

QUARTERLY  
OF  
APPLIED MATHEMATICS

EDITED BY

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# QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R.I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$30.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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## SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime (*'*), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent  $\frac{1}{2}$  rather than with the sign  $\sqrt{\quad}$ .

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol *exp* should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{a^2 + b^2, 1/2}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered, Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

**Cuts:** Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*)—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; (*for periodicals*)—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOKS RECEIVED—

*Decision information.* Edited by Chris P. Tsokos and Robert M. Thrall. Academic Press, New York, 1979. xiii + 528 pp. \$34.00.

Many of the papers published in this volume are based on talks given at a scientific workshop on Decision Information for Tactical Command and Control, held in September 1976 at Airlie House, Virginia and sponsored by the United States Air Force Office of Scientific Research, with the participation of the other three military services. There are also some invited contributions from scientists whose work complements the selected presentations from the workshop. The papers are divided into three groups: deterministic models, stochastic models and military models.

*Selected papers of Chiing-Shen Chern.* Springer-Verlag, New York, 1978. xxxi + 476 pp. \$24.80.

The work of C. S. Chern has influenced modern (differential) geometry profoundly. This collection, published on the occasion of his sixty-eighth birthday, contains approximately one-third of Chern's total mathematical work to date. The author has made the selection of papers for this volume, giving preference to his shorter and less accessible works. The book features three introductory articles. The first by Philip Griffiths and the second by André Weil describe various aspects of Chern's life and work. Then Chern offers some perspectives of his own in an article entitled "Summary of my scientific life and works."

*The algebra of econometrics.* By D. S. G. Pollock. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1979. xxi + 360 pp. \$45.00.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is intended as a text for graduate and advanced undergraduate courses in econometrics and as a reference book for research workers in econometrics and statistics. It aims to provide a unified treatment of the subject of theoretical econometrics by using certain general themes of the algebra of vector spaces and by pursuing the statistical theme of maximum-likelihood estimation. In the first four chapters of the book and in the appendix, there is a thorough treatment of the mathematical and statistical theory which is requisite to the advanced study of econometrics. The major part of the text consists of a broad coverage of the classical methods of econometrics and of the methods for treating time-series problems in the time domain. Chapter headings: 1. Vector spaces; 2. Linear transformations; 3. Metric spaces; 4. Extensions of matrix algebra; 5. The algebra of econometrics; 6. The Gauss-Markov model; 7. The classical linear model; 8. Models with errors in variables; 9. The Gauss-Markov model with a singular dispersion matrix; 10. The Gauss-Markov model with linear restrictions on parameters; 11. Temporal stochastic processes; 12. Temporal regression models; 13. Sets of linear regressions; 14. Systems of simultaneous equations; 15. Quasi-Gaussian methods; 16. Maximum likelihood methods; 17. Appendix on statistical theory.

*R. A. Fisher: an appreciation.* Edited by S. E. Fienberg and D. V. Hinkley. Springer-Verlag, New York, 1980. ix + 208 pp.

This is volume 1 of Lecture Notes in Statistics. It contains edited versions of presentations delivered as part of a faculty-student seminar and during a special lecture series in the School of Statistics, University of Minnesota, during the spring of 1978. The special lectures printed here were given by Joan Fisher Box, William Cochran and David Wallace, and there are introductory remarks on R. A. Fisher as well as two other papers by David Hinkley. Twelve additional papers from the faculty-student seminar are included, as well as a list of Fisher's publications.

Continued from Page 466

*Mathematical statistics and probability theory.* Proceedings of the 6th International Conference, Wisła, Poland. Edited by W. Klonecki, A. Kozek and J. Rosinski. Springer-Verlag, New York, 1980. xvi + 373 pp. \$20.00.

This is volume 2 of Lecture Notes in Statistics. It is dedicated to Jerzy Neyman and includes talks by R. Bartoszyński (some thoughts about Jerzy Neyman), K. Doksum (some remarks on the achievements of Professor Neyman in the United States) and K. Krickeberg (the role of Jerzy Neyman in the shaping of the Bernoulli Society).

*Modern trends in the theory of condensed matter.* Edited by A. Pekalski and J. Przystrawa. ix + 597 pp. Springer-Verlag, Berlin, Heidelberg, New York, 1980. \$40.20.

This is volume 115 of Lecture Notes in Physics, and is the proceedings of the 16th Karpacz Winter School of Theoretical Physics, February–March 1979. What was meant by “modern trends” was to a large extent left to the lecturers to decide; therefore the School was expected to cover a broad spectrum of topics ranging from classical liquids to perfect solids. Lecturers were invited in a way which reflected the local interest of Polish scientific communities; nevertheless, it was hoped that the scope was broad enough to evoke general interest and yet not too broad to put off specialists. The thirty-one papers are divided into five groups: quantum liquids, classical liquids, random systems, phase transitions, and other topics.

*Introduction to operations research.* 3rd edition. By Frederick S. Hillier and Gerald J. Lieberman. Holden-Day, Inc., San Francisco, 1980. xiv. + 829 pp. \$27.95.

This is the third edition of a text first published in 1967. Several new topics concerned with practical applications have been added. The emphasis is still on motivation and simplicity of explanation rather than rigorous proofs and technical detail.

*The Radon transform.* By Sigurdur Helgason. Birkhäuser, Boston, 1980. vii + 192 pp. \$12.00.

This is volume 5 of the series Progress in Mathematics. It consists of a revision of the author's notes for lectures given at MIT in the fall of 1966, based mostly on his papers during 1959–1965 on the Radon transform and its generalization. The topic is one in geometric analysis which has its origin in results of Funk (1916) and Radon (1917) determining, respectively, a symmetric function on the two-sphere from its great circle integrals and a function on the plane from its line integrals. Recent developments, in particular applications to partial differential equations, X-ray technology (CAT scanners) and radio-astronomy, have widened interest in the subject. The term “Radon transform” seems to be due to F. John (1953). Radon's now somewhat inaccessible pioneering paper is reproduced in an appendix.

*Duality in measure theory.* By Corneliu Constaninescu. Springer-Verlag, New York, 1980. iv + 197 pp. \$14.00.

This is volume 796 of Lecture Notes in Mathematics. There are four chapters: 1. Preliminaries; 2. Representations; 3. Duals of spaces of measures; 4. Vector measures.

*Algebraic geometry.* Edited by K. Lonsted. Springer-Verlag, Berlin, Heidelberg, New York, 1979. 658 pp.

This is volume 732 of Lecture Notes in Mathematics and represents the proceedings of the Copenhagen Summer Meeting in Algebraic Geometry held in August, 1978. In addition to the papers presented at lectures, this volume also includes a few by other participants and some by authors who could not attend. There are 34 papers in all.

Continued from Page 478

*Reversibility and stochastic networks.* By F. P. Kelley. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1979. viii + 230 pp. \$39.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. Its main topic is the study of the behavior in equilibrium of vector stochastic processes, or stochastic networks. Such processes have a wide range of applications: e.g. the components of the vector may represent queue sizes in a queueing network, gene frequencies in a population, or the condition of fruit trees in an orchard. When a stochastic network is reversible its analysis is greatly simplified, and the first chapter is devoted to a discussion of the concept of reversibility. Two themes emerge from the remainder of the book: first, the various uses of reversibility in the study of the output from a queue, the flow of current in a conductor, the age of an allele, or the equilibrium distribution of a polymerization process; second, the extent to which the assumption of reversibility can be relaxed without destroying the associated tractability.

The main prerequisite is an understanding of Markov processes at about the level of Feller's *Introduction to probability theory and its applications*, Volume I. The necessary material is very briefly reviewed, primarily to establish terminology and notation.

*Numerical methods for partial differential equations.* Edited by Seymour V. Parter. Academic Press, New York, 1979. ix + 332 pp.

This is publication no. 42 of the Mathematics Research Center, Madison, Wisconsin, and constitutes the proceedings of an advanced seminar held there in October 1978. There are ten papers, surveying some of the ideas and techniques currently of interest in applications such as continuum mechanics and fluid dynamics, and also presenting a glimpse of a variety of research problems and results.

*Conjugate direction methods in optimization.* By Magnus Hestenes. Springer-Verlag, New York, 1980. x + 325 pp. \$29.80.

This is volume 12 of the series Application of Mathematics. The author originated the method of conjugate gradients in the early 1950s at the UCLA Institute for Numerical Analysis of the National Bureau of Standards and the preface puts this event into its historical perspective, giving the origin of the method and its more recent history. The purpose of the present book is to present a relatively full account of conjugate direction methods in optimization. The author begins with an analysis of the Newton's method and the gradient method, then studies the quadratic case in depth together with extensions to the nonquadratic case via a modified Newton's method.

*An introduction to the theory of numbers.* 4th edition. By Ivan Niven and Herbert S. Zuckerman. John Wiley & Sons, New York, 1980. xii + 335 pp. \$19.95.

The purpose of this book is to present a reasonably complete introduction to the theory of numbers within the compass of a single volume. The basic concepts are presented in the first part of the book, followed by more specialized material in the final three chapters. Paralleling this progress from general topics to more particular discussions, the authors have attempted to begin the book at a more leisurely pace than followed later. Thus the later parts of the book are set forth in a more compact and sophisticated presentation than are the earlier parts. This book is intended for seniors and beginning graduate students. It contains a least enough material for a full year course. Chapter headings: 1. Divisibility; 2. Congruences; 3. Quadratic reciprocity; 4. Some functions of number theory; 5. Some Diophantine equations; 6. Farey fractions and irrational numbers; 7. Simple continued fractions; 8. Elementary remarks on the distribution of primes; 9. Algebraic numbers; 10. The partition function; 11. The density of sequences of integers; special topics.

Continued from Page 498

*Quantum theory and gravitation*. Edited by A. R. Marlow. Academic Press, New York, 1980. x + 267 pp. \$18.50.

This volume contains seventeen papers delivered at the second in a series of meetings at Loyola University, New Orleans, between physicists and mathematicians concerned with the fundamental questions of modern theoretical physics. The authors are J. A. Wheeler (who spoke on pregeometry), W. K. Wootters, C. H. Brans, A. R. Marlow, D. Finkelstein, R. Hermann, A. Komar, P. E. Parker, L. Castell, J. S. Anandan, G. F. Chapline, S. A. Fulling, M. J. Dupre, J. A. Goldstein, J. G. Miller and C. F. Blakemore. The premise from which the conference arose was the belief that mathematicians and physicists talking and working together can accomplish more than either group separately. Amongst other participants were A. M. Gleason, R. J. Greechie, P. R. Halmos, C. DeWitt-Morette and B. S. DeWitt.

*Variational inequalities and complementarity problems: theory and applications*. Edited by R. W. Cottle, F. Giannessi and J.-L. Lions. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1980. xvii + 408 pp. \$48.95.

This volume contains the proceedings of an International School of Mathematics devoted to variational inequalities and complementarity problems in mathematical physics and economics, held in June 1978 in Erice, Sicily. It is dedicated to the memory of Professor G. Stampacchia, one of the organizers of the School. Many of the variational inequalities papers deal with free boundary problems such as arise in flow through porous media, hydrodynamic lubrication and elastoplastic analysis. The papers on complementarity problems deal with algorithms, existence theory, and the relationships between complementarity problems, variational inequalities, mathematical programming, and monotone operator theory. Others cover calculus of variations, stochastic optimization and control, convex analysis, and systems of nonlinear equations.

*Electromagnetic interactions in elastic solids*. Edited by H. Parkus. Springer-Verlag, Vienna & New York, 1979. vi + 425 pp. \$38.10.

This is volume 257 in the series Courses and Lectures of the International Centre for Mechanical Sciences in Udine, Italy, and is the proceedings of a course organized at the Centre. The lectures are: General theory of electro- and magneto-elasticity, by J. B. Alblas; Foundations of linear piezoelectricity, by W. Nowacki; Thermodynamic aspects in field-matter interactions, by K. Hutter; Classical magnetoelasticity in ferromagnets with defects, by G. A. Maugin; Electromagnetic interactions in elastic solids: some relativistic aspects, by A. Prechtl; Applications of electromagnetic interaction theory, by H. Parkus.

*Approximation theory and numerical methods*. By G. A. Watson. John Wiley & Sons, New York, 1980. x + 229 pp. \$24.95.

This book treats approximation theory in classical style, with the methods and language of the theory of functions, to ensure that the material is accessible to the widest possible audience. There is, however, also a general treatment of the characterization of best approximations in normed linear spaces, using the modern methods of functional analysis. There are ten chapters. After an introductory chapter, the next six chapters are devoted to the treatment of standard linear problems with the different  $L_p$  norms. Chapter 8 is concerned with piecewise polynomials, and is an introduction to what is a very important and still rapidly growing subject. The last two chapters are devoted to nonlinear problems, with Chapter 9 dealing in particular with the case of rational approximation.

*Combinatorial optimization.* Edited by N. Christofides, A. Mingozzi, P. Toth and C. Sandi. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1979. viii + 425 pp. \$47.00.

This book is based on a series of lectures given at the Summer School in Combinatorial Optimization held in Urbino, Italy in June 1977. The term "combinatorial optimization" is meant to describe those areas of mathematical programming concerned with the solution of optimization problems having a pronounced combinatorial or discrete structure. The title is used as a unifying term covering integer programming, graph theory, parts of dynamic programming, etc., and the areas covered are of increasing importance because of the large number of practical problems that can be formulated and solved as combinatorial optimization problems. The chapters are by various authors and their headings are 1. Branch and bound methods for integer programming; 2. The theory of cutting-planes; 3. Subgradient optimization; 4. A partial order in the solution space of bivalent programs; 5. The complexity of combinatorial optimization algorithms and the challenge of heuristics; 6. The traveling salesman problem; 7. Set partitioning—a survey; 8. The graph-coloring problem; 9. The 0-1 knapsack problem; 10. Complexity and efficiency in minimax network location; 11. The vehicle routing problem; 12. Loading problems; 13. Minimizing maximum lateness on one machine: algorithms and applications; 14. The crew scheduling problem: a traveling salesman approach; 15. Graph-theoretic approaches to foreign exchange operations.

*Spectra of graphs: theory and applications.* By Dragos M. Cvetkovic, Michael Doob and Horst Sachs. Academic Press, New York, 1980. 368 pp. \$45.00.

From the preface: "This book has been written for mathematicians working in the area of graph theory and combinatorics, for chemists who are interested in quantum chemistry, and, at least partly, for physicists and electrical engineers using graph theory in their work. The book is almost entirely self-contained; only a little familiarity with graph theory and matrix theory is assumed. The theory of graph spectra can, in a way, be considered as an attempt to utilize linear algebra including, in particular, the well-developed theory of matrices for the purposes of graph theory and its applications. However, that does not mean that the theory of graph spectra can be reduced to the theory of matrices; on the contrary, it has its own characteristic features and specific ways of reasoning fully justifying it to be treated as a theory in its own right." Chapter headings: 0. Introduction; 1. Basic properties of the spectrum of a graph; 2. Operations on graphs and the resulting spectra; 3. Relations between spectral and structural properties of graphs; 4. The divisor of a graph; 5. The spectrum and the group of automorphisms; 6. Characterization of graphs by means of spectra; 7. Spectral techniques in graph theory and combinatorics; 8. Applications in chemistry and physics; 9. Some additional results.

*Geometry and convexity: a study in mathematical methods.* By Paul J. Kelly and Max L. Weiss. John Wiley & Sons, New York, 1979. x + 261 pp. \$21.95.

This is a volume in the series Pure and Applied Mathematics, founded by Richard Courant. From the preface: "This book was written because we believe that the subject matter of basic convex body theory is ideally suited to give pregraduate students an unusual and valuable mathematical experience. The geometric theme is apparently a simple one, namely, to identify the convex bodies and surfaces of Euclidean  $n$ -space and to establish some of their basic properties. This program, from the outset, involves the student with viewpoints and methods that both generalize and unify much of his previous background. It is this eclectic aspect of the material that we feel is especially valuable. Thus, although the setting and the central questions of the book are geometric, what is stressed throughout—as the subtitle indicates—is the natural way in which the development involves the interplay of concepts and methods from topology, analysis, and linear and affine algebra.  $N$ -dimensional convex body theory has important applications in probability and statistics, combinatorial mathematics, and optimization theory." Chapter headings: 1. Introductory metric topology and Euclidean  $n$ -space; 2. The structure of Euclidean  $n$ -space; 3. Dimension and basic structure of convex bodies and surfaces; 4. The general geometry of convex bodies; 5. Convex spans and independence, related classical theorems; 6. Linear combinations of sets and the Hausdorff metric.

*The excitation and propagation of elastic waves.* By J. A. Hudson. Cambridge Monographs on Mechanics and Applied Mathematics, Cambridge University Press, New York. vi + 224 pp. \$32.50.

This book is an attempt to collect together the fundamental results of linearized elastodynamics primarily from the point of view of the propagation of transient pulses in an isotropic material which may be regarded as unbounded. Analysis of surface waves and of the reflection and refraction of plane waves proceeds in terms of harmonic time-dependence. Most of the results are what would usually be described as "classical"; however, the theory of ray path transmission of a disturbance in a solid (although known in the 19th century and described here) was first published in 1956. The book is intended to be "fundamental"—designed to enable a student to tackle (say) problems of diffraction and scattering. Chapter headings: 1. The linearized equations of motion; 2. Compressional waves and shear waves; 3. Surface and interface waves; 4. Wavefronts and ray paths; 5. The general initial-boundary-value problem; 6. Time-harmonic problems; 7. A line source in a half-space (the Lamb problem); 8. The Lamb problem with a harmonic source; 9. Linear viscoelasticity.

*Combinatorial algorithms for computers and calculators.* 2nd edition. By Albert Nijenhuis and Herbert S. Wilf. Academic Press, Inc. New York, 1978. xii + 302 pp. \$19.50.

This is the second edition of a volume in the series Computer Science and Applied Mathematics first published in 1975. Several chapters have been substantially rewritten to take account of theoretical or algorithmic improvements, and to clarify the presentation. The book is divided into two parts: 1. Combinatorial families; 2. Combinatorial structures.

*Linear algebra and its applications.* 2nd edition. By Gilbert Strang. Academic Press, New York, 1980. vii + 414 pp. \$17.95.

This is the second edition of a well-received text first published in 1976. From the preface: "I believe that the teaching of linear algebra has become too abstract. This is a sweeping judgment, and perhaps it is too sweeping to be true. But I feel certain that a text can explain the essentials of linear algebra, and develop the ability to reason mathematically, without ignoring the fact that this subject is as useful and central and applicable as calculus. It has a simplicity which is too valuable to be sacrificed." Chapter headings: 1. Gaussian elimination; 2. The theory of simultaneous linear equations; 3. Orthogonal projections and least squares; 4. Determinants; 5. Eigenvalues and eigenvectors; 6. Positive definite matrices; 7. Computations with matrices; 8. Linear programming and game theory; Appendices: A. Linear transformations, matrices, and change of basis; B. The Jordan form; C. Computer codes for linear algebra.

*Vibrations and stability of multiple parameter systems.* By K. Huseyin. Noordhoff International Publishing Co., Alphen aan den Rijn, Netherlands, 1978. x + 216 pp.

This is volume six in the Noordhoff series Mechanics of Elastic Stability. It presents a general linear theory for the free vibrations and stability of multiple-parameter systems; the immediate applications are to structural mechanics. Chapter headings: 1. Mathematical preliminaries; 2. Fundamental concepts of stability; 3. Conservative systems; 4. Pseudo-conservative systems; 5. Gyroscopic systems; 6. Circulatory systems.

*Algorithms: their complexity and efficiency.* By Lydia J. Kronsjo. John Wiley & Sons, New York, 1979. xii + 361 pp. \$41.25.

This book is concerned with the study of algorithms and evaluation of their performance. It is intended as a text for an intermediate course in computer science or computational mathematics which focuses on the basic principles and concepts of algorithms. It requires general familiarity with computers, preferably some courses on programming and introductory computer science. Each chapter is devoted to one particular class of problems and their algorithms. Chapter headings are: 1. Introduction; 2. Evaluation of polynomials; 3. Iterative processes; 4. Direct methods for solving sets of linear equations; 5. The fast Fourier transform; 6. Fast multiplication of numbers: Use of the convolution theorem; 7. Internal sorting; 8. Large-scale data processing: external sorting using magnetic tapes; 9. Searching.

*The finite difference method in partial differential equations.* By A. R. Mitchell and D. F. Griffiths. John Wiley & Sons, New York, 1980. xii + 272 pp. \$24.95.

An earlier version of this text, written by one of the present authors (A.R.M.), was published in 1969 under the title *Computational methods in partial differential equations*. This edition has been extensively revised to incorporate many of the developments of the last decade. From the preface: "This book is primarily concerned with finite difference techniques and these may be thought of as having evolved in two stages. During the fifties and early sixties many general algorithms were produced and analysed for the solution of standard partial differential equations. Since then the emphasis has shifted toward the construction of methods for particular problems having special features which defy solution by more general algorithms. This approach often necessitates a greater awareness of the different physical backgrounds of the problems such as free and moving bounding problems, shock waves, singular perturbations and many others, particularly in the field of fluid dynamics. The present volume attempts to deal with both aspects of finite difference development with due regard to non-linear as well as linear differential equations. Often the solution of the sparse linear algebraic equations which arise from finite difference approximations forms a major part of the problem and so substantial coverage is given to both direct and iterative methods including an introduction to recent work on preconditioned conjugate gradient algorithms."

*Game theory: mathematical models of conflict.* By A. J. Jones. John Wiley & Sons, Somerset, N.J., 1980. 309 pp. \$45.00.

This is a volume in the series *Mathematics and its Applications*. It is an informal introduction to game theory which can be understood by non-mathematicians. Chapter headings: 1. The name of the game; 2. Non-cooperative games; 3. Linear programming and matrix games; 4. Cooperative games; 5. Bargaining models.

*Discrete structures of computer science.* By Leon S. Levy. John Wiley & Sons, New York, 1980. 309 pp. \$17.95.

The notes on which this textbook is based have been used for a second-year, one-semester course in discrete structures of computer science at the University of Delaware. Chapter headings: 0. An essay on discrete structures; 1. Sets, functions, and relations; 2. Directed graphs; 3. Algebraic systems; 4. Formal systems; 5. Trees; 6. Programming applications.