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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

BOOK REVIEW

Picture languages: Formal models for picture recognition. By Azriel Rosenfeld. Academic Press Inc., New York, San Francisco, London, 1979. xiii + 225 pp.

Pictorial pattern recognition is a well-established and active field, with numerous applications in such areas as document processing (character recognition), industrial automation (assembly, inspection), medicine (cytology, radiology), and remote sensing, among many others. Efficient picture recognition systems are thus a matter of great practical interest. In recent years, there have appeared several papers on two-dimensional languages. However, relatively little is known about the automata theory of two-dimensional patterns.

This is a pioneering and leading textbook in this field. It consists of the following eight chapters. Chapter 1. Introduction: this chapter describes the intention and relationship of this book to picture recognition and automata theory. Chapter 2. Digital geometry: this chapter reviews a number of topics in *digital geometry*—the theory of geometrical properties of subsets of digital pictures. The theory provides the foundations for a number of basic array automata algorithms that are developed in later chapters. The theorems of this chapter are interesting and useful from the point of view of mathematics and practical applications. Chapter 3. String acceptors: this chapter reviews one-dimensional (string) automata and cellular automata, with emphasis on results whose two-dimensional analogues are treated in the subsequent chapters. Chapter 4. Sequential array acceptors: in this chapter, (sequential) array automata are treated for rectangular or connected arrays. Chapter 5. Cellular array acceptors: this chapter deals with cellular array automata. Various speed comparisons in acceptance are presented. Chapter 6. Pyramid cellular acceptors: a special kind of cellular array acceptors, called pyramid acceptors, is introduced, and its accepting powers are considered in relation to ordinary cellular acceptors. Chapter 7. Special types of acceptors: this chapter discusses three specialized acceptor models: (a) pebble acceptors, (b) pushdown acceptors, (c) parallel/sequential acceptors, and some properties of those acceptors are shown. Chapter 8. Grammars: this chapter is concerned with array grammars (including a brief review of string grammars), both sequential and parallel; that is, systems to generate two-dimensional pictures. The corresponding acceptors to grammars are discussed.

The content of this book is basically mathematical, but an informal approach is taken for accessibility. There are some typographical or trivial errors. Nevertheless, this is an excellent and must book for those interested in pattern recognition or automata theory.

AKIRA NAKAMURA (*Tokyo*)

—BOOKS RECEIVED—

Lecture Notes in Pure and Applied Mathematics. Marcel Dekker, Inc. New York and Basel.

Problems and propositions in analysis, vol. 49. By Gabriel Klambauer. 1979. 492 pp. $24.50.

This volume contains nearly 500 problems with complete solutions. The problems are designed to stimulate interest in solving nontrivial mathematical problems. Emphasis is on basic matter of real analysis. Chapter 1 deals with questions in number theory and combinatorics. Chapter 2 is on inequalities, Chapter 3 studies sequences and series, and Chapter 4 is devoted to real function theory.

Analytic arithmetic of algebraic function fields, vol. 50. By John Knopfmacher. 1979. 130 pp.

This monograph is intended as an introduction to a branch of the topic of the author's book *Abstract analytic number theory.*

Binary time series, vol. 52. By Benjamin Kedem. 140 pp. $23.50.

The objective of this book is to report some new developments in the analysis of stationary time series. The analysis is based to a large degree on *counting* methods which lead to the determination of various distributions of interest and to the fast computation of estimates of parameters in time series models. Particular attention is focused on level-crossings, their properties, and their applications in estimation and in the evaluation of the asymptotic distribution of extremes in stationary series.

Hypoelliptic boundary-value problems, vol. 53. By J. Barros-Neto and Ralph A. Artino. 1980. 90 pp. $19.75.

This book provides a survey of the theory of hypoelliptic boundary-value problems in the constant coefficients case. Reproducing kernels for hypoelliptic boundary-value problems are constructed and the different regularity properties of these kernels discussed. The necessary and sufficient conditions for the existence of the reproducing kernels are proven. Semi-elliptic boundary-value problems are presented as natural generalizations of elliptic boundary-value problems.

Introduction to mathematical consensus theory, vol. 59. By Ki Hang Kim and Fred W. Roush. 1980. 192 pp. $25.00.

This book presents the theory of social welfare functions and game theory in terms of the mathematical background of combinatorics and matrix theory. The book begins by defining the types of binary relations used in this field and describes the application of Boolean matrix methods to the study of binary relations. Summaries of the most recent work and the collection of proofs in social consensus theory take the reader to the frontier of knowledge in consensus theory.

Measures of noncompactness in Banach spaces, vol. 60. By Jozef Banas and Karzimierz Goebel. 1980. 112 pp. $17.50.

The techniques based on the non-compactness of Banach spaces enable one to extend the classical results of compact and completely continuous mappings to a much wider class of transformations that satisfy miscellaneous regularity conditions. These measures also facilitate applications to fixed point theory and the theory of differential equations. The book is intended for mathematicians and students involved in the branches of analysis in which measures of noncompactness are applicable: ordinary and partial differential equations, functional differential equations, equations with deviating arguments, integral equations, optimal control theory, operator equations in Banach spaces, and others.

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Ordered groups, vol. 62. Edited by Jo E. Smith, G. Otis Kenny and Richard N. Ball. 1980. 192 pp. $25.50.

The original papers collected in this book reflect the work of the leading mathematicians on the field of ordered structures. Special emphasis is placed on fundamental discussions of the recent impact of ideals from mathematical logic and model theory of lattice-ordered groups.

Statistical methods. By George W. Snedecor and William G. Cochran. Seventh edition. Iowa State University Press, Ames, Iowa, 1980. xvi + 507 pp. $21.50.

This new edition of a well-known text has been extensively revised. The new Chapter 1 gives examples of three of the most common activities in statistics—controlled experiments, sample surveys, and nonexperimental comparative studies. The observational studies include an indication of the kinds of problems encountered in planning and reporting. The rearrangement of most of the remaining topics from the previous Chapter 1 into Chapters 2 and 3 allows a more gradual immersion into the subject matter. The following sections are new or rewritten to include recent work: 1. Simultaneous study of the different effects of a transformation; 2. Interpretation and use of the table of expected values of mean squares in the analysis of variance; 3. Selection of variates for prediction in regression; 4. Nonsampling errors in sample surveys.

ORACLS: a design system for linear multivariable control. (Control and Systems Theory Series, Volume 10.) By Ernest S. Armstrong. Marcel Dekker, Inc., New York, 1980. 256 pp. $35.00.

This book describes a unique package of FORTRAN subroutines (entitled ORACLS) for designing linear control laws and filters for systems with linear time-invariant multivariable differential and difference equation models. ORACLS (Optimal Regulator Algorithms for the Control of Linear Systems) represents an application of some of today's most reliable and efficient numerical linear algebra procedures, such as singular value and Schur decompositions, to implement linear-quadratic-Gaussian (LQG) methodology. The book includes procedures for computing eigensystems of real matrices; factored forms of matrices; the solution, and least-squares approximations to solutions, of certain matrix algebraic equations; the controllability properties of a linear time-invariant system; and the steady-state covariance matrix of an open-loop stable force by white noise. Algorithms are provided for solving both the continuous and discrete optimal linear regulator problems and the sampled-data optimal linear regulator problems. For measurement noise, duality theory and the optimal regulator algorithms are used to construct continuous and discrete Kalman-Bucy filters. Subroutines are also included which generate multivariable servomechanism control laws.

Analysis and computation of fixed points. Edited by Stephen M. Robinson. Academic Press, New York, 1980. ix + 413 pp. $22.50.

These are the proceedings of a symposium conducted by the Mathematics Research Center, University of Wisconsin, Madison, in May 1979. There are nine papers.

Complementary variational principles. By A. M. Arthurs. Second edition. Oxford University Press, New York, 1980. vii + 154 pp. $49.00.

This is the second edition of the book first published in 1970. It has been largely rewritten. For instance, the author has replaced the local theory with a global treatment based on simple ideas of convexity and monotone operators. Also, the class of problems treated is much wider than the Dirichlet type originally discussed. In addition, the variational results are given a geometrical formulation that includes the hypercircle, and error estimates for variational solutions are also described. The number of applications to boundary-value problems has been doubled to thirty cases arising in physics, chemistry, engineering and biology. There are several recent results.

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The nature of computation : an introduction to computer science. By Ira Pohl and Alan Shaw. Computer Science Press, Inc., Rockville, Maryland, 1980. xii + 397 pp. $16.95.

Intended for a first course in computer science, this book deals with the core concepts of the subject paralleled with an introduction to programming. It has the following three major objectives: a survey of the field, an initial literacy in the language and methods of computer science, and a historical, philosophical and social perspective. Since computer science is the study of algorithms, the book covers the technical foundations and applications by constructing and studying a number of basic algorithms for each topic. The history, applications, and social implications of various technical developments are discussed, and significant contemporary work is described. Also introduced are some of the controversies surrounding advances in computing, including those related to noncomputability, artificial intelligence, computer modeling, and data banks and privacy.

Contents: Computer science—origins and core topics; Algorithms, numbers, and data; Boolean algebra and applications; Computer organization and design; programming languages and systems; data types and structured data; Compilers; Operating systems; Turing machines and computability; Analysis of algorithms; Artificial intelligence; Computer applications and implications—medicine, decision making, information retrieval, forecasts.

Cluster analysis. By Brian Everitt. Second edition. Halsted Press, John Wiley & Sons, New York, 1980. iv + 136 pp. $16.95.

This is the second edition of the text first published in 1974. It seeks to provide a non-mathematical account of the techniques of cluster analysis. There are some minor changes, more real-life examples are considered and the literature since 1973 is referenced.

Advanced number theory. By Harvey Cohn. Dover Publications, New York, 1980. xi + 276 pp. $5.00.

This is a reprint of the 1962 text. The prerequisite is a standard one-semester course in number theory.

Elementary differential equations with applications. By William R. Derrick and Stanley I. Grossman. Second edition. Addison-Wesley, Reading, MA, 1980. 609 pp. $20.95.

This test for a one-semester undergraduate course has been largely rewritten for this edition. There is a wealth of examples from the biological and social as well as the physical sciences. Chapter headings: 1. Introduction; 2. First-order equations; 3. Linear differential equations; 4. The existence and uniqueness of solutions; 5. Power series solutions of differential equations; 6. Laplace transforms; 7. Numerical methods.

Conformal mapping on Riemann surfaces. By Harvey Cohn. Dover Publications, New York, 1980. xiv + 325 pp. $6.00.

This is a reprint of the 1967 monograph. It is in five parts: a review of complex analysis, Riemann manifolds, derivation of the existence theorems, real existence proofs, and algebraic applications.

Theory and applications of Fourier analysis. (Pure and Applied Mathematics: A series of Monographs and Textbooks, Volume 59.) By Charles Sparks Rees, S. M. Shah and Caslav V. Stanojevic. Marcel Dekker, Inc., New York, 1981. $37.50.

This book approaches Fourier analysis via the Lebesgue theory at the textbook level. It covers: trigonometric series; convergence and summability; multiple Fourier series; the Fourier transform; distributions and generalized functions; generalizations of the trigonometric system to orthogonal systems and series, with an emphasis on the classical orthogonal polynomials and Bessel functions and Fourier-Bessel series.

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Nonnegative matrices in the mathematical sciences. By A. Berman and R. J. Plemmons. Academic Press, Inc., 1979. xviii + 316 pp. $32.00.

This is a volume in the series Computer Science and Applied Mathematics. It is intended to be used as a reference book or a graduate textbook by mathematical economists, mathematical programmers, statisticians and computer scientists, as well as mathematicians. Each chapter is elf-contained insofar as possible. The theory of nonnegative matrices is discussed geometrically and algebraically; generalizations of inverse-positivity and M-matrices are studied in detail and selected applications to numerical analysis, probability, economics and operations research are described.

The natural philosophy of time. 2nd edition. By G. J. Whitrow. Oxford University Press, New York, 1980. ix + 399 pp. $39.50.

This is a completely revised second edition of the work first published by Nelson in 1961. It is its object to provide as up-to-date, comprehensive, and unified an account as possible of our knowledge concerning the concept of time and its role in the different sciences, particularly psychology, neurophysiology, biology, physics, and cosmology. The field covered lies mainly in between the purely philosophical aspects of time and the practical aspects of its measurement, although neither of these is completely ignored. The problems studied are placed in their historical perspective. The book is intended for scientists, philosophers, and all who take an interest in the subject of time and in the attempts that have been and are being made to understand this fundamental but puzzling concept. Chapter headings: 1. Universal time; 2. Human time; 3. Biological time; 4. Mathematical time; 5. Relativistic time; 6. Space-time and cosmic time; 7. The nature of time.

Oscar Zriski: collected papers. Volume IV: Equisingularity on algebraic varieties. Edited by J. Lipman and B. Teissier. MIT Press, Cambridge, 1979. xxvi + 651 pp. $50.00.

This volume contains fourteen papers, published originally between 1964 and 1979. The first two volumes were published in 1973 and the third was published in 1978. These four volumes are part of the series Mathematicians of Our Time. Other volumes in the series included papers by Paul Erdös, Einar Hille, Mark Kac, Charles Loewner, P. A. MacMahon, George Polya, Hans Rademacher, Stanislaw Ulam, and Norbert Wiener.

Mathematics and the real world. Edited by B. Booss and M. Niss. Interdisciplinary Systems Research 68. Birkhauser Boston Inc., Mass., 1979. 140 pp. $22.00.

These are the proceedings of an international workshop held at Roskilde University Centre (Denmark) in 1978.

Numerical techniques for stochastic systems. Edited by F. Archetti and M. Cugiani. North-Holland Publishing Co., Amsterdam and New York, 1980. viii + 406 pp. $53.75.

This volume collects the invited papers and some of the contributed papers presented at the conference "Numerical Techniques for Stochastic Systems" held in Gargnano, Italy in September 1979. The main aim of this book is to present some recent research results obtained in the interface between numerical methods and stochastic systems. This research area can be divided under two headings: the first connected with the development of numerical methods for the identification, estimation and modelling of stochastic systems, and the second with the applications of ideas from probability and statistics to the design and analysis of numerical methods. These two areas are strongly correlated and similar tools are often used in both domains of research. The conference aimed at bringing together scholars from each area in order to focus the most promising aspects of their common methodologies and to identify trends of future development.