QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXXVI JANUARY • 1979 NUMBER 4

QUARTERLY OF APPLIED MATHEMATICS

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Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia
Publication number 808680

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXXVI 1978/9

Printed by the
WILLIAM BYRD PRESS, INC.
Richmond, Virginia

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BOOK REVIEWS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime (′), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Review Section.

Characterizations of the normal probability law. By A. M. Mathai and G. Pederzoli. John Wiley & Sons, New York, London, Sydney, Toronto, 1978. x + 149 pp. $8.50.

This book deals mainly with the characterization of the normal probability law, i.e. with the investigation of the unique properties enjoyed by that distribution. The material is developed from the very elementary level to the research level. Under each topic a typical theorem is discussed in detail and the remaining material is presented in the form of exercises at the end of the chapter.

Chapter 1 gives a general introduction and a quick review of some elementary topics mainly for the purpose of recalling some technical terms which appear very often in later chapters.

Chapter 2 also contains elementary material. The purpose of this chapter is to motivate the reader by showing that the normal probability law is a natural aftermath in some practical situations. Topics discussed are the hypotheses of Herschel, Maxwell, and Hagen connected with problems in the physical sciences and the maximization of entropy.

Chapters 3 to 8 are short chapters dealing with the structural setup of conditional distributions, characterizations through independence of linear and quadratic statistics, and characterizations with the help of differential equations, functional equations, etc.

Chapter 9 looks into the possible applications of the various characterization theorems and gives an illustration by applying the theory to the problem of reducing composite statistical hypotheses into equivalent simple ones.

While chapters 1 to 9 deal with univariate normal law, chapter 10 considers characterization results on the multivariate normal law.

Vector and tensor analysis (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Volume 48). By Eutiquio C. Young. Marcel Dekker, Inc., 1978. 544 pp. $37.50. (A special adoption price of $24.50 is available on this title when 5 or more copies are ordered.)

This textbook is designed to provide an introduction to the fundamental concepts of vector and tensor analysis together with their corresponding physical and geometric applications. Throughout the book emphasis has been placed on the development of intuitive understanding. The development of basic techniques and computational skills is stressed, while highly complex proofs are deliberately deemphasized. The book contains a discussion of the various definitions of a vector, and tensors are treated fully and in depth. The definitions of line and surface integral are motivated by geometric and/or physical concepts; the various integral theorems are stated in simple geometric settings and illustrated by physical examples. Separate treatments are devoted to cartesian and general tensors, making the transition from vectors to tensors a smooth and natural one. There are a large number of examples drawn from the physical sciences, and each chapter contains exercises of varying degrees of difficulty.

Lectures on nonlinear-differential-equation models in biology. By J. D. Murray. Clarendon Press, Oxford, 1977. xiii + 370 pp. $24.50.

This book offers a selection of topics from the general area of nonlinear differential equation models in biology, chosen such that both the mathematics and the applications are non-trivial and interesting. The author attempts to show how, by way of example, a model of a real biological situation evolves, and how the mathematics can contribute positively in both design and interpretation of experiment and in basic understanding. The five chapter headings are: 1. Enzyme kinetics; 2. Facilitated diffusion; 3. Reduction of dimensionality in diffusion processes: antenna receptors of moths; 4. Biological oscillators I: homogeneous temporal oscillations; 5. Biological oscillators II: spatial structure and non-linear wave phenomena. There are also five mathematical appendices: A1. Singular perturbation theory: matched expansion procedures; A2. Boundary conditions and facilitated diffusion: mathematical analysis; A3. Linear diffusion equation: special solutions; A4. Hopf bifurcation theorem and limit cycles; A5. Some mathematical results for reaction-diffusion systems.

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Forecasting methods and applications. By Spyros Makridakis and Steven C. Wheelwright. John Wiley & Sons, Santa Barbara, New York, Chichester, Brisbane, Toronto, 1978. xvi + 713 pp. $19.95.

This book is addressed to management students and presents alternative forecasting methods in such a way that a minimum of technical background is required to understand each technique. It is divided into six parts: 1. Background and perspective; 2. Smoothing and decomposition time-series methods; 3. Regression methods; 4. Autoregressive/moving average (ARMA) time-series methods; 5. Qualitative and technological methods; 6. Integrating forecasting and planning in the organization.

An introduction to linear programming and matrix game theory. By M. J. Fryer. John Wiley & Sons, Inc., New York. vi + 120 pp. $7.95.

This book is addressed to students with only a very basic (manipulative) understanding of the algebra of simultaneous linear equations and the geometry of the line and plane.

The Minkowski multidimensional problem. By A. V. Pogorelov. John Wiley & Sons, Inc., V. H. Winston & Sons, Washington, D.C., 1978. v + 106 pp. $13.75.

The Minkowski problem is that of finding a closed convex surface in R^3 whose Gaussian curvature is a given positive function of the exterior unit normal. In addition to the regular solution of the problem, the book covers a number of related questions in geometry and partial differential equation theory.

Wave propagation and underwater acoustics (Lecture Notes in Physics, vol. 70). Edited by Joseph B. Keller and John S. Papadakis. Springer-Verlag, 1977. viii + 287 pp. $11.50.

This book contains the texts of five survey papers presented at a workshop on the subject sponsored by ONR and held in Mystic, Conn., in November 1974. The papers are: 1. Survey of wave propagation and underwater acoustics (Joseph B. Keller); 2. Exact and asymptotic representations of the sound field in a stratified ocean (Daljit S. Ahluwalia and Joseph B. Keller); 3. Horizontal rays and vertical modes (Robert Burridge and Henry Weinberg); 4. Wave propagation in a randomly inhomogeneous ocean (Werner Kohler and George C. Papanicolaou); and 5. The parabolic approximation method (Fred D. Tappert).

Problems of stellar convection (Lecture Notes in Physics, vol. 71). Edited by E. A. Spiegel and J. P. Zahn. Springer-Verlag, 1977. vii + 363 pp. $14.30.

This volume constitutes the proceedings of colloquium no. 38 of the International Astronomical Union, held at the Nice Observatory in August, 1976. The papers are divided into nine groups: 1. Mixing-length theory; 2. Linear theory; 3. Observational aspects; 4. Numerical solutions; 5. Rotation and magnetic fields; 6. Penetration; 7. Special topics; 8. Waves; 9. Turbulence, and there is an appendix containing a paper by D. O. Gough on stellar convection.

Introduction to modern algebra (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Volume 47). By Marvin Marcus. Marcel Dekker, Inc., 1978. 512 pp. $19.50.

This is a textbook for a basic one-year course in algebra at the advanced undergraduate or beginning graduate levels. The presentation is oriented towards the applications of algebra to other branches of mathematics and to science in general. This point of view is reflected in the choice of topics, in the selection of constructive methods of proof and in the space devoted to current applications. Thus modules over a principal ideal domain are studied via elementary operations on matrices. Considerable space is devoted to such topics as permutation groups and the Polya counting theory, polynomial theory, canonical forms for matrices, applications of linear algebra to differential equations, and representations of groups. There are 390 exercises which constitute an integral part of the book. Problems requiring intricate arguments are accompanied by complete solutions. The book can be used by upper-division mathematics majors or by graduate students.

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Moving boundary problems. Edited by D. G. Wilson, Alan D. Solomon and Paul T. Boggs. Academic Press, Inc., New York, 1978. x + 329 pp. $15.00.

This volume contains the proceedings of a symposium sponsored by the Army Research Office in Gatlinburg, Tennessee, in September 1977. It is divided into three parts: Theory papers (by J. R. Cannon; Avner Friedman; Bernard A. Fleishman, Ross Gingrich, and Thomas J. Mahar; David Kinderlehrer and Louis Nirenberg); Methods papers (by Gunter H. Meyer; Colin W. Cryer; George J. Fix; J. R. Ockendon; J. T. Oden and N. Kikuchi; N. Shamsundar; A. D. Solomon); Applications papers (by Bruno A. Boley; R. W. Dutton and D. A. Antoniadis; Michael F. Malone and Gabriel Horvay; J. A. Wheeler; Yin-Chao Yen).

Locally solid Riesz spaces. By C. D. Aliprantis and O. Burkinshaw. Academic Press, New York, 1978. xii + 198 pp.

This book is divided into seven chapters: 1. The lattice structure of Riesz spaces; 2. Locally solid topologies; 3. Lebesgue and pre-Lebesgue topologies; 4. Fatou topologies; 5. Metrizable locally solid Riesz spaces; 6. Locally convex-solid Riesz spaces; 7. Laterally complete Riesz spaces.

This is the first book completely devoted to topological Riesz spaces, a subject going back to Riesz' 1928 address at the International Congress in Bologna; it presents a unified approach to the subject of its title and emphasizes the relationship between the order structure and topological structure.

Optimal control and differential equations. Edited by A. B. Schwarzkopf, Walter G. Kelley and Stanley B. Eliason. Academic Press, Inc., New York, 1978. xi + 335 pp. $17.00.

The articles in this volume were presented at a conference of the same title at Norman, Oklahoma, in March 1977, on the occasion of the retirement of W. T. Reid and George M. Ewing from the faculty of the University of Oklahoma. The section on optimal control has papers by McShane, Komkov, Ewing, Berkovitz, Warga, Jacobs and Pickel, and Hestenes, and that on differential equations by W. T. Reid, Pitcher, Etgen and Lewis, Redheffer, Hartman, Muldowney and Seifert.

Tensor geometry: the geometric viewpoint and its uses (Surveys and Reference Works in Mathematics, No. 1). By C. T. J. Dodson and T. Poston. Fearon-Pitman Publishers, Inc., Belmont, California, 1978. xiii + 598 pp. $38.50.

This treatment of differential geometry and the mathematics required for general relativity makes the subject matter accessible to anyone familiar with elementary calculus in one variable and with a knowledge of some vector algebra. The emphasis throughout is on the geometry of the mathematics, which is enhanced by the many illustrations presenting figures of three and more dimensions as closely as book form will allow.

There are twelve chapters: 1. Real vector spaces; 2. Affine spaces; 3. Dual spaces; 4. Metric vector spaces; 5. Tensors and multilinear forms; 6. Topological vector spaces; 7. Differentiation and manifolds; 8. Connections and covariant differentiation; 9. Geodesics; 10. Curvature; 11. Special relativity; 12. General relativity.

Managerial applications of system dynamics. Edited by Edward B. Roberts. The MIT Press, Cambridge, Mass., 1978. xiii + 669 pp. $40.00.

This book provides an overview of past and continuing applications of systems dynamics philosophy and methodology (started by Jay W. Forrester in 1957) to managerial issues, especially in corporations. It is divided into six parts. The first part introduces the concepts, philosophy, and methodology of system dynamics. The remaining five parts deal with corporate functional areas of application, namely, manufacturing, marketing and distribution, research and development, and finance and control, with the final section treating societal problems. Each of the above six parts begins with an overview of the papers contained in that part and provides a summary of the circumstances leading up to the applications. These overviews reference related applications not included in this book, as well as some applications which are not documented at all in the public literature. Many of the papers provide complete descriptions of the modeling processes and results. In some cases model documentations are within the papers, in others they are included as appendices at the end of the book. In all, 36 chapters provide in-depth descriptions of over 25 applications to business and industrial situations and several applications to societal problems. Nine models are fully documented by listings in the chapters and the four appendixes.

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The collected papers of Paul Samuelson, vol. IV. Edited by Hiroaki Nagatani and Kate Crowley. The MIT Press, Cambridge, Mass., 1978. xii + 977 pp. $35.00.

This is volume IV of the Collected Papers. The 85 papers are divided into ten parts (the total number of papers in the four volumes is 292): 1. Theory of consumption and production; 2. Theory of capital and growth; 3. On Marxian economics; 4. Mathematical biology and economics of population; 5. Stochastic theory of economics; 6. International economics; 7. Welfare economics; 8. Theory of money and inflation; 9. Lectures and essays on current economic problems; 10. Essays on the evolution of economics.

Introduction to college mathematics with a programming language. By E. J. LeCuyer. Springer-Verlag, New York, Heidelberg, Berlin, 1978. xii + 420 pp. $14.80.

This is an introduction to college mathematics for liberal arts students with the unique and distinguishing feature that the developments are presented in both conventional mathematical notation and in Kenneth Iverson's APL notation in parallel throughout the text. Thus, the power of the APL notation is used to reinforce understanding of the mathematics, and, if there is access to an appropriate computer terminal, many of the processes can be immediately executed. This is thus not the usual programming handbook but a remarkable effort at showing how APL, the most concise, versatile and powerful programming language yet invented, can be used as an integral part of the educational process. There are twelve chapters (an introduction to APL is included as an appendix): 1. Set theory; 2. Logic; 3. Vectors and matrices; 4. Systems of linear equations; 5. Determinants; 6. Functions and graphing; 7. Exponential and logarithmic functions; 8. Differential calculus; 9. Integral calculus; 10. Probability; 11. Statistics; 12. The trigonometric functions.

Correlational procedures for research. By Robert M. Thorndike. John Wiley & Sons, Inc., New York, 1978. xi + 340 pp. $21.50.

This book covers that area of statistics dealing with measures of the relationships among variables. Topics include a detailed and extensive development of bivariate correlation, special correlation indices, multiple correlation, canonical correlation, discriminant function, and factor analysis. The focus of the discussion is on a conceptual understanding of the procedures, usually in geometric terms, rather than on the development of the mathematical models. Although some matrix algebra is included, the algebra is given geometric explanation.

The book is divided into three parts and twelve chapters: 1. Bivariate relationships (the nature of data, summary indices for bivariate relationships, other indices of relationship for two variables, terminology of matrices and geometry); 2. External factor analysis (part and partial correlation, multiple correlation, canonical analysis, discriminant analysis); 3. Internal factor analysis (cluster analysis, the general model for factor analysis, modification of the factor analysis model, other issues in factor analysis).

Conflicting objectives in decisions (International Series on Applied Systems Analysis, vol. I). Edited by D. E. Bell, R. L. Keeney, and H. Raiffa. John Wiley & Sons, Chichester, New York, Brisbane, Toronto, 1977. x + 442 pp. $32.00.

This is volume I in the International Series on Applied System Analysis, published by the International Institute for Applied Systems Analysis, a nongovernmental research institution located in Laxenburg, Austria. Its member organizations are academies of science or equivalent institutions in seventeen nations. This volume contains the edited versions of some of the papers presented and discussions that took place at a workshop on decision making with multiple conflicting objectives held at the Institute on October 20–24, 1975. It is in three parts, Methodology (eleven papers), Applications (seven papers) and General Discussion. There is a concentration of papers on decision analysis.

Statistical techniques in business and economics. 4th ed. By Robert D. Mason. Richard D. Irwin, Inc., Homewood, Ill., 1978. xx + 596 pp. $15.50.

This is the fourth edition of the text first published in 1967. New material in this edition includes two-way analysis of variance and an expansion of nonparametric tests of hypotheses to include the Mann-Whitney test. The chapters on multiple regression and correlation and on decision making have been revised and expanded.

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precise set of rules is in the long run essential, for those mathematical techniques which make instincts rigorous provide stepping-stones to new achievements (new instincts?). Soon the rules for determinancy and unfolding will themselves become as commonplace as the knowledge that if the matrix $H(f) = (\partial^2 f/\partial x_i \partial x_j)$ is positive definite at a critical point p, then f has a minimum at p. Catastrophe theory is in essence no more than an elaboration of this kind of algebraic hold on geometry, together with reasonable observations of the kind that if $H(f)$ becomes singular you do not expect (in the absence of constraints) $f(p)$ to vanish at the same time.

The mathematics of catastrophe theory consists of tools for handling Taylor series, together with *transversality* ideas that underlie this philosophical ingredient concerning what you *expect* to see in a catastrophe model. If you find what you expect, you have not learned much, although you may have saved yourself some work. If you find not what you expect, but something else instead, you have learned a lot.

Here is a book from which any mathematician or scientist can indeed learn a great deal. It is lively and well-presented, a survey of an impressively wide stretch of the scenery of mathematics as it is and will be applied. Its only drawback is its price. Surely a paperback version must be on the way.

DAVID CHILLINGWORTH (*Southampton*)

Probability measures on locally compact groups. By H. Heyer. Ergebnisse der Mathematik und ihrer Grenzgebiete 94, Springer-Verlag, Berlin, Heidelberg, New York, x + 531 pp. $55.20.

The study of probabilistic limit theorems on the real line or finite-dimensional Euclidean spaces constitutes one of the most glorious chapters of modern probability theory. The elegance of the results as well as their great applicability have tempted many mathematicians to extend these results to more general spaces. Such extensions were made quite early by Levy and in particular by Ito and Kawede. In the early 1960s one could see the emergence of a theory, still quite incomplete, dealing with probability measures on abstract spaces of varying generality. At that time appeared the books *Probabilities on algebraic structures* by this reviewer and *Probability measures on metric spaces* by Parthasarathy.

In reading Heyer's book it becomes clear that the early attempts have now been substantially strengthened and deepened by the work of many mathematicians, among them Heyer himself, resulting in a cohesive logical structure with an abundance of new results. This is true especially for the algebraic-topological background chosen for this book: the locally compact groups.

The author devotes most of his attention to limit theorems for infinitesimal triangular systems and embeddings into one-parameter semigroups. The analytical tools needed are presented in two introductory chapters, one on general harmonic analysis and another on convolution sequences of probability distributions. They could be read alone for their own sake.

Chapter III deals with the embedding of infinitely divisible measures in one-parameter semigroups and the role of the infinitesimal generator, while the fourth chapter examines the Levy-Khinchin representation in Heyes's more general setting. This leads up to the major subject: the central limit theorem in all its many appearances. They are treated in the last two chapters of the book.

It is impossible in a short review to do justice to the scholarly way in which the author has succeeded in presenting the material, much of which is his own work. The exposition is highly technical, which is unavoidable for a work on this mathematical level, and reading it is not easy. However, once one gets into the heart of the matter, after about 150 pages, the treatment becomes more lucid. It is also helpful to read the accompanying references and comments attached to each chapter; in them the author points out limitations of the results, open problems, as well as connections between seemingly isolated results.

He is always careful in pointing out gaps in the theory. Nevertheless, it is this reviewer's opinion that he has given as complete an account of his subject as is possible today. Our understanding of triangular systems which are not infinitesimal is still not satisfactory; this is perhaps the most important gap that remains to be filled.

The author should be congratulated for carrying out his herculean task. This work will be consulted by probabilists with abstract inclinations for many years to come.

A final comment. We now know a great deal, if not everything we want, about probabilistic limit theorems on groups. What happens for other algebraic structures lacking the group property? Recent work in pattern theory has led to such questions, where the relevant space, the image algebra, forms a partial universal algebra. Do they have laws of large numbers, central limit theorems, or infinitely divisible measures? Fourier analysis based on group representations will then not be applicable, but the probability operators approach may conceivably be useful.

ULF GRENANDER (*Providence*)

Kinematics and mechanisms design. By C. H. Suh and C. W. Radcliffe. John Wiley & Sons, New York, 1978. xxiii + 434 pp. $23.95.

This textbook presents modern kinematics with emphasis on computer-aided design of mechanisms. The chapter headings: basic concepts, vector methods in plane kinematics, matrix methods in kinematics, kinematic analysis of spatial mechanisms, mobility analysis of mechanisms, rigid body guidance, function generation, path generation, optimal synthesis of mechanisms, differential geometry of motion, dynamics of mechanisms, computer programs.

Differential equations for engineers. By Thomas M. Creese and Robert M. Haralick. McGraw-Hill Book Company, New York, 1978. xvi + 551 pp. $17.50.

This is a textbook for engineering students. Contents: 1. Introduction—first-order linear differential equations; 2. Models of engineering systems; 3. Transform methods for solving linear differential equations with constant coefficients; 4. Linearity explained and used.

Random excitation of structures by earthquakes and atmospheric turbulence. Edited by H. Parkus. Volume 25 of Courses and Lectures, International Center for Mechanical Sciences. Springer-Verlag, Vienna, New York, 1977. 307 pp. $25.00.

These are the texts of a series of courses given in Udine in July 1976, under the auspices of CISM. (A severe earthquake shook the neighbourhood shortly before the conference, killing more than 1000 people and making the subject of the meeting particularly topical!)

There are five courses: 1. Seismic safety assessment, by E. H. Van Marcke; 2. An approach to characterizing, modeling and analyzing earthquake excitation records, by F. Kozin; 3. Aseismic reliability and first-passage failure, by R. Grossmayer; 4. Applications of digital simulation of Gaussian random processes, by M. Shinozuka; and 5. Structural response under turbulent flow excitation, by Y. K. Lin.

Representation theory of algebras: proceedings of the Philadelphia Conference (Lecture Notes in Pure and Applied Mathematics, volume 37). Edited by Robert Gordon. Marcel Dekker, Inc., 1978. 480 pp. $35.00.

The papers featured here were presented at a National Science Foundation-sponsored research conference on representation theory. The book covers the gamut of the representation theory of finite-dimensional algebras. Two major articles by M. Auslander form the focal point of the collection. One of these develops a general theory of morphisms determined by modules. The other concerns applications of the general theory to Artin algebras and concludes with a discussion of certain variants of the Second Brauer-Thrall Conjecture. These articles synthesize Auslander's contributions to the field of representation theory.

Geometric probability. By Herbert Solomon. CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 28. S.I.A.M., 1978. 174 pp. $14.50.

The scope of the topics selected for this presentation is well summarized by the chapter headings: 1. Buffon needle problem, extensions, and estimations of π; 2. Density and measure for random geometric elements; 3. Random lines in the plane and applications; 4. Covering a circle circumference and a sphere surface; 5. Crofton's theorem and Sylvester's problem in two and three dimensions; 6. Random chords in the circle and the sphere.

Complex analysis: proceedings of S.U.N.Y. Brockport Conference (Lecture Notes in Pure and Applied Mathematics, Vol. 36). Edited by Sanford S. Miller. Marcel Dekker, Inc., 1978. 192 pp. $18.75.

This volume presents the proceedings of the 1976 conference on complex analysis held at the State University of New York at Brockport. The main emphasis of the book is on the recent developments in the theory of univalent functions and the theory of entire functions. Also included are papers on Riemann surfaces, cluster set theory, $H\rho$ functions, and approximation theory.

Complex Fourier transformation and analytic functionals with unbounded carriers (Mathematical Centre Tracts No. 89). By J. W. De Roever. Mathematisch Centrum, Amsterdam, 1978. xvii + 200 pp. f24.00.

This book continues the history of the generalizations of the Paley-Wiener theorem and is a rather complete account of analytic functionals and complex Fourier transforms.

The first chapter is an intriguing essay on causality and localizability of particles in quantum field theory. Recent developments have shown the need for real carried analytic functionals which are the Fourier transforms of distributions or so-called ultra-distributions.

Properties of analytic functionals with real unbounded carriers are investigated in the second chapter where in particular the Paley-Wiener theorem and the Edge of the Wedge theorem are generalized for ultra-distributions. Chapter III is devoted to analytic functionals with unbounded carriers in C_n and chapter IV to the Fundamental Principle of Ehrenpreis.

The author deals also with some concrete applications of the theory. Fourier transformation is a widely-used tool for solving differential equations with constant coefficients. The generalization to systems of partial differential equations with constant coefficients is not easy as it involves the solution of a matrix equation in a ring. It is with the aid of the generalization of Ehrenpreis' principle that the author derives in chapter V a Fourier representation of all weak solutions of the system in certain spaces which are the duals of spaces whose Fourier transforms consist of non-entire functions.

Homology and cohomology theory: an approach based on Alexander-Spanier cochains (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Vol. 46). By William S. Massey. Marcel Dekker, Inc., 1978. 432 pp. $29.75.

The textbook is divided into three parts. The first part is concerned with cohomology and homology for locally compact spaces. The second part deals with homology and cohomology theories for arbitrary spaces. In the third part, the simultaneous use of these two kinds of homology and cohomology theories on locally compact spaces is considered.

Mathematical models in water pollution control. Edited by A. James. John Wiley & Sons, Inc., 1978. xiv + 420 pp. $36.00.

These are the revised proceedings of a conference on the subject held at the University of Newcastle-upon-Tyne in September 1973. There are four parts (techniques, application to polluted environments, application to waste treatment, application to water resources) and nineteen chapters in all. The authors have a wide spectrum of backgrounds—engineers, mathematicians, chemists and biologists.

Introduction to functions of a complex variable (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Vol. 44). By J. H. Curtiss. Marcel Dekker, 1978. 416 pp. $16.50.

This introductory textbook in complex analysis falls at the rigorous end of the spectrum of books of its kind: the exposition aims at a high level of logical completeness. The book is largely self-contained—only some knowledge of the real number system and elementary calculus is assumed—and begins at a basic level, gradually increasing in complexity. The central theorem of complex analysis is the Cauchy Integral Theorem. In this book the theorem is proved first only for a starlike region. Despite this restricted validation, a large number of the core results of complex function theory can be correctly derived. There is emphasis on approximations by rational functions and polynomials, exemplified by the extensive exposure given to Runge's Theorem and its polynomial specializations as well as the very early introduction of power series and a later discussion of deep applications of conformal mapping to polynomial approximations. The book includes several hundred exercises, many of which contain important extensions and developments of the theory.

Introduction to set theory (Pure and Applied Mathematics: A Series of Monographs and Textbooks, Vol. 45). By Karel Hrbacek and Thomas Jech. Marcel Dekker, 1978. 200 pp. $12.50.

This introductory textbook presents an axiomatic but informal approach to set theory. In the course of the development of the theory, the reasons for adopting each axiom are carefully pointed out, and in controversial cases (such as the Axiom of Choice) detailed discussions are provided. Meanwhile, logical apparatus is kept to a minimum, while logical formalism is completely avoided.

The authors show that set theory is powerful enough to serve as an underlying framework for mathematics by using it to develop the beginnings of the theory of natural, rational, and real numbers. The development is carried only as far as is useful to illustrate the general idea and to motivate set-theoretic generalizations of basic concepts. A substantial part of the book is then devoted to the study of ordinal and cardinal numbers. In the final chapter some recent developments in set theory and their significance for other areas of mathematics are discussed.

Modern methods in partial differential equations. By Martin Schechter. McGraw-Hill Book Company, New York, 1977. xiv + 245 pp. $28.50.

It is the aim of this book to introduce the student to the modern techniques and methods that have been used in the newer theory of partial differential equations as it has been developed over the past thirty years, without swamping him with minute details which hide the basic ideas behind the methods. The author takes a middle road—he attacks problems of greater generality than those considered in the classical theory, but does not require the most general and refined machinery available. He is able to do this without requiring a background beyond advanced calculus, by using great didactic skill. The chapter headings are: 1. Existence of solutions; 2. Regularity (constant coefficients); 3. Regularity (variable coefficients); 4. The Cauchy problem; 5. Properties of solutions; 6. Boundary value problems in a half-space (elliptic); 7. Boundary value problems in a half-space (non-elliptic); 8. The Dirichlet problem; 9. General domains; 10. General boundary-value problems.

Vibrations and waves in physics. By Iain G. Main, Cambridge University Press, 1978. xiv + 336 pp. $37.50 hard cover; $8.95 paperback.

This is a textbook for an undergraduate course on vibrations and waves, stressing the physics rather than the mathematics. The table of contents is: 1. Free vibrations; 2. Free vibrations in physics; 3. Damping; 4. Damping in physics; 5. Forced vibrations; 6. Forced vibrations in physics; 7. Anharmonic vibrations; 8. Two-coordinate vibrations; 9. Non-dispersive waves; 10. Non-dispersive waves in physics; 11. Fourier theory; 12. Dispersion; 13. Water waves; 14. Electromagnetic waves; 15. De Broglie waves; 16. Plane waves at boundaries; 17. Diffraction.

The finite element method for elliptic problems. By Phillippe G. Ciarlet. Studies in Mathematics and its Applications, vol. 4. North-Holland Publishing Company, Amsterdam and New York, 1978. xviii + 543 pp. $56.95.

The objective of this book is to analyze within reasonable limits the basic mathematical aspects of the finite-element method. At the same time, it is also intended to be a working textbook for advanced courses in numerical analysis. The various finite-element methods currently used by engineers in solving second- or fourth-order problems are described. In addition, a thorough analysis is given of the convergence properties of such methods in various norms, including the latest results on uniform convergence. The table of contents is: 1. Elliptic boundary value problems; 2. Introduction to the finite element method; 3. Conforming finite element methods for second-order problems; 4. Other finite element methods for second-order problems; 5. Application of the finite element method to some nonlinear problems; 6. Finite element methods for the plate problem; 7. A mixed finite element method; 8. Finite element methods for shells.

Annual review of fluid mechanics, Vol. 9. Edited by M. Van Dyke, J. V. Wehausen, and J. L. Lumley. Annual Reviews, Inc., Palo Alto, 1977. v + 509 pp. $17.00.

Table of contents: Recollections from an earlier period in American aeronautics (R. T. Jones); Steady non-viscometric flows of viscoelastic liquids (A. C. Pipkin and R. I. Tanner); Compressible turbulent shear layers (P. Bradshaw); On the liquidlike behavior of fluidized beds (J. F. Davidson, D. Harrison and J. D. F. Guedes de Carvalho); History of boundary-layer theory (Itiro Tani); Incompressible boundary-layer separation (James C. Williams, III); Bubble dynamics and cavitation (Milton S. Plesset and Andrea Prosperetti); Underwater explosions (Maurice Holt); Hydrodynamics of the universe (Ya. B. Zel-dovich); Pulmonary fluid dynamics (T. J. Pedley); Flow and transport in plates (M. J. Canny); Particle capture from low-speed laminar flows (Lloyd A. Spielman); Electrokinetic effects with small particles (D. A. Saville); Fluid mechanics of propulsion by cilia and flagella (Christopher Brennen and Howard Winet); Optimum wind-energy conversion systems (Ulrich Hütter); Finite-element methods in fluid mechanics (Shan-fu Shen); Aeroacoustics (J. E. Ffowcs-Williams); Study of the unsteady aerodynamics of lifting surfaces using the computer (S. M. Belotserkovskii).

Annual review of fluid mechanics, Vol. 10. Edited by M. van Dyke, J. V. Wehausen and J. L. Lumley. Annual Reviews, Inc., Palo Alto, 1978. v + 475 pp. $17.00.

Table of contents: Some notes on the study of fluid mechanics in Cambridge, England (A. M. Binnie); Monte Carlo simulation of gas flows (G. A. Bird); Hydrodynamic problems of ships in restricted waters (E. O. Tuck); Drag reduction by polymers (Neil S. Berman); Viscous transonic flows (Oley S. Ryzhov); Dust explosions (Wayland C. Griffith); Objective methods for weather prediction (C. E. Leith); River meandering (R. A. Callander); Rossby waves—long-period oscillations of oceans and atmospheres (Robert E. Dickinson); Flows of nematic liquid crystals (James T. Jenkins); The structure of vortex breakdown (Sidney Leibovich); Flow through screens (E. M. Laws and J. L. Livesey); Turbulence and mixing in stably stratified waters (Frederick S. Sherman, Jorg Imberger, and Gilles M. Corcos); Prospects for computational fluid mechanics (G. S. Patterson, Jr.); Relativistic fluid mechanics (A. H. Taub); Turbulence-generated noise in pipe flow (Gerhard Reethof); River ice (George D. Ashton); Numerical methods in water-wave diffraction and radiation (Chiang C. Mei); Numerical methods in boundary-layer theory (Herbert B. Keller); Magnetohydrodynamics of the earth's dynamo (F. H. Busse).

Lecture Notes in Control and Information Sciences, Springer-Verlag, 1978.

Volume 1: *Distributed parameter systems: modelling and identification.* Edited by A. Ruberti. 458 pp. $18.50.

These are proceedings of an IFIP working conference held in Rome, Italy, June 1976. There are three survey papers (by A. V. Balakrishnan, J. L. Lions, and J. H. Seinfeld and M. Koda) and 23 contributed papers. They are devoted to recent advances in the mathematical description of DPS's and their structural properties; identification, with practical applications; filtering and state estimation; control and problems concerning the practical implementation of control schemes.

Volume 2: *New trends in systems analysis.* Edited by A. Bensoussan and J. L. Lions. 759 pp. $22.60.

These are the Proceedings of an International Symposium, held at Versailles, in December 1976, under the auspices of I.R.I.A. There are 42 papers, grouped under six headings: control of distributed parameter systems, industrial robotics and applications of microprocessors, systems analysis in problems of energy, applications of control theory, control of economic systems, environment and pollution.

Volume 3: *Differential games and applications.* Edited by P. Hagedorn, H. W. Knoblich, and G. J. Olsder. 236 pp. $11.50.

This volume contains all the invited lectures presented at a workshop held at Enschede, Netherlands, March 1977. The material can be divided into four groups:

1. The largest group (Bernhard, Blaquiere, Breakwell, Leitmann) represents what may be viewed as the continuation of Isaac's work. It provides an informative cross-section about the tools which are presently available in order to attack concrete problems in pursuit and evasions games, zero-sum games, cooperative and other types of dynamical games.
2. The second group (Elliott, Hajek) is devoted to basic questions. It illustrates some of the present efforts to deal with existence problems and to clarify the concepts of solutions and strategies.
3. The third group (Olsder, Roxin) concerns problems not described by ordinary differential equations and indicates present activities in neighbouring fields.
4. The fourth group (Case, Vincent) contains a selection of unconventional problems which arise in applications. These papers illustrate how attractive dynamic games can be for those who like the straightforward and intuitively motivated approach.

Volume 4: *An introduction to the regenerative method for simulation analysis.* By M. A. Crane and A. J. Lemoine. 111 pp. $8.30.

The regenerative method is a new technique developed to deal with the statistical analysis of the output of simulation experiments, and to solve the problems when to begin collecting data, how long to run the simulation and how to deal with highly correlated output. This book is a tutorial presenting the basic ideas and results of the method in a manner which may be easily understood by potential users. The chapter headings are: 1. Introduction; 2. Basic examples and motivation; 3. The regenerative method; 4. More examples of regenerative processes; 5. The regenerative approach and discrete-event simulations; 6. Approximation techniques; 7. Alternative ratio estimators; 8. Some other results.

Volume 5: *Singular optimal control: The linear-quadratic problem.* By David J. Clements and Brian D. O. Anderson. 93 pp. $8.00.

This is a monograph aimed at advanced graduate students, researchers and users of singular optimal control methods. It presumes prior exposure to the standard linear-quadratic regulator problem, and a general maturity in linear systems theory. A number of advances in singular, linear-quadratic control have taken place very recently. This book is intended to present an up-to-date account of many of these advances. At the same time, the book attempts to present a unified view of various approaches to singular optimal control, many of which are apparently unrelated.

Volumes 6 and 7: *Optimization techniques: parts I and II.* Edited by J. Stoer. 528 pp. (Part I), 512 pp. (Part II), $21.00 each part.

These two volumes are the proceedings of the 8th IFIP conference on optimization techniques held in Wurzburg, September 1977. The first volume contains most of the papers on optimal control, the second those on mathematical programming and on various application areas.

There are five invited lectures by A. V. Balakrishnan, M. R. Hestenes, J. L. Lions, G. I. Marchuk and H. J. Sussmann. There is a report on a round table discussion on world models. And the contributed papers are grouped as follows: computational techniques in optimal control, stochastic optimal control; differential games, optimal control of p.d.e.'s; immunology, disease and control theory; environmental and energy systems; mathematical programming, theory; nonlinear and stochastic programming; integer programming, networks; urban systems; economics; operations research; computer and communication networks, software problems.

—BOOK REVIEW SECTION—

Statistics of random processes, Vols. I; II. By R. S. Liptser and A. N. Shiryaev. English translation by A. B. Aries. Springer, 1977; 1978. x + 394 pp.; x + 339 pp. $29.50 per volume.

The books provide the first reasonably complete textbook accounts of the probabilistic background to and the theory of nonlinear filtering, via martingale methods. A good introduction is given to martingale representation and decomposition theory. Many of the results have previously appeared only in journals, and on the whole the treatment is quite readable. One might argue whether the given treatments are always the clearest and most efficient and whether the forest is sometimes obscured for all the trees, but the books contain an enormous amount of good material and remain essential reading for anyone interested in the current work on detection, filtering and optimal control for Itô and point process models. A good background in probability is required. The index is virtually nonexistent.

Volume I includes a very brief introduction to probability theory, and discusses martingales, sub- and supermartingales, stochastic differential equations, decomposition and properties of locally square integrable martingales, representation of martingales that are functionals of Brownian motion, the Girsarov theorem, problems related to absolute continuity of measures (for diffusion-like problems) and related results in detection and linear and nonlinear filtering theory, for systems where the corrupting observation noise is a Wiener process.

Volume II discusses filtering problems for conditionally Gaussian processes, stationary linear filtering theory, and the theory and applications of random point processes to filtering theory.

HAROLD J. KUSHNER *(Providence)*

Catastrophe theory and its applications. By T. Poston and I. N. Stewart. Fearon-Pitman, Belmont, California, 1978. 500 pp. $49.75.

From its very beginnings, catastrophe theory has been accompanied by an air of mystery and controversy, perhaps an inevitable product of the obscurity of Thom's original treatise *Stabilité Structurelle et Morphogénèse* and the almost overcompensating enthusiasm of Zeeman's lucid interpretations (*Catastrophe Theory: Selected Papers 1972–1977*). This book is a strenuous effort towards demystifying the subject. It shows how the mathematical techniques of catastrophe theory (rather than simply the over-exposed classification *theorem* for elementary catastrophes) offer new insights and new tools for calculus users, emphasizing the essential straightforwardness of much of the material as part of mainstream calculus. Catastrophe theory is not a completely new turning such as was Newton's invention of calculus itself.

The authors show catastrophe theory at work in many areas of the 'hard' sciences (ship stability, fluid flow, optics, elastic buckling, phase transitions, laser physics) and also discuss its possible role in ecology, developmental biology and social modelling. Sometimes it appears simply as an efficient pedagogic aid, sometimes as a generator of models offering qualitative and quantitative predictions, sometimes as a toolkit for extracting material from a given model—but invariably it inspires arresting and informative pictures. The book is lavishly illustrated, often with vivid three-dimensional effects (although no pop-up displays), and is itself a pedagogic *tour de force*.

The heart of the book, as the authors say, is the chapter on determinancy and unfoldings, which is the mathematical base of a large part of the theory. Full proofs of the key results are not all included, for the purpose is to show *why* these results should be true and to leave the ultimate technicalities to be pursued elsewhere if desired. After an excursion through this well-motivated account and many carefully-worked examples, the reader emerges firmly in control of rigorous methods for handling Taylor series of parameterized families of functions, without having abdicated responsibility by the common device of ignoring what would rather not be seen, and without being bemused by any magic. But then, Thom has remarked that 'geometry is successful magic' . . . .

Of course, experienced workers in any field usually know what sort of maltreatment of the mathematics is justifiable and what is not. But, as Poston and Stewart aptly remark, no one is born with years of experience. A