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A UNIQUENESS THEOREM FOR THE DYNAMIC INITIAL-DISPLACEMENT BOUNDARY-VALUE PROBLEM IN THE THEORY WITH INTERNAL STATE VARIABLES*

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1. Introduction. In nonlinear continuum mechanics a relatively small number of theorems on uniqueness for boundary-value and initial-boundary value problems have been proved to date. Quite recently a general uniqueness theorem for displacements in finite elastodynamics was demonstrated by Wheeler [1]. To the author's knowledge, in the dynamics of dissipative (three-dimensional) media a uniqueness theorem has been established in one paper [2]. It was done by Wheeler and Nachlinger in the case of a viscoelastic body with rate viscosity.

The purpose of this article is to present uniqueness of the solution to the displacement initial-boundary value problem appropriate to the dynamics of *dissipative bodies* described by the equations of the theory with *internal state variables*. In the literature uniqueness of that problem has not been stated. There are, however, some papers dealing with a static problem [3] or quasi-static problems at small strains (cf. the papers by Nguyen Quoc Son [4], John [5] and Nečas and Kratochvíl [6]) and with a dynamic problem of the one-dimensional theory [7].

Constitutive relations of the theory with internal state variables in the case of isothermal processes reduce to

$$\psi = \Psi^*(\mathbf{C}, \mathbf{q}), \qquad \mathbf{T}^* = 2\frac{\partial}{\partial \mathbf{C}}\Psi^*(\mathbf{C}, \mathbf{q}), \qquad \dot{\mathbf{q}} = \mathbf{a}^*(\mathbf{C}, \mathbf{q}), \tag{1.1}$$

where Ψ^* represents the free energy function per unit volume, $\mathbf{C}$ is the right Cauchy–Green strain tensor, $\mathbf{T}^*$ is the second Piola–Kirchhoff stress tensor and $\mathbf{q} = (\mathbf{q}_{(i)})$, $i = 1, 2, \ldots, k$, represents a set of internal state variables. The quantity $\dot{\mathbf{q}}$ is the material time derivative of $\mathbf{q}$. A pair $(\mathbf{C}, \mathbf{q})$ is called the *intrinsic state* and $\mathbf{a}^*$ is called the *preparation function* [8, 9].

The internal state variables may be tensors, scalars or vectors depending on the physical interpretations given for them. If they are tensors or vectors they must remain invariant with rotation of the spatial system of reference [10] but may transform with rotation of the material system [11]. For the purpose of the present paper we assume that $\mathbf{q}$ behaves as a scalar in the spatial system.

The reader interested in physical interpretations of the internal variables and examples of their applications may consult articles by Kratochvíl, Kröner, Mandel, Perzyna, Mróz, Teodosiu, and Valanis in reference [12] as well as Truesdell's monograph [13] and the literature cited there.

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2. Displacement initial-boundary value problem. Let us assume that a body occupies at time $t = 0$ a compact region $\mathscr{D}$ in three-dimensional Euclidean space. Let the boundary $\partial\mathscr{D}$ of $\mathscr{D}$ be such that the Gauss–Ostrogradski divergence theorem holds for vector fields in class $\mathscr{C}^1$ on $\mathscr{D}$. We identify a typical point of $\mathscr{D}$, denoted by $\mathbf{x}$, with a particle of the body.

Denoting the deformation gradient by $\mathbf{F}$ and the first Piola–Kirchhoff stress tensor by $\mathbf{T}$, we introduce two new functions as follows:

$$\Psi(\mathbf{F}, \mathbf{q}) := \Psi^*(\mathbf{F}^{\mathrm{T}}\mathbf{F}, \mathbf{q}), \qquad \mathbf{a}(\mathbf{F}, \mathbf{q}) := \mathbf{a}^*(\mathbf{F}^{\mathrm{T}}\mathbf{F}, \mathbf{q}).$$

In terms of these functions the *dynamic initial-displacement boundary value problem* on the interval $[0, t_f)$, where $0 < t_f \leq \infty$, is formulated by the following system of equations:

$$\mathbf{T} = \frac{\partial}{\partial \mathbf{F}} \Psi(\mathbf{F}, \mathbf{q}), \qquad \dot{\mathbf{q}} = \mathbf{a}(\mathbf{F}, \mathbf{q}),$$
$$\operatorname{Div} \mathbf{T} + \rho \mathbf{b} = \rho \ddot{\mathbf{u}}, \qquad \mathbf{T}\mathbf{F}^{\mathrm{T}} = \mathbf{F}\mathbf{T}^{\mathrm{T}} \tag{2.1}$$

and the set of the initial and boundary conditions

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{0}, \qquad \dot{\mathbf{u}}(\mathbf{x}, 0) = \mathbf{v}^0(\mathbf{x}), \qquad \mathbf{q}(\mathbf{x}, 0) = \mathbf{q}^0(\mathbf{x}), \qquad \mathbf{x} \in \mathscr{D},$$
$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}^1(\mathbf{x}, t), \qquad (\mathbf{x}, t) \in \partial\mathscr{D} \times [0, t_f). \tag{2.2}$$

In Eqs. (2.1) and (2.2) $\mathbf{u}$ is a field of the displacement vector generating the deformation $\mathbf{F}$ according to the formulae

$$\mathbf{F}(\mathbf{x}, t) - \mathbf{1} := \operatorname{Grad} \mathbf{u}(\mathbf{x}, t) = \frac{\partial}{\partial \mathbf{x}} \mathbf{u}(\mathbf{x}, t),$$
$$\det \mathbf{F}(\mathbf{x}, t) > 0, \qquad (\mathbf{x}, t) \in \mathscr{D} \times [0, t_f), \tag{2.3}$$

and $\mathbf{b}$ denotes the body force, ρ stands for the mass density at time $t = 0$; ρ is assumed to be continuous and positive on $\mathscr{D}$. The quantities $\mathbf{v}^0$, $\mathbf{u}^1$ and $\mathbf{q}^0$ are prescribed sufficiently smooth fields.

By a *solution* of this dynamic problem we mean a pair of fields $\{\mathbf{u}, \mathbf{q}\}$ defined on $\mathscr{D} \times [0, t_f)$, such that $\mathbf{u}$ is in class $\mathscr{C}^2$ and $\mathbf{q}$ is in class $\mathscr{C}^1$ in their domains, and satisfying the relations (2.1)–(2.3).

In the next parts we prove uniqueness of the solution under some smoothness assumptions concerning the functions Ψ and $\mathbf{a}$.

3. Introductory lemmas. Let $\{\mathbf{T}', \mathbf{u}'\}$ and $\{\mathbf{T}'', \mathbf{u}''\}$ be two pairs of fields satisfying the equation of motion $(2.1)_1$ in the domain $\mathscr{D}$, and $\mathbf{F}'$, $\mathbf{F}''$ be the corresponding deformation gradient tensors. Then the following integral identity holds (cf. Wheeler and Nachlinger [2]):

$$\int_{\mathscr{D}} \left\{ (\mathbf{T}'' - \mathbf{T}') \cdot (\dot{\mathbf{F}}'' - \dot{\mathbf{F}}') + \frac{\rho}{2} \frac{\partial}{\partial t} |\dot{\mathbf{u}}'' - \dot{\mathbf{u}}'|^2 \right\} dv = \int_{\partial\mathscr{D}} \{((\mathbf{T}'' - \mathbf{T}')\mathbf{n}) \cdot (\dot{\mathbf{u}}'' - \dot{\mathbf{u}}')\} \, ds. \tag{3.1}$$

Throughout this article the symbol $|\cdot|$ denotes a norm either in Euclidean vector space or in second-order or four-order tensor spaces. The dot "$\cdot$" denotes inner product in these spaces.

We formulate and prove the following.

LEMMA 1. Let the second derivative $\partial_{\mathbf{F}} \partial_{\mathbf{F}} \Psi(\mathbf{F}, \mathbf{q})$ be a continuous function of its variables $\mathbf{F}(\mathbf{x}, t)$ and $\mathbf{q}(\mathbf{x}, t)$ which are assumed to be continuous functions on $\mathscr{D} \times [t_0, t_1]$, where $0 \leq t_0 < t_1 < \infty$. Assume that there exists a positive constant c such that

$$\partial_{\mathbf{F}} \partial_{\mathbf{F}} \Psi(\mathbf{F}(\mathbf{x}, t), \mathbf{q}(\mathbf{x}, t)) \cdot (\boldsymbol{\alpha} \otimes \boldsymbol{\beta} \otimes \boldsymbol{\alpha} \otimes \boldsymbol{\beta}) \geq c |\boldsymbol{\alpha}|^2 |\boldsymbol{\beta}|^2 \tag{3.2}$$

for every $(\mathbf{x}, t) \in \mathscr{D} \times [t_0, t_1]$, and every pair $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ of vectors. Then there exist two constants $k_0 > 0$ and k_1 such that for each $t \in [t_0, t_1]$ the integral inequality

$$\int_{\mathscr{D}} \{\partial_{\mathbf{F}} \partial_{\mathbf{F}} \Psi(\mathbf{F}, \mathbf{q}) \cdot (\operatorname{Grad} \mathbf{l} \otimes \operatorname{Grad} \mathbf{l})\}(\mathbf{x}, t)\, dv \geq k_0 \int_{\mathscr{D}} |\operatorname{Grad} \mathbf{l}|^2(\mathbf{x}, t)\, dv + k_1 \int_{\mathscr{D}} |\mathbf{l}|^2(\mathbf{x}, t)\, dv \tag{3.3}$$

holds for any continuous vector field $\mathbf{l}$ on $\mathscr{D} \times [t_0, t_1]$ such that Grad $\mathbf{l}$ is continuous and $\mathbf{l} = \mathbf{0}$ on $\partial\mathscr{D} \times [t_0, t_1]$.

Proof. Under our hypothesis the fourth-order tensor

$$\mathbf{A}(\mathbf{x}, t) := \partial_{\mathbf{F}} \partial_{\mathbf{F}} \Psi(\mathbf{F}(\mathbf{x}, t), \mathbf{q}(\mathbf{x}, t)) \tag{3.4}$$

is the continuous function on $\mathscr{D} \times [t_0, t_1]$ and satisfies the inequality

$$\mathbf{A}(\mathbf{x}, t) \cdot (\boldsymbol{\alpha} \otimes \boldsymbol{\beta} \otimes \boldsymbol{\alpha} \otimes \boldsymbol{\beta}) \geq c |\boldsymbol{\alpha}|^2 |\boldsymbol{\beta}|^2 \tag{3.5}$$

uniformly in its domain. Hence, in view of Wheeler's lemma in [1] we obtain (3.3).

LEMMA 2. Let the following assumptions be satisfied:

a) the preparation function $\mathbf{a}(\mathbf{F}, \mathbf{q})$ is Lipschitz-continuous on every compact subset of its domain,

b) the free energy function $\Psi(\mathbf{F}, \mathbf{q})$ is three times continuously differentiable.

Let $\{\mathbf{u}', \mathbf{q}'\}$ and $\{\mathbf{u}'', \mathbf{q}''\}$, two solutions of the problem (formulated in Sec. 2), be identical on an interval $[0, t_0]$, for some $t_0 \geq 0$. If $t_1 \in (t_0, t_f)$, then for any two positive constants δ and γ there exists a constant $M > 0$ (which may depend on t_1 as well as on $\mathbf{u}'$, $\mathbf{u}''$, $\mathbf{q}''$ and $\mathbf{q}'$) such that for every $(\mathbf{x}, t) \in \mathscr{D} \times [t_0, t_1]$ the inequality[1]

$$\int_{t_0}^{t} \mathbf{S} \cdot \dot{\mathbf{H}}\, d\tau \geq \tfrac{1}{2} \partial_{\mathbf{F}} \partial_{\mathbf{F}} \Psi(\mathbf{F}'', \mathbf{q}'') \cdot (\mathbf{H} \otimes \mathbf{H}) - \delta^2 |\mathbf{H}|^2/2 \cdot |\mathbf{q}|^2(\gamma\delta^2 - |\partial_{\mathbf{q}} \partial_{\mathbf{F}} \Psi(\mathbf{F}'', \mathbf{q}'')|^2)/2\delta^2 - M \int_{t_0}^{t} (|\mathbf{H}|^2 + |\mathbf{q}|^2)\, d\tau \tag{3.6}$$

holds, where

$$\mathbf{S} := \mathbf{T}'' - \mathbf{T}', \qquad \mathbf{H} := \mathbf{F}'' - \mathbf{F}' = \operatorname{Grad}(\mathbf{u}'' - \mathbf{u}'), \qquad \mathbf{q} := \mathbf{q}'' - \mathbf{q}'.$$

Proof. Assumption (b) together with the continuity of both the solutions imply that the identity

$$\mathbf{S} = \partial_{\mathbf{F}} \partial_{\mathbf{F}} \Psi(\mathbf{F}'', \mathbf{q}'')[\mathbf{H}] + \partial_{\mathbf{q}} \partial_{\mathbf{F}} \Psi(\mathbf{F}'', \mathbf{q}'')[\mathbf{q}] + \mathbf{U}_F + \mathbf{U}_q \tag{3.7}$$

[1] In [7] Nachlinger and Nunziato have proved a similar inequality in one-dimensional theory.

holds and that there exist two positive constants N_1 and N_2 such that

$$|\mathbf{U}_F(\mathbf{F}'', \mathbf{q}''; \mathbf{F}', \mathbf{q}')| \leq N_1(|\mathbf{H}|^2 + |\mathbf{q}|\,|\mathbf{H}|),$$
$$|\mathbf{U}_q(\mathbf{F}'', \mathbf{q}''; \mathbf{F}', \mathbf{q}')| \leq N_2(|\mathbf{H}|\,|\mathbf{q}| + |\mathbf{q}|^2)$$

on $\mathscr{D} \times [t_0, t_1]$. The last two terms in Eq. (3.7) give the remainder in the Taylor expansion of the stress function. Taking the inner product of (3.7) with $\dot{\mathbf{H}}$, we get an expression in which the following two terms occur:

$$\mathbf{A}'' \cdot (\dot{\mathbf{H}} \otimes \mathbf{H}) := \partial_{\mathbf{F}}\, \partial_{\mathbf{F}} \Psi(\mathbf{F}'', \mathbf{q}'') \cdot (\dot{\mathbf{H}} \otimes \mathbf{H});\ \mathbf{B}'' \cdot (\dot{\mathbf{H}} \otimes \mathbf{q}) := \partial_{\mathbf{q}}\, \partial_{\mathbf{F}} \Psi(\mathbf{F}'', \mathbf{q}'') \cdot (\dot{\mathbf{H}} \otimes \mathbf{q}).$$

We have also the following identities:

$$\mathbf{A}'' \cdot (\dot{\mathbf{H}} \otimes \mathbf{H}) = \frac{1}{2}\frac{\partial}{\partial t}(\mathbf{A}'' \cdot (\mathbf{H} \otimes \mathbf{H})) - \frac{1}{2}\frac{\partial \mathbf{A}''}{\partial t} \cdot (\mathbf{H} \otimes \mathbf{H}),$$

$$\mathbf{B}'' \cdot (\dot{\mathbf{H}} \otimes \mathbf{q}) = \frac{\partial}{\partial t}(\mathbf{B}'' \cdot (\mathbf{H} \otimes \mathbf{q})) - \frac{\partial \mathbf{B}''}{\partial t} \cdot (\mathbf{H} \otimes \mathbf{q}) - \mathbf{B}'' \cdot (\mathbf{H} \otimes \dot{\mathbf{q}}).$$

Using both the Schwartz inequality in a tensor space and the inequality $a \geq -|a|$, we obtain

$$\mathbf{S} \cdot \dot{\mathbf{H}} \geq \frac{\partial}{\partial t}\{\mathbf{A}'' \cdot (\mathbf{H} \otimes \mathbf{H})/2 + \mathbf{B}'' \cdot (\mathbf{H} \otimes \mathbf{q})\} - \mathbf{B}'' \cdot (\mathbf{H} \otimes \dot{\mathbf{q}})$$
$$- \frac{\partial \mathbf{A}''}{\partial t} \cdot (\mathbf{H} \otimes \mathbf{H})/2 - \frac{\partial \mathbf{B}''}{\partial t} \cdot (\mathbf{H} \otimes \mathbf{q}) - |\mathbf{U}_F|\,|\dot{\mathbf{H}}| - |\mathbf{U}_q|\,|\dot{\mathbf{H}}|. \quad (3.8)$$

Assumption (a) together with the continuity of both the solutions on compact domains imply the existence of a Lipschitz constant L of the set $\mathscr{D} \times [t_0, t_1]$ such that

$$|\dot{\mathbf{q}}| = |\mathbf{a}(\mathbf{F}'', \mathbf{q}'') - \mathbf{a}(\mathbf{F}', \mathbf{q}')| \leq L(|\mathbf{H}| + |\mathbf{q}|). \quad (3.9)$$

By virtue of the smoothness of $\mathbf{H}$ and $\mathbf{q}$ and assumption (b) there exist the positive constants

$$2C_1 = \max\left|\frac{\partial \mathbf{A}''}{\partial t}\right|, \qquad C_2 = \max\left|\frac{\partial \mathbf{B}''}{\partial t}\right|$$
$$C_0 = \max|\mathbf{B}''|, \qquad B_0 = \max|\dot{\mathbf{H}}|, \qquad \text{on } \mathscr{D} \times [t_0, t_1].$$

Application of the inequality

$$ab \leq a^2/2 + b^2/2 \quad (3.10)$$

to the right-hand side of (3.8) yields

$$\mathbf{S} \cdot \dot{\mathbf{H}} \geq \frac{\partial}{\partial t}\{\mathbf{A}'' \cdot (\mathbf{H} \otimes \mathbf{H})/2 + \mathbf{B}'' \cdot (\mathbf{H} \otimes \mathbf{q})\} - M_1(|\mathbf{H}|^2 + |\mathbf{q}|^2)/2, \quad (3.11)$$

where

$$M_1 := 3(C_0 L + B_0 P_2 + P_1), \qquad P_1 = \max(2C_1/3, C_2/3), \qquad P_2 = \max(N_1, N_2).$$

Using (3.9) and (3.10), we may write the inequalities

$$\frac{\gamma}{2}\frac{\partial}{\partial t}|\mathbf{q}|^2 \le \gamma|\dot{\mathbf{q}}|\,|\mathbf{q}| \le 3\gamma L(|\mathbf{H}|^2 + |\mathbf{q}|^2)/2$$

$$|\mathbf{B}''\cdot(\mathbf{H}\otimes\mathbf{q})| \le |\mathbf{B}''|\,|\mathbf{H}|\,|\mathbf{q}| \le |\mathbf{B}''|^2|\mathbf{q}|^2/2\delta^2 + \delta^2|\mathbf{H}|^2/2 \tag{3.12}$$

for any $\delta > 0$, $\gamma > 0$. Since both the solutions are equal at t_0, then integrating (3.11) over (t_0, t) and using (3.12) we obtain (3.6) with $2M = M_1 + 3\gamma L$, which completes the proof.

4. Uniqueness theorem. The last result is necessary in the proof of the following main

THEOREM. Let the hypotheses of Lemma 1 and assumptions (a) and (b) of Lemma 2 be satisfied. Then the solution of the dynamic problem (2.1)–(2.3) is unique.

Proof. Assume that the theorem is not true; it means that there exist two solutions $\{\mathbf{u}', \mathbf{q}'\}$ and $\{\mathbf{u}'', \mathbf{q}''\}$ and there exists a compact interval $[t_0, t_1] \subset [0, t_f)$ on which the solutions are distinct. As a point t_0 we choose the number

$$t_0 = \min\{\sup\{0 \le \tau < t_f\colon h = 0 \text{ on } [0, \tau]\}, \sup\{0 \le s < t_f\colon g = 0 \text{ on } [0, s]\}\}. \tag{4.1}$$

The functions h and g in (4.1) are given by

$$h(t) := \int_{\mathscr{D}} |\mathbf{H}|^2(\mathbf{x}, t)\,dv, \qquad g(t) := \int_{\mathscr{D}} |\mathbf{q}|^2(\mathbf{x}, t)\,dv,$$

where $\mathbf{H} := \mathrm{Grad}(\mathbf{u}'' - \mathbf{u}')$, $\mathbf{q} := \mathbf{q}'' - \mathbf{q}'$.

The existence of t_0 follows from the fact that $h(0) = 0$, $g(0) = 0$. Furthermore, since the field $\mathbf{u} := \mathbf{u}'' - \mathbf{u}' = \mathbf{0}$ on $\partial\mathscr{D} \times [0, t_f)$, the relation (3.1) reduces to

$$\int_{\mathscr{D}}\left\{\frac{\rho}{2}|\dot{\mathbf{u}}|^2(\mathbf{x}, t) + \int_{t_0}^{t}(\mathbf{S}\cdot\dot{\mathbf{H}})(\mathbf{x}, \tau)\,d\tau\right\}dv = 0, \qquad t \in [t_0, t_f). \tag{4.2}$$

Application of the result of Lemma 2 to the inequality (4.2) yields

$$\int_{\mathscr{D}}\{\rho|\dot{\mathbf{u}}|^2 + \mathbf{A}''\cdot(\mathbf{H}\otimes\mathbf{H}) - \delta^2|\mathbf{H}|^2 + (\gamma\delta^2 - |\mathbf{B}''|^2)|\mathbf{q}|^2/\delta^2\}(\mathbf{x}, t)\,dv$$
$$\le 2M\int_{\mathscr{D}}\int_{t_0}^{t}(|\mathbf{H}|^2 + |\mathbf{q}|^2)(\mathbf{x}, \tau)\,d\tau\,dv, \qquad t \in [t_0, t_1]. \tag{4.3}$$

Using Lemma 1 we obtain

$$m_0 k(t) + k_0^* h(t) + m_1 g(t) \le 2M\int_{t_0}^{t}(h(\tau) + g(\tau))\,d\tau + |k_1|\int_{t_0}^{t}\int_{\mathscr{D}}|\mathbf{u}|^2(\mathbf{x}, \tau)\,d\tau\,dv,$$

where

$$m_0 = \min_{\mathscr{D}}\rho(x), \qquad k_0^* := k_0 - \delta^2, \qquad m_1 := \gamma - (C_0/\delta)^2, \qquad k(t) := \int_{\mathscr{D}}|\dot{\mathbf{u}}|^2(\mathbf{x}, t)\,dv.$$

By the Hölder inequality, and because $\mathbf{u}(\cdot, t_0) = \mathbf{0}$,

$$|\mathbf{u}|^2(\mathbf{x}, t) = \left|\int_{t_0}^{t}\dot{\mathbf{u}}(\mathbf{x}, \tau)\,d\tau\right|^2 \le (t - t_0)\int_{t_0}^{t}|\dot{\mathbf{u}}|^2(\mathbf{x}, \tau)\,d\tau,\ t \in [t_0, t_1]. \tag{4.4}$$

Hence (4.3), (4.4) furnish

$$m_0 k(t) + k_0^* h(t) + m_1 g(t) \leq M^* \int_{t_0}^{t_1} (k(\tau) + h(\tau) + g(\tau))\, d\tau, \tag{4.5}$$

with $M^* := \max\{2M, |k_1|(t_1 - t_0)\}$. Consequently the number

$$M_0 := M^*\{\min(m_0, k_0^*, m_1)\}^{-1}$$

is positive whenever

$$\delta^2 < k_0 \quad \text{and} \quad \delta^2 \gamma > C_0^2,$$

Now the number M_0 transforms (4.5) to the following Gronwall-type inequality:

$$k(t) + h(t) + g(t) \leq M_0 \int_{t_0}^{t} \{k(\tau) + h(\tau) + g(\tau)\}\, d\tau$$

on $[t_0, t_1]$, implying

$$k = 0,\ h = 0,\ g = 0 \text{ on } [t_0, t_1],$$

which contradicts our assumptions made in the beginning. The proof is now complete.

It should be pointed out that our uniqueness theorem holds under relatively weak assumptions. In fact, the inequality (3.3) corresponds to Wheeler's uniqueness condition of elastodynamics and assumption (a) ensures the uniqueness of solution of the evolution equation $(2.1)_2$. However, one can expect that these assumptions are insufficient for uniqueness of solution of initial-mixed boundary value problems.

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