. 5. The origins of Cauchy's theory of the derivative. 6. The origins of Cauchy's theory of the definite integral.

Techniques in operational research, Volume 2: Models, search and randomization. By Brian Conolly. John Wiley & Sons, New York, 1981. 338 pp. \$77.95.

Chapter headings: 1. Deterministic population models. 2. Stochastic population models. 3. Principles of detection and search. 4. Random versus deterministic search or transit. 5. Some notes on geometrical probability. 6. Computers in operational research. 7. The generation of pseudorandom numbers. 8. Linear congruential generators. 9. Non-uniform pseudo-generators.

Basic numerical mathematics, Volume 1: Numerical analysis. By John Todd. Birkhauser-Verlag, Basel, Stuttgart, 1979, Academic Press, New York, 1980. 253 pp.

Chapter headings: 1. The algorithms of Gauss, Borchardt and Carlson. 2. Orders of magnitude and rates of convergence. 3. Recurrence relations for powers. 4. The solution of equations. 5. Uniform convergence and approximation. 6. The acceleration processes of Aitken and Euler. 7. Asymptotic series. 8. Interpolation. 9. Quadrature. 10. Difference equations, differentiation and differential equations.

Advanced calculus: an introduction to modern analysis. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Volume 63.) By William L. Voxman and Roy H. Goetschel, Jr. Marcel Dekker, New York, 1981. 608 pp. \$55.00.

This text provides an introduction to the theory and applications of elementary analysis and aims to maintain a careful balance between abstract concepts and applied results of significance.

Towards a formal description of Ada. Edited by D. Bjorner and O. N. Oest. Springer-Verlag, Berlin, Heidelberg, New York, 1980. xiv + 630 pp. \$27.20.

This is volume 98 of Lecture Notes in Computer Science, edited by G. Goos and J. Hartmanis. Ada is a programming language developed on behalf of the U. S. Department of Defense by CII-Honeywell-Bull in Paris. It was named in honor of Ada Augusta, Lady Lovelace, a colleague and inspirer of Charles Babbage and daughter of Lord Byron. There are four chapters: 1. What is Ada? 2. The DDC Ada compiler and run-time system development method; 3. Quality assurance; 4. Software engineering support.

Graphs, networks, and algorithms. By M. N. S. Swamy and K. Thulasiraman. John Wiley & Sons, New York, 1981. xviii + 592 pp. \$37.50.

This book is addressed to students in mathematics, electrical engineering, and computer science. It is organized into three parts, dealing with the theory of graphs, electrical networks, and graph algorithms. In part I (Chapters 1 to 10) the authors discuss the theory of graphs, trees, Hamilton and Euler graphs, directed graphs, matrices of a graph, planarity, connectivity, matching, coloring, and an introduction to matroid theory. Among the matroid topics presented here are Minty's self-dual axiom system which makes obvious the duality between

circuits and cutsets of a graph, the arc-coloring lemma, and the greedy algorithm and its intimate relationship with matroids. Part II (chapters 11 to 13) is concerned with a discussion of those aspects of electrical network theory whose development is essentially graph-theoretic. In chapter 11 the authors discuss, among other things, the principal partition of a graph and its application in the mixed-variable method of networks. In chapter 12 they discuss several results in the theory of resistance networks and a method for realizing circuit and cutset matrices. In chapter 13 they develop topological formulas for network functions. Part III, which deals with graph algorithms, is organized into two chapters: chapter 14 on algorithms for the analysis of graphs and chapter 15 on algorithms that concern optimization problems on graphs. The authors' main concern here is the theory underlying the design, proof of correctness, and analysis of several graph algorithms. Among other things they include algorithms for flow graph reducibility, dominators, shortest paths, matchings, optimum binary search trees, network flows, and optimum branchings, as well as Hopcroft and Karp's analysis of a bipartite matching algorithm and Edmonds and Karp's analysis of Ford-Fulkerson's labeling algorithm. A major omission from this book is a discussion of NP-complete problems, which is beyond its scope.

Quadratic form theory and differential equations. By John Gregory. Academic Press, New York, 1980. xii + 237 pp.

This is volume 152 in the series Mathematics in Science and Engineering. Historically, quadratic form theory has been treated as a rich but misunderstood uncle. It appears briefly, almost as an afterthought, when needed to solve a variety of problems. A partial list of such problems includes the Hessian matrix in n -dimensional calculus; the second variational (Jacobi or accessory) problem in the calculus of variations and optimal control theory; Rayleigh-Ritz methods for finding eigenvalues of real symmetric matrices; the Aronszajn-Weinstein methods for solving problems of vibrating rods, membranes, and plates; oscillation, conjugate point, and Sturm comparison criteria in differential equations; Sturm-Liouville boundary-value problems; spline approximation ideas for numerical approximations; Gershgorin-type ideas (and the Euler-Lagrange equations) for banded symmetric matrices; Schrödinger equations; and limit-point-limit-circle ideas of singular differential equations in mathematical physics. A major purpose of this book is to develop a unified theory of quadratic forms to enable mathematicians to handle the mathematical and applied problems described above in a more meaningful way. The author's development is on four levels and should appeal to a variety of users of mathematics. For the theoretically inclined, he presents a new formal theory of approximations of quadratic forms/linear operators on Hilbert spaces. These ideas allow him to handle a wide range of problems. They also allow him to solve these problems in a qualitative and quantitative manner more easily than with more conventional methods. His second level of development is qualitative in nature. Using this theory, he can derive very general qualitative comparison results such as generalized Sturm separation theorems of differential equations and generalized Rayleigh-Ritz methods of eigenvalues. This theory is also quantitative in nature. He derives in level three an approximation theory that can be applied in level four to give numerical algorithms that are easy to implement and give good numerical results.

Geometric quantization. By Nicholas Woodhouse. Oxford University Press, New York, 1981. xi + 316 pp. \$74.00.

This systematic account of geometric quantization theory and its applications to theoretical physics is intended as a work of applied mathematics, suitable for graduate students and established workers in mathematical physics. The basic aim of the theory is to identify the structures that are needed for the reconstruction of a quantum system from its classical limit and to determine the extent to which these are singled out by the symplectic geometry of the classical phase space. The geometric method is developed not as a substitute for the analytic method of quantum theory, but to illuminate the relationship between classical and quantum mechanics. The book also clarifies the connection with twistor theory. Chapter headings: 1. Symplectic geometry. 2. Lagrangian and Hamiltonian mechanics. 3. Symmetry. 4. Polarizations and Hamilton-Jacobi theory. 5. Quantization. 6. The metaplectic correction. 7. Spinors and relativistic systems. There is also an appendix on useful background material.

Applied functional analysis. 2nd edition. By A. V. Balakrishnan. Springer-Verlag, New York, 1981. xiii + 373 pp. \$34.00.

This is the second edition of a work first published in 1976. Errors have been corrected and new material has been added reflecting relevant recent work.

Advances in control and dynamics systems, vol. 17. Edited by C. T. Leondes. Academic Press, New York, 1981. xvii + 424 pp. \$32.50.

This volume contains the following review articles: An overview of differential games (N. K. Gupta); Use of parameter optimization methods to determine the existence of game-theoretic saddle points (J. E. Rader); Solution techniques for realistic pursuit-evasion games (J. Shinar); Differential dynamic programming techniques in differential games (B. A. S. Järmark); Stochastic differential game techniques (B. Mons); Algorithms for differential games with bounded control and state (A. Chompaissal); Air combat systems analysis techniques (U. H. D. Lynch); Reachable set methods (N. K. Gupta); Zero-sum dynamic games (P. R. Kumar and T. H. Shiau); Singular differential game techniques and closed-loop strategies (K. Forouhar).

Mathematical programming for operations researchers and computer scientists. Edited by Albert G. Holzman. Marcel Dekker, Inc. New York, 1981. 392 pp. \$45.00.

This text consists of eleven chapters: 1. Linear programming. 2. Integer programming. 3. Game theory. 4. Goal programming. 5. Multicriteria decision making. 6. Quadratic programming. 7. Complementarity problems. 8. Geometric programming. 9. Fixed-point computing methods. 10. Classical optimization. 11. Nonlinear programming.

Theory and applications of some new classes of integral equations. By A. G. Ramm. Springer-Verlag, New York, 1980. xiii + 343 pp.

This book is intended for students, research engineers, and mathematicians interested in applications of numerical analysis. Pure analysts will also find some new problems to tackle. Most of the material can be understood by a reader with a relatively modest knowledge of differential and integral equations and functional analysis. There are eleven appendices and an extensive bibliography.

Finite rotations and Lagrangean description in the non-linear theory of shells. By Wojciech Pietraszkiewicz. Polis Scientific Publishers, Warszawa, Poland, 1979. 102 pp.

Chapter headings: 1. Introduction. 2. Introductory relations. 3. Deformation. 4. Theory of finite rotations. 5. Basic shell equations. 6. Small strain theory. 7. Theory of moderate rotations. 8. Concluding remarks.

A brief course of higher mathematics. By V. A. Kudryavtsev and B. P. Demidovich. Mir Publishers, Moscow, 1981. Distributed by Imported Publications, Inc., Chicago. 693 pp. \$16.00.

This book outlines those aspects of mathematics which are essential for students of the natural sciences. The twenty-six chapters range from geometry via calculus of one and several variables to probability theory and linear programming. There are many examples and exercises, with their solutions.

A book of problems in ordinary differential equations. By M. L. Krasnov, A. I. Kiselyou, and G. I. Makarenko. Translated by Vladimir Shokurov. Mir Publishers, Moscow, 1981. Distributed by Imported Publications, Inc., Chicago. 332 pp. \$6.80.

The topics in this book include the method of isoclines for equations of the first and second order, problems in finding orthogonal trajectories, the use of the method of superposition in solving linear differential equations of r th order, linear dependence and linear independence of a system of functions, problems in solving linear equations with constant and variable coefficients, boundary-value problems for differential equations, integrating equations in power series, asymptotic integration, integrating systems of differential equations, Lyapunov stability, and the operator method.

Intermediate mathematical statistics. By G. P. Beaumont. Chapman & Hall, London and Methuen, Inc., New York, 1980. 248 pp. \$10.95.

Chapter headings: 1. Sufficiency. 2. Unbiased point estimators. 3. Elementary decision theory and Bayesian methods. 4. Methods of estimation. 5. Hypothesis testing I. 6. Hypothesis testing II. 7. Interval estimation. Appendix 1. Functions of random variables. Appendix 2. The regular exponential family of distributions.

Introduction to multivariate analysis. By C. Chatfield and A. J. Collins. Chapman & Hall, London and Methuen, Inc., New York, 1980. x + 246 pp. \$25.00.

Chapter headings: 1. Introduction. 2. Multivariate distributions. 3. Preliminary data analysis. 4. Principal component analysis. 5. Factor analysis. 6. The multivariate normal distribution. 7. Procedures based on normal distribution theory. 8. The multivariate analysis of variance. 9. The multivariate analysis of covariance and related topics. 10. Multidimensional scaling. 11. Cluster analysis.

Intermediate statistical methods. By G. Barrie Wetherill. Methuen, Inc., New York, 1981. (Chapman & Hall, London). xvi + 390 pp. \$25.00.

Chapter headings: 1. Some properties of basic statistical procedures. 2. Regression and the linear model. 3. Statistical models and statistical inference. 4. Properties of the method of maximum likelihood. 5. The method of least squares. 6. Multiple regression: further analysis and interpretation. 7. Polynomial regression. 8. The use of transformations. 9. Correlation. 10. The analysis of variance. 11. Designs with regressions in the treatment effects. 12. An analysis of data on trees. 13. The analysis of variance: subsidiary analyses. 14. Components of variance. 15. Crossed classifications. 16. Further analysis of variance. 17. The generalized linear model.

Complex variable methods for linear multivariable feedback systems. Edited by A. G. J. MacFarlane. Methuen, New York, 1981. 368 pp. \$39.95.

This collection of reprints from the *International Journal of Control* summarizes an extensive investigation into ways of generalizing the classical Nyquist-Bode-Evans techniques used in the study of single-input, single-output feedback systems to the multivariable case. They are based on work carried out under Science Research Council support by the Control Group of the Cambridge University Engineering Department; this was a continuation of work started in the Control Systems Centre of the University of Manchester Institute of Science and Technology.

An introduction to nonlinear oscillations. By Ronald E. Mickens. Cambridge University Press, Cambridge, England, 1981. xiv + 224 pp. \$69.00.

Nonlinear oscillations of one-dimensional physical systems may be represented by a harmonic oscillator equation with the addition of a "small" nonlinear term. The book introduces the reader to a number of quantitative analytical approximation techniques which may be used to obtain solutions to this class of nonlinear differential equations. The approximation techniques considered include the Lindstedt-Poincaré perturbation method, the method of slowly varying amplitude and phase, the generalized method of Krylov-Bogoliubov-Mitropolsky, and the multi-time expansions. Chapter headings: 1. Nonlinear physical systems. 2. The perturbation method. 3. Method of slowly varying amplitude and phase. 4. Multi-time expansions. 5. Forced oscillations. 6. Advantages and disadvantages of various techniques. There are also nine appendices giving the needed mathematical background.

Modeling with differential equations. By D. N. Burghes and M. S. Borrie. John Wiley & Sons, New York, 1981. 172 pp. \$29.95.

This is an introduction at the level of first-year undergraduates. It describes applications to many field—biology, ecology, economics, advertising, as well as the traditional physical sciences.

Mechanics of continuous media and analysis of structures. By Roger Valid. North-Holland Publishing Co., Amsterdam and New York, 1981. xi + 356 pp. \$68.25.

This work stresses the importance of the mechanics of continuous media to the analysis of structures. The problems of mechanics are approached by variational methods, leading to their discretization by the finite-element method. The formalism of intrinsic differential geometry is used throughout. Chapter headings: 1. Mechanics of solid continuous media. 2. The finite-element method. 3. Variational principles in linear elasticity. 4. Vibration of linear structures. 5. Non-linear deformations—buckling. 6. Shell theory.

Topics in finite elasticity. By Morton E. Gurtin. SIAM, Philadelphia, 1981. 58 pp. \$9.50.

This monograph is based on a series of lectures delivered at the University of Tennessee in June 1979. Chapter headings: 1. Introduction. 2. Kinematics. 3. Stress. 4. Elastic materials. Change of observer. 5. Material symmetry. 6. Simple shear. 7. The Piola-Kirchhoff stress. 8. Hyperelasticity. 9. The elasticity tensor. 10. The boundary-value problem. 11. Variational formulation. 12. Stability and uniqueness. 13. Incompressible materials. 14. Deformations of a cube. 15. Anti-plane shear.

Finite elements, vol. 1: an introduction. By E. G. Becker, G. F. Carey and J. T. Iden. Prentice-hall, Inc., Englewood Cliffs, N. J., 1981. xii + 258 pp. \$24.95.

This is the first volume in a projected six-volume series on finite elements, written by members of the Texas Institute for Computational Mechanics at the University of Texas at Austin (Volume 2: a second course, 3: computational aspects, 4: mathematical aspects, 5: special problems in solid mechanics, 6: fluid mechanics). The purpose of this introductory volume is to provide the undergraduate student of engineering and science with a concise introduction to finite-element methods—one that will give a reader, equipped with little more than calculus, some matrix algebra, and ordinary differential equations, a clear idea of what the finite element method is, how it works, why it makes sense, and how to use it to solve problems of interest to him. Chapter headings: 1. A model problem. 2. One-dimensional problems. 3. Development of a finite-element program. 4. Two-dimensional problems. 5. Two-dimensional element calculations. 6. Extension.

Higher geometry. By Nikolai V. Efimov. Translated by P. C. Sinha. Mir Publishers, Moscow, 1980. Imported Publications, Inc., Chicago. 560 pp. \$11.00.

This is a translation of the sixth Russian edition published in 1977. It gives a systematic presentation of the fundamentals of Euclidean geometry, non-Euclidean geometry of Lobachevsky and Riemann, projective geometry and the geometrical aspects of special relativity theory. It also give a general introduction to constant curvature geometries. The author is a Professor at the Moscow State University.

Realization theory of continuous-time dynamical systems. By Tsuyoshi Matsuo. Springer-Verlag, Berlin, Heidelberg, New York, 1981. vi + 320 pp. \$17.40.

This is volume 32 of Lecture Notes in Control and Information Science. The realization problem consists in determining an intrinsic mathematical model (canonical dynamical system) from the input-output relations of a given causal black box, i.e., to understand fully the internal behavior of the black box from the experimental data to which it gives rise. There are four chapters: 1. Introduction. 2. Realization theory of (general) dynamical systems. 3. Realization theory of linear representation systems. 4. Realization theory of (algebraic) linear (time-constant) systems.

Mathematical modeling in epidemiology. By J. C. Frauenthal. Springer-Verlag, Berlin, 1980. viii + 118 pp. \$14.80.

This is a volume in the series Universitext. It is derived from courses taught by the author to fourth-year undergraduate students majoring in the mathematical sciences. Chapter headings: 1. Deterministic epidemic models. 2. Rumors and mousetraps. 3. Stochastic epidemic models. 4. Chain binomial models. 5. Branching process model. 6. Smallpox vaccination discontinuation. 7. Schistosomiasis eradication. 8. Gonorrhea. 9. Sickle-cell anemia.

Numerical solution of partial differential equations. By Theodor Meis and Ulrich Marcowitz. Springer-Verlag, New York, 1981. viii + 541 pp. \$24.00.

This book—volume 32 in Applied Mathematical Sciences—is the result of two courses of lectures given at the University of Cologne in Germany in 1974/75. The majority of the students were not familiar with partial differential equations and functional analysis, which explains why the book contains some basic material and results from these areas. The three parts of the book are largely independent of each other and can be read separately. Their topics are: initial-value problems, boundary-value problems, solutions of systems of equations. There is much emphasis on theoretical considerations and they are discussed as thoroughly as the algorithms which are presented in full detail, together with FORTRAN programs.

Electrodynamics and classical theory of fields and particles. By A. O. Barut. Dover Publications, New York, 1980. xv + 235 pp. \$4.50.

This is a corrected reprint of the original 1964 edition.

Stochastic monotonicity and queueing applications of birth-death processes. By Erik van Doorn. Springer-Verlag, New York, 1981. vi + 118 pp. \$9.80.

This is volume 4 of Lecture Notes in Statistics. Chapter headings: 1. Preliminaries. 2. Natural birth-death processes. 3. Dual birth-death processes. 4. Stochastic monotonicity: general results. 5. Stochastic monotonicity: dependence on the initial state distribution. 6. The M/M/s queue length process. 7. A queueing model where potential customers are discouraged by queue length. 8. Linear growth birth-death processes. 9. The mean of birth-death processes. 10. The truncated birth-death process.

Duality for crossed products of von Neumann algebras. By Yoshiomi Nakagami and Masamichi Takesaki. Springer-Verlag, Berlin, 1979. ix + 139 pp. \$9.00.

This is volume 731 of Lecture Notes in Mathematics.

Sturmian theory of ordinary differential equations. By William T. Reid. Springer-Verlag, New York, 1980. xv + 559 pp. \$28.50.

This is volume 31 of Applied Mathematical Sciences. The main text of the book was completed in September 1975 and at the time of Professor Reid's death (October 1977) the manuscript was still in the review process. Professor John Burns undertook the responsibility of seeing the book through the press. The prime purpose of the monograph is the presentation of a historical and comprehensive survey of the Sturmian theory for self-adjoint differential systems and the interrelations of this theory with the calculus of variations. Chapter headings: 1. Historical prologue. 2. Sturmian theory for real linear homogeneous second-order ordinary differential equations on a compact interval. 3. Self-adjoint boundary problems associated with second-order linear differential equations. 4. Oscillation theory on a non-compact interval. 5. Sturmian theory for differential systems. 6. Self-adjoint boundary problems. 7. A class of definite boundary problems. 8. Generalizations of Sturmian theory.

Gravitational curvature: an introduction to Einstein's theory. By Theodore Frankel. Freeman and Co., San Francisco. xviii + 172 pp. \$18.50 cloth; \$8.95 paper.

The author presents the basic aspects of general relativity by a "geometric" method of development: he prefers to compute curvatures on second fundamental forms rather than components of the Riemann tensor or Christoffel symbols. Knowledge of some basic differential geometry is assumed. Chapter headings: 1. Special relativity. 2. Clocks and gravitational potential. 3. A heuristic derivation of Einstein's equations. 4. The geometry of Einstein's equations. 5. The Schwarzschild solution. 6. The classical motion of a continuum. 7. The relativistic equations of motion. 8. Light rays and Fermat's principle. 9. Electromagnetism in three-space and Minkowski space. 10. Electromagnetism in general relativity. 11. The interior solution. 12. Cosmology.