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The Editors
regret to announce the death of

William Prager

Founder of the
Quarterly of Applied Mathematics

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)1/2}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\,\dfrac{\pi x}{2b}}{\cos\,\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Review Section.

Topics in functional analysis: essays dedicated to M. G. Krein on the occasion of his 70th birthday. (Advances in mathematics: supplementary studies series.) Edited by I. Gohberg and M. Kac. Academic Press, New York, 1978. xi + 395 pp. $41.00.

This book contains a biography of M. G. Krein and the following articles: Quantum Cesàro operators, by Louis de Branges and David Trutt; Inverse scattering, orthogonal polynomials and linear estimation, by K. M. Case; On recovering the mass distribution of a string from its spectral function, by Harry Dym and Haftali Kravitsky; Spectral analysis of families of operator polynomials and a generalized Vandermonde matrix, 1. The finite-dimensional case, by I. Gohberg, M. A. Kaashoek, and L. Rodman; Orbit structure of the Möbius transformation semigroup acting on H^∞ (broadband matching), by J. William Helton; On the asymptotic number of bound states for certain attractive potentials, by M. Kac; Generalized Krein–Levinson equations for efficient calculation of Fredholm resolvents of nondisplacement kernels, by Thomas Kailath, Lennart Ljung, and Marin Morf; Trotter's product formula for an arbitrary pair of self-adjoint contraction semigroups, by Tosio Kato; The time delay operator and a related trace formula, by Peter D. Lax and Ralph S. Phillips; Boussinesq's equation as a Hamiltonian system, by H. P. McKean; On the set of minimal extensions of a subadditive functional, by D. P. Milman; Symmetric singular integral operators with arbitrary deficiency, by Joel D. Pincus; Change of variables formulas with Cayley inner functions, by Marvin Rosenblum and James Rovnyak; The complexwave representation of the free boson field, by I. E. Segal; Families of pseudodifferential operators, by Harold Widom.

Constructive functional analysis. By D. S. Bridges. (Research Notes in Mathematics, No. 28.) Fearon–Pitman, San Francisco, 1979. 203 pp. $15.00.

The aim of this book is to lead the professional mathematician as quickly as possible (and with a minimum of logic and philosophy) to constructive proofs of major results of functional analysis and measure theory. Chapter headings: 1. The foundations of constructive mathematics; 2. Metric spaces; 3. Normed spaces and linear functionals; 4. The algebra $C(X, \mathbb{F})$; 5. Integration on a locally compact space; 6. Hilbert space and the functional calculus; Epilogue: On best approximation theory; Appendix: On constructive calculus.

Elongational flows: aspects of the behavior of model elasticoviscous fluids. (Research Notes in Mathematics, No. 29.) By C. J. S. Petrie. Fearon–Pitman, San Francisco, 1979. 254 pp. $17.50.

This work is concerned with the mathematical description of polymers. From the basic kinematics and dynamics and constitutive equations, it proceeds through theoretical results for uniaxial stretching to applications of interest to specialists in non-fluid mechanics and rheology and to engineers concerned with fibres. Table of contents: Introduction. Experimental background. Basic kinetics and dynamics. General theoretical results. Constitutive equations. Theoretical results for uniaxial stretching. Theory of spinning. Theory of plane and biaxial stretching. Applications. Bibliography.

Nonlinear analysis and mechanics: Heriot–Watt symposium vol. III. (Research Notes in Mathematics, No. 30.) Edited by R. J. Knops. Fearon–Pitman, San Francisco, 1979. 173 pp. $16.50.

This book contains three articles: Population dynamics with age dependence, by M. E. Gurtin and R. C. MacCamy; Semi-groups of linear contractions and their asymptotic behavior, by A. Pazy; Damped conservation laws in continuum mechanics, by M. Slemrod.

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Computers and intractability: a guide to the theory of NP-completeness. By Michael R. Garey and David S. Johnson. W. H. Freeman & Co., San Francisco, 1979. vii + 338 pp. $18.50 cloth; $10.00 paper.

This book is intended as a detailed guide to the theory of NP-completeness, emphasizing those concepts and techniques which seem most useful for applying the theory to practical problems. The first part, chapters 1 through 5, covers the basic theory of NP-completeness. Chapter 1 presents a relatively low-level introduction to some of the central notions of computational complexity and discusses the significance of NP-completeness in this context. Chapters 2 through 5 provide the detailed definitions and proof techniques necessary for thoroughly understanding and applying the theory. The second part, chapters 6 and 7, provides an overview of two alternative directions for further study. Chapter 6 concentrates on the search for efficient "approximation" algorithms for NP-complete problems, an area whose development has seen considerable interplay with the theory of NP-completeness. Chapter 7 surveys a large number of theoretical topics in computational complexity, many of which have arisen as a consequence of previous work on NP-completeness. The third part of the book is the appendix, which contains an extensive list (more than 300 main entries) of NP-complete and NP-hard problems. Annotations to the main entries discuss what is known about the complexity of subproblems and variants of stated problems.

Transformations: mathematical approaches to culture change. Edited by Colin Renfrew and Kenneth L. Cooke. Academic Press, New York, 1979. xxii + 515 pp. $39.50.

The editors, an archaeologist and a mathematician, have brought together here the contributions of leading anthropologists and theoretical biologists in order to rethink the application of mathematical techniques to the study of the development of human societies. The papers are grouped into six parts: 1. Mathematics and culture changes (four papers); 2. Hierarchy and social space (three papers); 3. Simulation and the measurement of change (five papers); 4. Systems and subsystem interaction (two papers); 5. The decision nexus and early economics (three papers); 6. Holistic behavior and catastrophe theory (four papers).

Relativistic quantum fields. By C. Nash. Academic Press, London, 1978. viii + 223 pp. $31.00.

This book presents several new techniques important in field theory calculations, such as the short distance expansion of Wilson, the dimensional regularization method, and the renormalization group methods, in a form comprehensible to graduate students. There are also chapters on functional integration and differentiation. Chapter headings: 1. Renormalization, functional differentiation and integration, and the Schwinger–Dyson equations; 2. Dimensional regularization and $\lambda\phi^4/4!$ theory; 3. Dimensional regularization of quantum electrodynamics; 4. The gauge and infrared properties of quantum electrodynamics; 5. Asymptotic behavior and renormalization group methods.

The H-function with applications in statistics and other disciplines. By A. M. Mathai and R. K. Saxena. John Wiley & Sons, New York, 1978. xii + 192 pp. $9.95.

This book deals with H-functions, known in the literature as generalized Mellin–Barnes functions or generalized G-functions or Fox's H-functions. All the recent developments on H-functions are given, with key results in the text and other results in the exercises at the end of each chapter. Applications of the results in statistics and other disciplines as well as functions of matrix argument are discussed. Chapter headings: 1. H-function; 2. Generalized H-function and integrals of H-function; 3. Finite and infinite series for the H-function; 4. Applications in statistics and other disciplines; 5. Special functions of matrix argument; Appendix.

Operations research support methodology (Industrial Engineering Series, vol. 2). Edited by Albert G. Holzman. Marcel Dekker, Inc., New York, 1979. ix + 664 pp. $39.75.

The topics covered in this book can be classified into four categories: mathematical foundations; mathematical methods; operations-research related concepts and solution methodology; and linguistics and behavioral concepts. The articles, written by different experts, discuss fundamentals as well as unsolved problems and research approaches to problem solutions.

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Theory of extremal problems. By A. D. Ioffe and V. M. Tihomirov, North-Holland Publishing Co., Amsterdam and New York, 1979. xii + 468 pp. $71.00.

This is volume 6 of *Studies in Mathematics and Its Applications*, translated from the Russian edition of 1974 by K. Makowski. It surveys the theory and shows how to apply it in specific situations. Among the topics treated are: Mathematical foundations (differential calculus in Banach spaces, differential equations, measurable multifunctions, convex analysis including that of integral functionals), necessary conditions for problems of various analytical natures, sufficient conditions, and existence of solutions, as well as main classes of extremal problems such as general problems of the calculus of variations and optimal control, linear and nonlinear programming, quadratic problems in Hilbert space, discrete-time, optimal control problems and others. There are numerous examples and solutions of concrete problems of various kinds (geometry, mechanics, engineering), along with more than 100 exercises, partly accompanied by solutions.

Probability theory: independence, interchangeability, martingales. By Yuan Shih Chow and Henry Teicher. Springer-Verlag, New York, 1978. ix + 455 pp. $24.80.

The concern of this book is with the measure-theoretic foundations of probability theory and the body of laws and theorems that emerge therefrom. Only the topics of independence, interchangeability and martingales are treated. Sums of independent random variables and interchangeable random variables are dealt with extensively. Particular emphasis is placed on stopping times—as tools in proving theorems and as objects of interest in themselves: randomly stopped sums, optimal stopping problems, and limit distributions of sequences of stopping rules are of special interest. Wald's equation and its second-moment analogue show the usefulness of such stopped sums in renewal theory and elsewhere in probability. Martingales provide a natural vehicle for stopping times. Many of the proofs given and a few of the results are new. Measure and probability are intertwined (as in Billingsley's book, see following notice). The book is intended to serve as a graduate text in probability theory, although no knowledge of measure and probability is presupposed—an elementary background in the latter would, however, be useful. Chapter headings: 1. Classes of sets, measures, and probability spaces; 2. Binomial random variables; 3. Independence; 4. Integration in a probability space; 5. Sums of independent random variables; 6. Measure extensions, Lebesgue-Stieltjes measure, Kolmogorov consistency theorem; 7. Conditional expectation, conditional independence, introduction to martingales; 8. Distribution functions and characteristic functions; 9. Central limit theorem; 10. Limit theorems for independent random variables; 11. Martingales; 12. Infinitely divisible laws.

Probability and measure. By Patrick Billingsley. John Wiley & Sons, New York, 1979. xiv + 515 pp. $24.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. Its aim is to interweave the two subjects so that probability motivates measure theory and measure theory generates further probability.

To introduce the idea of measure the book opens with Borel's normal number theorem, proved by calculus alone, and there follow short sections establishing the existence and fundamental properties of probability measures, including Lebesgue measure on the unit interval. For simple random variables—ones with finite range—the expected value is a sum instead of an integral. Measure theory, without integration, therefore suffices for a completely rigorous study of infinite sequences of simple random variables, and this is carried out in the remainder of chapter 1, which treats laws of large numbers, the optimality of bold play in gambling, Markov chains, large deviations, the law of the iterated logarithm. These developments in their turn motivate the general theory of measure and integration in chapters 2 and 3.

Measure and integral are used together in chapters 4 and 5 for the study of random sums, the Poisson process, queues, convergence of measures, characteristic functions, central limit theory. Chapter 6 begins with derivatives according to Lebesgue and Radon-Nikodym—a return to measure theory—then applies them to conditional expected values and martingales. Chapter 7 treats such topics in the theory of stochastic processes as Kolmogorov's existence theorem and separability, all illustrated by Brownian motion.

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Modeling, estimation, and their applications for distributed parameter systems. (Lecture Notes in Control and Information Sciences, Vol. II.) By Y. Sawaragi, T. Soeda and S. Omatu. Springer-Verlag, Berlin, 1978. vi + 269 pp.

This book treats estimation and control problems for linear stochastic distributed-parameter systems. Background material in probability theory and stochastic processes in Hilbert spaces are given in chapter 2. Chapters 3-4 are devoted to a study of optimal estimation problems for a linear stochastic distributed-parameter system and chapter 5 presents the optimal sensor location problems. Chapter 6 develops the optimal control problems for a linear stochastic distributed-parameter system.

Equations of evolution. (Monographs and studies in mathematics.) By H. Tanabe. Fearon-Pitman, San Francisco, 1979. xii + 260 pp. $42.00.

This book presents a systematic account of the theory of equations of evolution, based mainly on the results from semigroups of linear operators, as developed by Hille and Yosida. The author begins with an introduction from functional analysis which is frequently used in later chapters. This is followed in succession by a description of dissipative operators and fractional powers of operators and of the theory of the semigroups of linear operators and its applications. Subsequent chapters develop the method of solving temporally inhomogeneous equations and apply the results to the initial and mixed problems of hyperbolic and parabolic equations. Also included is a discussion of semilinear hyperbolic equations which is important in physics, as is a brief survey of the theory of monotone operators. Finally, a general indication of the optimal control of an evolution equation is given. The reader is expected to be familiar with linear operators in Banach and Hilbert spaces and distributions.

Applied time series analysis. Edited by David F. Findley. Academic Press, New York, 1976. x + 345 pp.

These are the proceedings of a symposium held at the University of Tulsa, Oklahoma, in May 1976. It featured speakers from econometrics, electrical engineering, geophysics, mathematics and statistics. The authors are H. Akaike, R. F. Engle, C. W. J. Granger, H. L. Gray, A. G. Houston, R. H. Jones, J. H. Justice, R. T. Lacoss, S. J. Laster, A. V. Oppenheim, E. Parzen, E. A. Robinson, S. Treitel, and G. S. Watson.

Stochastic abundance models with emphasis on biological communities and species diversity. (Monographs on Applied Probability and Statistics.) By S. Engen. Chapman and Hall (a Halsted Press Book), London, 1978. vi + 126 pp. $13.95.

This monograph deals with the analysis of populations of elements. Each element is a member of one and only one class, and the book is mainly concerned with populations with a large number of classes. The theory has its origin in ecology, where the elements represent individual animals or plants, while the classes are the various species of the ecological community under consideration. Some basic ideas go back to R. A. Fisher's classical contribution. The book is directed towards biometricians and statisticians but should also be of interest to ecologists, sociologists and linguists. It is divided into two parts: 1. Theoretical treatment (with chapters on sampling from a population of classes, abundance models, sample coverage, indices of diversity and equistability, and 2. Ecological applications (abundance models in ecology, with examples). Some background in probability theory and statistics is assumed.

Computing methods in applied sciences and engineering, 1977, I. Third International IRIA Symposium, 1977. Edited by R. Glowinski and J. L. Lions. Lecture Notes in Mathematics, vol. 704. Springer-Verlag, Berlin, 1979. vi + 391 pp. $17.80.

This volume is divided into six parts with papers on: Generalities (two papers); Numerical algebra and optimization (four papers); Finite elements (five papers); Time-dependent problems (two papers); Nonlinear problems, bifurcation (five papers); Homogenization (five papers); Medical applications (one paper).

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Matrices and linear programming with applications. By Toshinori Munakata. Holden-Day, San Francisco, 1979. ix + 469 pp. $18.00.

This is an elementary introduction, presupposing only highschool algebra.

A course in mathematical physics, vol. 1: classical dynamical systems. By W. Thirring. Translated by Evans M. Harrell. Springer-Verlag, New York, 1978. xii + 258 pp.

This course presents mathematical physics in its chronological order; the other one-semester lecture series are: 2. Classical field theory; 3. Quantum mechanics of atoms and molecules; 4. Quantum mechanics of large systems. Mathematical knowledge at the level of a beginning graduate student is assumed. Several additions and corrections were made in the English translation. Chapter headings: 1. Introduction; 2. Analysis on manifolds; 3. Hamiltonian systems; 4. Nonrelativistic motion; 5. Relativistic motion; 6. The structure of space and time.

Seminar on singularities of solutions of linear partial differential equations. (Annals of Mathematics Studies, No. 91.) Edited by Lars Hörmander. Princeton University Press, Princeton, 1979. ix + 283 pp. Cloth $18.50, paper $7.50.

This book consists of the notes of a seminar held at the Institute of Advanced Study in 1977–78, with some modifications and additions. There are chapters by Hörmander (two), Sjöstrand, Menikoff, Melrose, Hanges, Boutet de Monvel, Guillemin and Widom.

Singular perturbations of hyperbolic type. (Mathematical Centre Tracts, No. 98.) By R. Geel. Mathematisch Centrum, Amsterdam, 1978. xxii + 184 pp. Dft. 23.00.

This work deals with initial-value problems and initial-boundary value problems in $\mathbb{R}^2$ which are governed by a differential equation of the type $\varepsilon L_2[u] + L_1[u] = f(x, t)$, where ε is a small positive parameter, L_2 is a linear hyperbolic differential operator of the second order and L_1 is a non-vanishing, linear or quasilinear first-order differential operator. Such an equation describes the propagation of waves and occurs in many problems, for example traffic flow, the flow of water in long rivers, glacier flow, chemical exchange processes, the process of sedimentation in rivers, the motion of a vibrating string in a highly viscous medium and the propagation of radiation in a highly absorbing medium. There are six chapters: 1. Initial-value problems for linear ordinary differential equations; 2. Initial-value problems for nonlinear ordinary differential equations; 3. Initial-value problems for linear hyperbolic differential equations; 4. Initial-value problems for nonlinear hyperbolic differential equations; 5. Initial-boundary value problems for linear hyperbolic differential equations; 6. Characteristic boundary-value problems for linear hyperbolic differential equations.

A course in elasticity. (Applied Mathematical Sciences, Vol. 29.) By B. M. Fraeijs de Veubeke. Springer-Verlag, New York, 1979. xi + 330 pp. $16.80.

This book is based on lecture notes of the late Professor de Veubeke. The subject is presented at a level suitable for graduate students in engineering, physics or mathematics. The first third contains the fundamentals of the theory and the remainder is devoted to three classic applications: Saint-Venant's theory of prismatic beams, plane deformations, and the bending of plates. These are first presented and analyzed in general, then rounded out with numerous specific and sometimes novel examples.

Modelling and simulation in practice. Edited by M. Cross, R. D. Gibson, M. J. O'Carroll, and T. S. Wilkinson. John Wiley & Sons, New York, 1979. 358 pp.

These are the proceedings of "Polymodel 1," the inaugural conference of the North East Polytechnics Mathematical Modelling and Computer Simulation Group, held at Sunderland Polytechnic, 24–25th May 1978. The papers are non-mathematical and the emphasis is on the approach to and process of model building. The first session assessed models of coking, iron ore sintering, turbogenerators and renal functions; the second session discussed modelling and simulation packages, the third philosophical approaches to modelling and the final one examined various aspects of modelling and simulation in practice.