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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime (′), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Review Section.

Lecture Notes in Biomathematics, Springer-Verlag, Boston, Heidelberg, New York.

Volume 16: *Stochastic models for spike trains of single neurons.* By G. Sampath and S. K. Srinivasan, 1977. 188 pp.

These notes are aimed at collecting in one volume the vast and scattered literature on stochastic models of spontaneous activity in single neurons. An attempt has been made to make the treatment self-contained by providing an introduction to neurophysiology as well as the mathematical background for each kind of model. The mathematical aspects of a model are stated as a series of lemmas and theorems; this gives the relative importance of the different results and also facilitates easy reference. The proofs are often sketchy and sometimes omitted, to make the notes compact and easier to read. There are ten chapters: 1. Some basic neurophysiology; 2. Signals in the nervous system; 3. Stochastic modelling of a single neuron spike trains; 4. Superposition models; 5. Deletion models; 6. Diffusion models; 7. Counter models; 8. Discrete state models; 9. Continuous state models; 10. Real neurons and mathematical models.

Volume 17: *Stochastic problems in population genetics.* By Takeo Maruyama, 1977. 245 pp.

These are notes based on courses in theoretical population genetics given at the University of Texas at Houston during the winter quarter, 1974, and at the University of Wisconsin during the fall semester, 1976. They explore problems of population genetics and evolution involving stochastic processes. Biological models and various mathematical techniques are discussed. Special emphasis is given to the diffusion method and an attempt is made to emphasize the underlying unity of various problems based on the Kolmogorov backward equation. A particular effort was made to make the subject accessible to biology students who are not familiar with stochastic processes. There are twelve chapters: 1. Orientation; 2. Population genetics models; 3. Classification of boundaries; 4. Expectation of integration along sample paths; 5. Modification of processes; 6. Numerical integration of the Kolmogorov backward equation; 7. Eigenvalues and eigenvectors of the KBE; 8. Approximation methods; 9. Geographical structure of populations; 10. Geographically invariant properties; 11. Gene frequency distributions and random drift in geographically structured population; 12. Some special problems.

Volume 18: *Mathematics and the life sciences.* Edited by David E. Matthews, 1977. 385 pp.

These are the texts of some of the lectures at a seminar held at the Canadian Mathematical Congress in August 1975. They are: Problems of statistical inference in the life sciences (G. A. Barnard); Bioassay (D. J. Finney); Quantitative analysis of complex systems (R. Levins); Mathematical models in population biology (J. Maynard Smith); The generation and recognition of patterns in biological systems (R. Rosen); Catastrophe theory and the modelling of biological systems (A. E. R. Woodcock).

Volume 19: *Measuring selection in natural populations.* Edited by F. B. Christiansen and T. M. Fenchel, 1977. 564 pp.

The present volume constitutes the proceedings of the symposium: "Measuring Selection in Natural Populations", held in memory of the late Professor Ove Frydenberg. The symposium took place in Sandbjerg Manor House in Southern Jutland on May 10-14, 1976. The purpose of the symposium was to reflect contemporary research on the mechanisms of biological evolution.

The lectures are grouped into five parts: 1. Study of selection (9 papers); 2. Study of polymorphism (9 papers); 3. Sex and evolution (5 papers); 4. Ecology and evolution (6 papers); 5. Human evolution (3 papers).

Mathematical techniques for biology and medicine. By William Simon. The MIT Press, 1977. xii + 291 pp. $9.95.

This is a paperback edition of the book originally published in 1972 by Academic Press as *Mathematical techniques for physiology and medicine.* The eleven chapters are 1. Review of differential calculus; 2. How differential equations arise; 3. Guessing the solution of a differential equation; 4. The Laplace transform; 5. Compartmental problems; 6. Numerical methods; 7. Regulation and oscillation; 8. Diffusion; 9. The theory of blood flow measurement; 10. Curve fitting; 11. Tracer experiments (added for this edition).

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Statistical survey techniques. By Raymond J. Jessen. John Wiley & Sons, Inc., 1978. vii + 520 pp. $24.95.

This volume in the Wiley series in Probability and Mathematical Statistics describes techniques basic to the advances in sampling and other survey methodology which have taken place in such fields as public opinion measurement, sociology, political science, economics, business, biology, engineering, urgan planning and measurement. The chapter headings are as follows: 1. The statistical survey; 2. Elements of random sampling; 3. Why variance?; 4. Choice of sampling unit; 5. Estimation; 6. Building sampling frames; area sampling; 7. Stratification; 8. Elements of probability sampling; 9. Subsampling (multistage); 10. Double sampling (multiphase); 11. Lattice sampling (multistratification); 12. Miscellaneous survey techniques; 13. Coverage, canvassing, and measurement; 14. Analysis and presentation.

Mathematics, statistics, and systems for health. By Norman T. J. Bailey. John Wiley & Sons, Inc. 1978. x + 222 pp. $15.50.

This book, a volume in the Wiley series in Probability and Mathematical Statistics, is primarily concerned to make available to a wide reading public a greater understanding of the role of quantitative methods in helping to develop and improve medical care and health services throughout the world. Special attention is paid to the significance and potentialities of mathematics, statistics, modelling, computers, operational research, systems analysis, and system dynamics. It keeps purely mathematical aspects to a minimum and is more concerned with general notions and principles than with mathematical details. The chapter headings are: 1. Human health problems; 2. The scientific approach; 3. The necessity of mathematics; 4. The role of probability and statistics; 5. The art of modelling; 6. The use and abuse of computers; 7. Operational research; 8. Systems analysis and system dynamics; 9. Teams, projects, and organizations; 10. Future prospects.

Ordinary differential equations. By Tyn Myint-U. Elsevier North-Holland, New York, 1977. xii + 295 pp. $18.50.

This text is designed for students with a good background in calculus and a knowledge of matrices. The basic concepts and definitions of ordinary differential equations as well as mathematical models in population dynamics and celestial mechanics are presented in the first chapter. First-order equations with an emphasis on Picard's existence theorem are treated in chapter 2. The third chapter is concerned with classical methods for determining solutions of second-order linear equations together with the qualitative behavior of solutions of some types of equations. In chapter 4, equations with analytic coefficients are treated with Legendre and Bessel equations presented as examples of practical importance. Chapter 5 contains an analysis of systems of equations using matrices. Chapters 6 and 7 deal with boundary-value and eigenvalue problems, respectively. Stability of autonomous systems and the method of Lyapunov are the central themes of chapter 8. Chapter 9 describes the Laplace transform and its applications. Numerical solutions and error analyses are the main topics in the final chapter.

Simple-periodic and non-periodic Lamé functions. By J. K. M. Jansen (Mathematical Centre Tracts, No. 72). Stichting Mathematisch Centrum, Amsterdam, 1977. vi + 104 pp. Dfl. 13.—.

This monograph is a result of antenna research carried out at the Technological University Eindhoven. In particular, it investigates the electromagnetic field inside a conical horn with an elliptical cross-section and an arbitrary flare angle, bounded by a perfectly conducting surface. The Helmholtz equation is solved in an appropriate coordinate system and Algol 60 procedures are available from the author for calculating the periodic and non-periodic solutions of the Lamé equations which are needed.

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Continuation methods. Edited by Hansjorg Wacker. Academic Press, New York, 1978. ix + 336 pages. $19.50.

This volume contains the proceedings of a symposium held at Johannes Kepler University in Linz, Austria, in October 1977. The aim of the symposium was to bring together people who have contributed to the development of continuation methods in various fields. The mathematical disciplines represented range from topology to numerical analysis.

An introduction to numerical analysis. By Kendall E. Atkinson. John Wiley & Sons, New York, 1978. xiii + 587 pages. $19.95.

This introduction is written for students in mathematics, the physical sciences and engineering, at the upper undergraduate to beginning graduate level, assuming a knowledge of elementary calculus, linear algebra and an introduction to differential equations. It is suitable for a one-year course. The chapter headings are: 1. The sources and propagation of errors; 2. Rootfinding for nonlinear equations; 3. Interpolation theory; 4. Approximation of functions; 5. Numerical integration; 6. Numerical methods for differential equations; 7. Linear algebra; 8. Numerical solution of systems of linear equations; 9. The matrix eigenvalue problem.

Mathematics for econometrics. By Phoebus J. Dhrymes. Springer-Verlag, New York, 1978. viii + 136 pages. $7.90.

This booklet contains a collection of mathematical results employed in the author's text *Introductory econometrics,* but will also be useful as a reference to students of econometrics irrespective of the text employed. Chapter headings are: 1. Vectors and vector spaces; 2. Matrix algebra; 3. Linear systems of equations and generalized inverses of matrices; 4. Vectorization of matrices and matrix functions: Matrix differentiation; 5. Systems of difference equations with constant coefficients.

Numerical methods in laminar and turbulent flows. Edited by C. Taylor, K. Morgan and C. A. Brebbia. A Halsted Press Book New York, 1978. $55.00.

These are the proceedings of the First International Conference on Numerical Methods in Laminar and Turbulent Flow, held at Swansea, Wales, in July 1978. The papers are grouped into nine sections, as follows: 1. General viscous flow; 2. Turbulent flow; 3. Boundary layer analysis; 4. Flow with heat transfer; 5. Free surface flows and lubrication; 6. Turbomachinery and airfoil flow; 7. Two phase flow and meteorology; 8. Mass transport and convection; 9. Numerical and mathematical concepts.

Methods in electromagnetic wave propagation. By D. S. Jones. Clarendon Press, Oxford, 1979. xiii + 887 pages. $59.00.

This book aims to describe some of the recent developments in the field of electromagnetism, including not only those achieved by traditional methods but also from applications of functional analysis, and numerical techniques. No background is assumed in the latter fields. The first chapter describes various parts of numerical analysis used in the remainder of the book. Chapters 2, 3 and 4 have as their main topic the propagation in waveguides: waveguides and difference equations, operators and eigenvalues, variational methods and optimization. Treatments of errors analysis of variational methods, associated integral equations and finite elements comprise chapter 5. The radiation and scattering of waves by wire and solid antennas is discussed in chapter 7: harmonic waves in chapter 7 and arbitrary time variation in chapter 8, the inverse problem of finding the source from variation in chapter 8. The inverse problem of finding the source from a given radiation pattern is studied in chapter 9, as are the related questions of holography and adaptive arrays. There are numerous exercises dispersed throughout the text.

Infinite loop spaces. By J. F. Adams. Princeton University Press. New Jersey, 1978. x + 214 pages. $14.00 cloth; $5.50 paper.

This is volume 90 of the Annals of Mathematics and derives from a series of the Hermann Weyl Lectures given at the Institute of Advanced Study in 1975. These lectures have as their aim to provide broad surveys of various topics in mathematics, accessible to nonspecialists. There is a bibliography of 156 items.