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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos\frac{\pi x}{2b}}{\cos\frac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEW SECTION—

Stability of fluid motions, I (282 pp.) and II (274 pp.). By Daniel D. Joseph. Springer Verlag, Berlin, Heidelberg, New York. $39.80 each.*

It has been wellnigh one hundred years since Reynolds discovered turbulence in fluid flow, and although the increasingly extensive and intensive research on turbulence since the beginning of this century has provided much information on the *effects* of turbulence, it has not to this day really succeeded in illuminating our *understanding* of the phenomenon of turbulence. Efforts at understanding it have proceeded either along the avenue of statistical theories, with some success for homogeneous and isotropic turbulence but much less for shear flows, or along the road of the theory of hydrodynamic stability, in the hope of approaching turbulent flows from laminar ones through repeated bifurcations. It is this noble if distant aim that has attracted many able researchers to the field of hydrodynamic stability, among whom it is our good fortune to have Daniel D. Joseph, the author of the book under review.

There are few books on the same subject. One by C. C. Lin (1956) treats mainly the Orr-Sommerfeld equation and the intricacies of its solutions, although to a lesser extent it also deals with convective, inertial, and geophysical instabilities, and certain nonlinear aspects. An extensive book by S. Chandrasekhar (1961) deals exclusively with the linear theory of convective, inertial, and magnetohydrodynamic instabilities, especially those with geophysical and astrophysical applications. A more recent book by Betchov and Criminali (1971) is a computer-oriented treatment of the Orr-Sommerfeld equations. Against such a background, Joseph's book, in its reliance on the energy method and its insistence on taking the amplitude of the disturbance into account, is unique. It enriches our knowledge of hydrodynamic stability and is indispensable to university libraries and to serious researchers and students in the field.

The book consists of two volumes. Volume I contains seven chapters and five appendices. In Chapter I the basic ideas on global instability and well-known results on the uniqueness of the Navier-Stokes equations are presented. In Chapter II instability and bifurcation are discussed in a general way. These two chapters serve to orient the reader and to prepare him for things to come. In Chapters III and IV the author presents many interesting recent results on the stability of Poiseuille flow and of parallel flow through annular ducts. Chapter V discusses the global stability of Couette flows, and Chapters VI and VII treat spiral Couette-Poiseuille flows and flows between concentric rotating spheres. Appendices A, B, C, and D give useful mathematical results. Appendix E, on nearly parallel flows, could have constituted another chapter. It is curious that it is presented as an appendix.

Volume II contains five chapters dealing exclusively with gravitational convection, one of which concerns convection in porous media. One chapter treats viscoelastic fluids, and the final chapter is principally a study of the effect of surface tension on the stability of superposed fluids of different densities in a gravitational field.

Throughout the book the superbly-presented energy method runs like the warp of a loom. This method can be traced back to the work of Orr (1907), Hopf (1941), Thomas (1942), and Serrin (1959), and consists in expressing the Reynolds number, via the Navier-Stokes equations, as a ratio of two integrals, of which the integrands contain, quadratically, the unknown velocity (or temperature) of the disturbance, and in finding the maximum of this ratio as the unknown quantity is allowed to roam through a function space, subject only to the requirement of the equation of continuity and to the boundary conditions. Then if the actual Reynolds number is less than this maximum ratio, the flow is monotonically stable.

Since the finding of the maximum is by means of the Euler-Lagrange equation of the calculus of variations, which is linear, the method is finally a linear analysis emerging and arising from a baptism in the holy water of nonlinearity. The attending angel absolved linear analysis of its original sin, whispered in its ears one truth, then flew away, revealing no more secrets. The one truth enables it to provide a lower bound for the Reynolds number, below which the fluid is monotonically stable. Of course, since the function space in which the unknown roams is not very restricted, this bound is usually not very sharp. For Bénard cells, however, it coincides with the

* This book review was written during the tenure of a Senior Scientist Award given by the Alexander von Humboldt Foundation of the German Federal Republic, which enabled me to read the book at leisure at the University of Karlsruhe. To the Humboldt Foundation and my Karlsruhe hosts I wish to express my sincere appreciation.

upper bound (above which the fluid is unstable) provided by a linear analysis ab initio—a very satisfying fact.

If the energy method does not provide detailed description of fluid motion, and avoids direct confrontation with the difficulties of nonlinearity, the same cannot be said of the valiant attempts in determining the stability of perturbed flows (Chapter III and IV). For this purpose the Floquet theory is used, since the perturbed flows treated are periodic in time. The results obtained, taken together with the theorem (henceforth called Joseph's theorem in this review) given in Chapter II that flows with small subcritical disturbances are stable, are extremely interesting, exciting, and bewildering. I shall return to this point later. Here I only want to underline the importance of Joseph's theorem, and to point out one direct way of confronting nonlinearity admirably given in this book.

The simple and precise style of writing, emerging partly from the author's mathematical discipline, acquires, in the long run, an elegance. This fact emerges in spite of the occasionally unfortunate choice of mathematical symbols, and of one peculiar manner of presentation. Of the latter the reader should be warned. A theorem is often stated without promise of proof, and just when one, after mistaking it to be obvious and making a few vain attempts to prove it in his mind, is about to give up the effort, he finds the proof in the next paragraph or in the next three or five pages, in which he may repeat the experience at another level.

Since the game is, understandably, squeezing as much juice out of the lemon as one can, unnecessary insufficiencies are extremely rare in this book. I found only one, in Chapter V. In extending Synge's (1938) proof of the Rayleigh criterion for the stability of Couette flow against axisymmetric disturbances, the author imposed the unnecessary restriction that Ω_1 (angular velocity on the inner cylinder) be greater than Ω_2 (angular velocity on the outer cylinder), and, curiously enough, also attributes this restriction to Synge's proof for infinitesimal disturbances. Neither Rayleigh, Kármán, nor Synge ever needed this restriction, and upon close examination of Joseph's proof I found that he did not need it either. For (40.17) stands without it, $A + Br^{-2}$ will be positive if Rayleigh's criterion is satisfied, and all one needs to do to broaden the base of validity of (40.17) is to replace B by $|B|$ in the G_v defined on p. 150.

I now turn to a discussion of the consequences of Joseph's theorem and the neutral bifurcation solutions he has found for perturbed flows. Lest the reader, upon careful reading of Joseph's proof of his theorem, think there is a fatal flaw in it, I note first that there is an oversight in the proof, but in the end it does not matter. He states, in p. 47 and p. 51, that σ_ν is positive. Actually it is positive only along the left branch of the neutral curve when it has two branches; along the right branch it is negative. However, when the sign of ν_ϵ is taken into account his theorem (pp. 47 and 51) stands intact. So the solid lines and dotted lines (indicating instability) in Fig. 34.1 on p. 111 are all properly drawn. As a consequence of his theorem, Joseph states that subcritical solutions are not observable.

But the host of questions they call forth, these dotted lines! First of all, comparing Fig. 34.1 with Fig. E1.6 in Appendix E, we find that most of Schubauer and Skramstad's experimental points lie in the subcritical region, and furthermore some points in the supercritical region are near the right branch, whereas according to Fig. 34.1 they should not be there at all. To be sure, Fig. E1.6 is only for nearly parallel flows. But I do not think that alone can explain the discrepancy.

On purely theoretical grounds, one might wonder what happens to unstable infinitesimal disturbances that fall within the loop of the neutral curve in Fig. 34.1. Do they all migrate downward when the amplitude ϵ is increased, to the region where supercritical disturbances can persist without growing or damping? That seems unlikely, since that region is so narrow. Even the application of the "principle of equi-conjecture" is not without risk. The author says that perhaps calculations to higher orders in ϵ might produce stable subcritical solutions, to which transient solutions might be attracted. This still leaves the going over from within the neutral loop at small ϵ to the surface representing stable subcritical solutions a cloudy affair. Furthermore, near 0 in Fig. 34.1, stable subcritical solutions should exist (by continuity) for small ϵ if they do for large ϵ. So why do the solid lines not continue beyond 0, if indeed stable subcritical solutions exist for some finite ϵ?

Joseph's theorem, together with his bifurcation solutions and their stability or instability, has led us to a point of the frontier separating conquered territory from terra incognita, on our way toward an understanding of the origin of turbulence. At this point of the frontier, we stand *verwirrt* if not quite *verirrt*. But it is a measure of Joseph's imaginativeness that the questions his work has raised not only do not detract from its value, but, on the contrary, bear testimony to the excitement it calls forth.

The two volumes contain a wealth of results and thought: interesting, stimulating, and often important. So if at this moment, soon after the appearance of the book, the author feels tempted to say "Verweile doch, du bist so schön!", we in our appreciation of his contribution rejoice with him. But before the last glass of champagne is poured, we already feel the urge to resume our journey, for we have "promises to keep, and miles to go."

CHIA-SHUN YIH
(*Ann Arbor*)

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Review Section.

Spectral synthesis. By John J. Benedetto. Academic Press, New York, 1976. 278 pages. $27.50.

The purposes of this book are: to trace the development of spectral synthesis from its origin in the study of Tauberian theorems; to draw attention to other mathematical areas related to spectral synthesis; to give a thorough treatment of spectral synthesis for the space of complex-valued integrable (with respect to Haar measure) functions on Hausdorff locally compact Abelian groups; and to introduce the "integration" and "structure" problems that have emerged because of the study of spectral synthesis. The "integration" problem—essentially posed by Beurling—is to find the relationship between spectral synthesis and integration theories, and the "structure" problem in that of finding the intrinsic properties of a distribution when its Fourier transform is known, and vice versa. There is a 14-page bibliography.

An anatomy of risk. By William D. Rowe. John Wiley & Sons, New York, 1977. xii + 488 pages. $26.95.

This book, a volume in the series "Systems Engineering and Applications", examines how society, or its agents in Congress and regulatory agencies, can determine levels of acceptable risk for technological systems and programs. It provides techniques for addressing the problems and focuses on the formal analysis of risk, as well as on the more subjective problems of risk acceptance and its interaction with estimated levels of risk. There are 20 chapters divided into five parts, as follows: the nature of risk, factors in risk valuation and evaluation, methodological problems and approaches in the qualification of risks, evaluation of revealed societal preferences for risk assessment, methodological approach to risk assessment. There are appendices on data bases, problems in data analysis, and crisis management, and the bibliography occupies 13 pages.

Motion geometry of mechanisms. By E. A. Dijksman. Cambridge University Press, New York, 1976. xvi + 288 pages. $22.50.

A mechanism can be regarded as a composition of rigid bodies linked as in a chain. The geometrical configuration so formed moves under forces applied on the mechanism. The study of such motion is called motion geometry or kinematics. An understanding of the motion is of key importance to designers of machines who can use the techniques described in this book to test efficiencies and operating capability extensively before embarking on the construction of prototypes. The author provides a simple introduction to the geometry of motion in a way suitable for use by undergraduate students of mechanical engineering or applied mathematics. Basic definitions and descriptions of simple motions are given to provide a framework within which to look at more complex machines. Frequent reference is made to real apparatus which uses the mechanism under discussion and the whole text is amply illustrated with diagrams.

(continued on p. 128)

(continued from p. 120)

The theory of information and coding: a mathematical framework for communication (*Encyclopedia of Mathematics and its Applications,* Volume 3). By Robert J. McEliece. Addison-Wesley Publishing Co., 1977. xvi + 302 pages. $21.50.

This is Volume 3 of the *Encyclopedia of Mathematics and Its Applications,* edited by Gian-Carlo Rota, and a volume in the section *Probability,* edited by Marc Kac.

The work is a self-contained introduction to the basic results in the theory of information and coding. It is aimed at non-specialists as well as specialists and is in three parts: introduction, information theory, coding theory. The introduction gives an overview of the whole subject. Part I has six chapters: entropy and mutual information; discrete memoryless channels and their capacity-cost functions; discrete memoryless sources and their rate-distortion functions; the Gaussian channel and source; the source-channel coding theorem; survey of advanced topics for Part I. Part II has five chapters: linear codes; BCH, Goppa and related codes; convolutional codes; variable-length source coding; survey of advanced topics for Part II. There are appendices giving a brief survey of probability theory; convex functions and Jensen's inequality (used frequently in Part I); finite fields; and an algorithm for counting paths in directed graphs.

To quote the section editor, this monograph "is an excellent introduction to the two aspects of communication, coding and transmission—the first an elegant illustration of the power and beauty of algebra, the second a part of probability theory, enriched by Shannon in novel and unexpected ways."

Fractals: form, chance, and dimension. By Beniot B. Mandelbrot. W. H. Freeman & Co., San Francisco, 1977. xvi + 365 pages. $14.95.

This beautiful book—beautiful both in content and in its production—is a unique geometrical investigation. Fractals are a class of highly irregular shapes that have myriad counterparts in the real world, such as islands, continents, coastlines and snowflakes.

This book brings several studies together into one integrated essay. It is an exposition claiming that the geometry of fractals is necessary to describe geometric shapes in nature. In the text, Mandelbrot examines fractal applications for describing the surface of the earth, the clusterings of stars, the shapes of clouds, and other forms appearing in nature.

Fractals is an informal mathematical treatment backed by adequate references to rigorous proofs. The illustrations are computer simulations of mathematically defined shapes; some nonrandom, others only seemingly random. Mandelbrot points out how these shapes relate to the world around us. Some of the figures resemble photographs taken by the crew of Apollo XI.

The classic fractals include Brownian paths, Cantor sets, and Koch curves. Hausdorff dimension is the main parameter of a fractal. The dimension serves as an excellent measure of irregularity and fragmentation and gives meaning to ideas like curves of dimension greater than 1 and surfaces of dimension greater than 2. The application of this concept of dimensions in natural science is absolutely new. However, understanding the material presented here involves only elementary mathematical prerequisites.

This is an exciting and original work. It can be read with delight by mathematicians, other scientists and educated laymen alike.

(continued on p. 201)

Groups, representations, and characters. By Victor E. Hill. Macmillan Publishing Co., New York, 1976. x + 182 pages. $12.95.

This book is addressed to undergraduate majors in mathematics, to students in the physical sciences who are likely to encounter groups in crystallography and quantum mechanics, and to mature physical scientists who would like to have some sense of the mathematics behind the techniques with which they have had experience in applications. It is a rapid survey and emphasizes examples and applications of the theorems, avoiding most of the longer and more difficult proofs.

Modeling and control in the biomedical sciences (Lecture Notes in Biomathematics, Vol. 6). By H. T. Banks. Springer-Verlag, Berlin, 1975. v + 114 pages. $7.40.

These notes are based on lectures given at a Canadian mathematical congress and concern the use of optimization and control theory in biomedical problems. The chapter headings are: 1. A brief review of enzyme kinetics, 2. Models of enzymatically active membranes, 3. Modeling of enzyme cascades, 4. Modeling and control of epidemics, 5. Modeling of the control system in glucose homeostasis, 6. Modeling and control of tumor growth, 7. A survey of recent efforts. There is a bibliography of 185 items.

Nonlinear diffusions (Research Notes in Mathematics, Vol. 14). Edited by W. E. Fitzgibbon and H. F. Walker. Pitman Publishing, London, 1977. vi + 232 pages. £7.50.

These are the proceedings of a conference held at the University of Houston in June 1976. The principal speaker was D. G. Aronson, whose ten one-hour lectures will be published separately. The book contains papers by D. G. Aronson, J. R. Cannon and J. A. Smoller, J. W. Evans, P. C. Fife, D. Henry, N. J. Kopell, R. M. Miura, P. Nelson, J. Rinzel, M. E. Schonbek, A. D. Snider and D. L. Akins. They cover topics in neurophysiology, ecology, chemical reaction theory, neutron diffusion, and mathematical aspects of nonlinear diffusion.

Unsolved problems concerning lattice points (Research Notes in Mathematics, Vol. 15). By J. Hammer. Pitman Publishing, London, 1977. vi + 101 pages. £5.00.

The aim of the book is to give a student of mathematics insight into the field of lattice points. It is divided into three parts. Part 1 deals with problems of a geometric nature, many closely related to the classical theorems of Minkowski. Part 2 contains problems related to combinatorics. Part 3—compactness theorems, abstract lattices—is concerned with sets of lattices and not with lattice points of a single lattice, as in the first two parts.

The book is organized in a unconventional way: it consists of problems, accompanied by background material and a description of related results; some of these problems and results are new. In general, proofs are not given, but reference to the original source is made. There is an extensive, annotated bibliography.

Introductory statistics. 3rd ed. By T. H. Wonnacott and R. J. Wonnacott. John Wiley & Sons, 1977. xxii + 650 pages. $15.95.

This is the third edition of a volume in the Applied Probability and Statistics section of Wiley's series in Probability and Mathematical Statistics. It is directed to the beginning student of statistics in a service course provided by a mathematics department. The text is kept simple, with optional, more difficult developments relegated to footnotes and starred sections. A student's and an instructor's manual are available; they contain solutions to the problems and explain some of the finer points.

Contractors and contractor directions: theory and applications; a new approach to solving equations (Lecture Notes in Pure and Applied Mathematics Series, Volume 32). By Mieczyslaw Altman. Marcel Dekker, Inc., New York, 1977. x + 290 pages. $24.75.

This research monograph contains a general theory of solving equations by analytical means. The concept of contractors and contractor directions provides a powerful tool for bridging methods and theories which previously had seemed to be far apart. From this point of view, many well-known procedures and algorithms, including the most important ones—the method of successive approximations, the Newton-Kantorovich method, the Newton-Altman method for nonlinear functionals, the method of steepest descent, and other gradient-type methods—turn out to be particular cases of a unified approach.

Symmetry and separation of variables. By Willard Miller, Jr. Addison-Wesley Publishing Co., 1977. xxx + 285 pages. $21.50.

This is Volume 4 of the *Encyclopedia of Mathematics and its Applications,* edited by Gian-Carlo Rota, and a volume in the section *Special Functions,* edited by Richard Askey.

This monograph gives the first systematic treatment of the method of separation of variables for solving differential equations. It is concerned with the relationship between symmetries of a linear second-order partial differential equation of mathematical physics, the coordinate systems in which the equation admits solutions via separation of variables, and the properties of the special functions that arise in this manner, relating the subject to the theory of Lie algebras: it is shown how group theory can be used to provide a foundation for much of special function theory. In particular, it turns out that for the most important linear equations, the separated solutions are characterized as common eigenfunctions of sets of second-order commuting elements in the universal enveloping algebra of the Lie symmetry algebra corresponding to the equations. There are five chapters: on the Helmholtz (2- and 3-variable), Schrödinger, heat, 3-variable Laplace and wave equations, and the hypergeometric function and its generalizations. Appendices give basic properties of special functions and summarize Lie groups and algebras, and elliptic functions.

Like all volumes in this Encyclopedia so far, this monograph does not merely give an encyclopedic treatment of its subject, but presents an original, highly readable and exciting account of a chapter in modern pure and applied mathematics.

Statistics of random processes I: general theory. By R. S. Liptser and A. N. Shiryayev. Translated by A. B. Aries. Springer-Verlag, New York, Inc., 1977. x + 394 pages. $29.80.

This is volume 5 in the Springer series "Applications of Mathematics". The original Russian edition was published in 1974. The problem discussed in this book is the following. On a certain probability space $(\Omega, \mathfrak{F}, P)$ a partially observable random process $(\theta, \xi) = (\theta_t, \xi_t)$, $t \geq 0$, is given with only the second component $\xi = (\xi_t)$, $t \geq 0$, observed. It is required to estimate the unobservable state θ_t from $\{\xi_s, 0 \leq s \leq t\}$. The book thus deals with the filtering problem. In the second volume, linear estimation of random processes will also be discussed. Chapter headings: 1. Essentials of probability theory and mathematical statistics; 2. Martingales and semimartingales: discrete time; 3. Martingales and semimartingales: continuous time; 4. The Wiener process, the stochastic integral over the Wiener process, and stochastic differential equations; 5. Square-integrable martingales, and structure of the functionals on a Wiener process; 6. Nonnegative supermartingales and martingales, and the Girsanov theorem; 7. Absolute continuity of measures corresponding to the Ito processes and processes of the diffusion type; 8. General equations of optimal nonlinear filtering, interpolation and extrapolation of partially observable random processes; 9. Optimal filtering, interpolation and extrapolation of Markov processes with a countable number of states; 10. Optimal linear nonstationary filtering.

Formulations and computational algorithms in finite element analysis. Edited by Klaus-Jurgen Bathe, J. Tinsley Oden, and Walter Wunderlich. The MIT Press, 1977. xvi + 1091 pp. $40.00.

These are the proceedings of a symposium on finite element methods held at M.I.T. on August 9-13, 1976. There are 37 papers grouped into five parts: 1. Linear static and dynamic analysis, including software considerations (seven papers); 2. Material and geometric nonlinear analysis (eleven papers); 3. Solutions of equations (six papers); 4. Mathematical aspects (seven papers); 5. Hydrodynamic and field problems (six papers).

Numerical methods for engineers and scientists. By A. C. Bajpai, I. M. Calus and J. A. Fairley. John Wiley & Sons, 1977. xii + 380 pp. $10.95.

This is a volume of algorithms and flowcharts for a course on numerical methods for undergraduates. It is divided in three units: 1. Equations and matrices; 2. Finite differences and their applications; 3. Differential equations, each with several chapters. The emphasis is on the practical sides of the topics and the more theoretical aspects have been omitted.

Catastrophe theory. Selected papers 1972-1977. By E. C. Zeeman. Addison-Wesley Publishing Co., 1977. x + 675 pp. Hardcover $26.50; Paperback $14.50.

This collection of selected papers on catastrophe theory and its applications by E. C. Zeeman and his collaborators is divided into six parts: 1. General introductory papers (including the draft for a *Scientific American* article), 2. Biological sciences (seven papers), 3. Social sciences (five papers), 4. Physical sciences (three papers), 5. Mathematics (two papers), 6. Discussion (three papers). There is a detailed index of sixteen pages.

Theoretical immunology (Immunology Series, Vol. 8). Edited by George Bell, Alan S. Perelson, and George H. Pimbley, Jr., 1978. Marcel Dekker, Inc., New York. 660 pages. $45.00.

This book is the first of its kind devoted solely to topics in theoretical immunology. By juxtaposing articles by renowned experimental immunologists with contributions by mathematicians and physical scientists, the editors have summarized much of the mathematical modeling done to date in immunology and present discussions of challenging experimental problems which might benefit from theoretical analysis.

The book contains an historical survey of theoretical contributions to immunology, discusses the role and philosophy of constructing theories in immunology, and covers specific examples of mathematical models which should be of benefit to experimental immunologists. Commonly used assays, such as the hemolytic plaque technique, are analyzed mathematically for their reliability and the types of information that can be extracted. The interactions of antigen with antibodies and immunoglobulin receptors on B cells are analyzed, as well as interactions among B cells, T cells, and macrophages. New network and dynamic models of the immune response are presented. Other topics covered include: immune surveillance, the generation of antibody diversity, lymphocytic traffic patterns, and models for the cellular infiltration of infected tissue.

Optimization methods in operations research and systems analysis. By K. V. Mital. John Wiley & Sons, 1977. xiii + 259 pages. $9.75.

This book is an elementary mathematical introduction to the deterministic problems of operations research. The chapter headings are: 1. Mathematical preliminaries; 2. Extrema of functions; 3. Linear programming; 4. Transportation problem; 5. Flow and potential in networks; 6. Nonlinear convex programming; 7. Dynamic programming; 8. Geometric programming; 9. Theory of games; 10. Direct search and gradient methods.

Computational methods in engineering and science with applications to fluid dynamics and nuclear systems. By Shoichiro Nakamura. John Wiley & Sons, 1977. xii + 457 pages. $25.00.

This book covers the computational methods for differential equations in science and engineering—finite difference, finite element and Monte Carlo methods. It includes eigenvalue problems for ordinary differential equations, and methods of solution for elliptic, parabolic and hyperbolic partial differential equations. It is intended to be a self-contained text for senior or graduate courses in engineering schools. There are two chapters on computational fluid dynamics describing finite difference techniques for fluid flow and for transonic aerodynamics, respectively. Other applications to flow problems are interspersed throughout the text.

Introductory functional analysis with applications. By Erwin Kreyszig. John Wiley & Sons, Inc., 1978. xiv + 688 pages. $21.50.

This is an elementary introduction, requiring only undergraduate linear algebra and calculus as prerequisites. Measure theory is neither assumed nor discussed, nor is knowledge of topology required. There are eleven chapters, containing: metric, normed, Banach, Hilbert spaces; Banach fixed-point theorem; approximation theory; spectral theory of linear operators and compact linear operators in normed spaces and of bounded self-adjoint linear operators; unbounded linear operators in Hilbert space and in quantum mechanics. There are appendices with review and reference material, and answers to problems.

Measure and integral: an introduction to real analysis (Pure and Applied Mathematics, Volume 43). By Richard L. Wheeden and Antoni Zygmund. Marcel Dekker, 1977. x + 274 pages. $16.75.

This volume develops the classical theory of the Lebesgue integral and some of its applications. Initially, the integral is presented in the context of n-dimensional Euclidian space, after the concepts of outer measure and measure have been thoroughly studied. A more general treatment of the integral, based on an axiomatic approach, is subsequently given.

Closely related topics in real variables (such as functions of bounded variation, the Riemann-Stieltjes integral, Fubini's theorem, L^p classes, and various results about differentiation) are examined in detail, and several applications of the theory to a specific branch of analysis—harmonic analysis—are also given. Among these applications are basic facts about convolution operators and Fourier series, including results for the conjugate function and the Hardy-Littlewood maximal function.

Requiring only basic familiarity with advanced calculus, this book provides an introduction to real analysis for students of mathematics, statistics, or probability.

Basic numerical mathematics. Vol. 2. By John Todd. Birkhauser Verlag, Basel, Stuttgart, 1977. 216 pp. SFr. 48.

This is volume 22 in the "International Series of Numerical Mathematics." The chapter headings of this volume are as follows: 1. Manipulation of vectors and matrices; 2. Norms of vectors and matrices; 3. Induced norms; 4. The inversion problem I: theoretical arithmetic; 5. The inversion problem II: practical computation; 6. The characteristic value problem—generalities; 7. The power method, deflation, inverse iteration; 8. Characteristic values; 9. Iterative methods for the solution of systems $Ax=b$; 10. Applications: solution of a boundary value problem; 11. Applications: least squares curve fitting; 12. Singular value decomposition and pseudo-inverses. There are many problems, with solutions given to selected ones. The reader is assumed to have absorbed the fundamental ideas of linear algebra: vector space, basis, matrix, determinant, characteristic values and vectors. The idea of "controlled computational experiments" is emphasized: the author checks the programs and gets some idea of errors by using them on problems of which the solutions are already known. Also, "bad examples" are exhibited, which show the difficulties of the subject.

Numerische Prozeduren aus Nachlass und Lehre von Prof. Heinz Rutishauser. By Walter Gander, Luciano Molinari and Hana Svecova. Birkhauser-Verlag, Basel, Stuttgart, 1977. 127 pp. SFr. 48.

This is volume 33 in the "International Series of Numerical Mathematics". It consists of five articles based on lectures and manuscripts by the late Heinz Rutishauser, a pioneer of modern numerical mathematics, and assembled by former collaborators. Their titles are: Spline-interpolation (W. Gander); Solution of systems of linear equations (H. Svecova); Method of least squares (L. Molinari); Gram-Schmidt orthogonalisation (L. Molinari); Stationary quotient-difference algorithm (W. Gander).

Each article is divided into the following sections: 1. Introduction (purpose of the algorithm); 2. Theoretical foundations; 3. Calling sequence and parameter list; 4. Listing of the algorithm in terms of ALGOL 60 procedure statements; 5. Remarks on organization and notation; 6. Numerical properties; 7. Applications; 8. Examples.

Numerical analysis. Edited by J. Descloux and Jurg Marti. Birkhauser Verlag, Basel, Stuttgart, 1977. 248 pp. SFr. 44.

This is volume 37 of the "International Series of Numerical Mathematics." It is the proceedings of a colloquium on numerical analysis held in Lausanne, Switzerland, October 11-13, 1976. There are twelve papers on the application of numerical methods to several areas of science, such as fluid dynamics, mechanics, neutron diffusion, magnetism, molecular dynamics, material science, viscoelasticity, and others.

Numerik und Anwendungen von Eigenwertaufgaben und Verzweigungs-problemen. Edited by E. Bohl, L. Collatz, K. P. Hadeler. Birkhauser Verlag, Basel, Stuttgart, 1977. 218 pp. SFr. 42.

This is volume 38 of the "International Series of Numerical Mathematics." It consists of eleven papers delivered at an Oberwolfach meeting, November 14-20, 1976. They are on: bifurcation diagrams (L. Collatz), eigenvalue problems nonlinear in the parameter (P. Lancaster), spline eigenvalue problems (W. Mackens), quasilinear diffusion equations (P. de Mottoni), intermediate problems (W. R. Richert), multiparametric eigenvalue problems (G. F. Roach), eigenvalue problems in elliptic p.d.e.s (F. Stummel), asymptotic stability of diffusion equations (A. Tesei), continuation methods (H. J. Wacker), bifurcation problems in circular elastic plates (H. J. Weinitschke), eigenvalue problems in p.d.e.s (W. Wetterling).

Finite elements in geomechanics. Edited by G. Gudehus. John Wiley & Sons, 1977. xii + 573 pages. $37.50.

This book is based on the international symposium held at Karlsruhe in September 1975. It is devoted to the use of finite element methods in rock and soil mechanics. There are sixteen papers: 1. A survey (G. Gudehus); 2. Background to mathematical modelling (C. M. Gerrard); 3. Mechanical properties of sand and clay (G. Gudehus et al.); 4. Unified approach to soil mechanics (O. C. Zienkiewicz et al.); 5. Isotropic yield surfaces (O. C. Zienkiewicz et al.); 6. Soft clay (C. P. Wroth); 7. Soil-structure interaction (C. S. Desai); 8. Time-dependent soil-structure interaction (I. M. Smith); 9. Pore-water pressure (A. Verruijt); 10. Foundations, joints and fluids (E. L. Wilson); 11. Jointed rocks (R. E. Goodman); 12. Underground openings (K. Kovari); 13. Underground openings in rock (W. Wittke); 14. Water flow and mechanics of soil and rock (C. Lovis et al.); 15. Accuracy considerations (R. H. Gallagher); 16. Boundary element methods (P. K. Bunerjee et al.).

Mathematical foundations of computer science 1977 (Lecture Notes in Computer Science, Vol. 53). Edited by J. Gruska. Springer-Verlag, Berlin-Heidelberg-New York, 1977. xi + 595 pages. $18.40.

These are the Proceedings of the 6th Symposium, held at Tatranska Lomnica, Czechoslovakia, September 5-9, 1977. They consist of the texts of 15 invited papers and 46 short communications. The invited lecturers and short titles of their papers were: G. Ausiello (NP-complete problems), A. Blikle (program verification methods), L. Boasson (context-free languages), C. C. Elgot (finite automation from a flowchart-scheme point of view), E. Engeler (models of computation), A. P. Ecshov and V. E. Itkin (correctness of mixed computation), F. Gecseg and P. E. Toth (algebra and logic in theoretical computer science), B. Kacevicz and H. Wozniakowski (analytical computational complexity), H. A. Maurer and T. Ottmann (tree-structures for set manipulation problems), A. Salwicki (applied algorithmic logic), C. P. Schnorr (improved lower bounds on the number of multiplications/divisions which are necessary to evaluate polynomials), B. A. Trakhtenbrot (frequency algorithms and computations), L. G. Valiant (graphtheoretic arguments in low-level complexity), G. Wechsung (properties of complexity classes), J. B. Wright, E. G. Wagner and J. W. Thatcher (uniform approach to inductive posets and inductive closure).

Statistics: a biomedical introduction. By Byron W. Brown, Jr. and Myles Hollander. John Wiley & Sons, 1977. xiii + 456 pages. $15.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It was written as a text for an introductory course for persons interested in learning the fundamental concepts and techniques of statistical inference for application to the planning and evaluation of scientific studies in medicine and biology. The chapters are centered on actual studies from the biomedical sciences. No mathematical knowledge beyond college algebra is needed. The chapter headings are: 1. Introduction; 2. Elementary rules of probability; 3. Populations, samples, and the distribution of the sample mean; 4. Analysis of matched pairs using sample means, 5. Analysis of the two-sample location problem using sample means, 6. Surveys and experiments in medical research, 7. Statistical inference for dichotomous variables, 8. Comparing two probabilities, 9. Chi-squared tests, 10. Analysis of k-sample problems, 11. Linear regression and correlation, 12. Analysis of matched pairs using ranks, 13. Analysis of the two-sample location problem using ranks, 14. Methods for censored data. There are appendices with a glossary, answers and solutions to problems, and tables.

Circuit analysis with computer application to problem solving. By S. C. Gupta, J. W. Bayless and B. Peikari. Matrix Publishers, 1977. xiii + 546 pages. $25.95.

The objective of this textbook is to develop the basic principles of circuit analysis while at the same time introducing the computer as an aid in solving linear, nonlinear and time-varying problems. The computer programs developed range from simple to quite complex, and throughout the student is encouraged to learn to write his own programs. The language used is FORTRAN.

Applied abstract analysis. By Jean-Pierre Aubin. John Wiley & Sons, 1977. xi + 263 pages. $21.95.

This is a volume in the Wiley Series "Pure and Applied Mathematics". It is the object of this book to introduce and study the principal notions of topology in the elementary framework of metric spaces rather than in the general case of topological spaces. This approach permits the early consideration of various applications (in particular in optimization and in game theory). It is the hope of the author to illustrate in this manner the advantages of an abstract approach to concrete problems and thus to encourage the student to appreciate applied mathematics. There are five chapters: 1. Metric spaces: definition and examples; 2. Topological properties of metric spaces; 3. Continuous functions; 4. Operations on metric spaces; 5. Special properties of metric spaces. The principal propositions and theorems of the text are grouped in a resumé at the conclusion, giving a concise review of the essential results under each of eleven headings. Readers are assumed to be familiar with elementary differential and integral calculus of one and several variables. There is a set of excercises.

Navier-Stokes equations: theory and numerical analysis. By R. Temam. North-Holland Publishing Co. 1977. xvi + 454 pages. $45.00.

This survey describes the present state of the mathematical theory of Navier-Stokes equations of viscous incompressible fluids and examines many recently-developed methods of numerical solution of these equations at moderate Reynolds numbers. The mathematical foundations of the methods are investigated as well as some practical questions. The book is self-contained in that it presents many classical results related to the mathematical theory of the equations. There are three chapters: 1. The steady-state Stokes equations (function spaces, existence and uniqueness, discretization, numerical algorithms, slightly compressible fluids); 2. The steady-state Navier-Stokes equations (existence and uniqueness, discrete inequalities and compactness theorems, approximations, bifurcation theory and non-uniqueness results); 3. The evolution Navier-Stokes equations (the linear case, compactness theorems, existence and uniqueness, discretization, approximations). There is a bibliography of over 250 items.

Scientific analysis of the pocket calculator. By Jon M. Smith. John Wiley & Sons, 1975. xii + 445 pages. $13.75.

This is the second edition of the book first published in 1975. It is revised and updated to reflect the changes in available models and in potential uses. There are five parts: 1. Introduction (two chapters); 2. Numerical evaluation of functions (two chapters); 3. Advanced analysis (six chapters: Fourier analysis, integration, linear systems simulation, Chebyshev and rational approximations, roots of functions, statistics and probability); 4. The programmable pocket calculator (three chapters); 5. Financial analysis (two chapters). There are four appendices on tricks, matrix analysis, complex numbers and functions, and common formulae, respectively.

Linear estimation and stochastic control. By M. H. A. Davis. Halsted Press, 1977. xii + 224 pages. $14.95.

This book deals with problems of estimation and control in linear stochastic dynamical systems, i.e. systems represented by linear vector differential equations of the form

$$\dot{x}_t = Ax_t + Bu(t) + Cw_t$$

where x_t is the state vector, $u(t)$ a control input and w_t a random disturbance function. The types of disturbances considered are white noise and processes which can be generated from it. The chapter headings are: 1. finite-dimensional linear estimation; 2. stochastic processes and linear estimation; 3. orthogonal increment processes; 4. estimation in dynamical systems; 5. linear stochastic control; 6. an outline of further developments. An appendix discusses independence and conditional expectation.

Statistics of random processes II: applications. By R. S. Liptser and A. N. Shiryayev. (Translated by A. B. Aries.) Springer-Verlag, New York, 1978. x + 339 page. $29.80.

This second volume of a treatise on filtering of random processes contains the following chapters: 11. Conditionally Gaussian processes; 12. Optimal nonlinear filtering: interpolation and extrapolation of components of conditionally Gaussin processes; 13. Conditionally Gaussian sequences: filtering and related problems; 14. Application of filtering equations to problems of statistics of random sequences; 15. Linear estimation of random processes; 16. Application of optimal nonlinear filtering equations to some problems in control theory and information theory; 17. Parameter estimation and testing of statistical hypotheses for diffusion type processes; 18. Random point processes: Stieltjes stochastic integrals; 19. The structure of local martingales, absolute continuity of measures for point processes, and filtering. There is a bibliography of 217 items.

Algebraic number theory. (Pure and Applied Mathematics, Volume 41). By Robert L. Long. Marcel Dekker, Inc., 1977. ix + 192 pages. $18.50.

This textbook gives an exposition of the central parts of algebraic number theory, presupposing a one-quarter course on the topic and familiarity with basic topological notions of metric spaces. Indications of the various directions in which the theory can be pursued are also given. Using group algebra techniques to study arithmetic properties of Galois extensions of number fields, the textbook develops and applies the notions of localization and completion, and particularly emphasizes Abelian extensions of the rational field. There are numerous exercises and a bibliography.

Integrals and measures (Pure and Applied Mathematics, Volume 42). By Washek F. Pfeffer. Marcel Dekker, 1977. ix + 259 pages. $19.75.

This monograph develops the theory of measure and abstract Lebesgue integral by a systematic extension of certain linear functionals, in particular, the Riemann integral of continuous functions. The relationship between integrals and measures is studied in both abstract and topological situations. The product measures are discussed from the linear functional point of view. The last chapter gives a detailed exposition of some recent results on the regularity of Borel measures in locally compact Hausdorff spaces.

Probability and statistics for engineers and scientists. 2d edition. By Ronald E. Walpole and Raymond H. Myers. Macmillan Publishing Co., 1978. xii + 580 pages. $16.95.

This is the second edition of a text first published in 1972. There are several additions, such as a separate chapter on nonparametric tests, and one on multiple linear regression.

Introduction to probability and statistics. 4th edition. By Bernard W. Lindgren, G. W. McElrath and D. A. Berry. Macmillan Publishing Co., 1978. xii + 356 pages. $13.95.

This is the fourth edition of the text first published in 1959, and again in 1966 and 1969. The material covered is the same as in previous editions, but the text has been reorganized and almost completely rewritten. The examples and problems are mainly new.