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OF
APPLIED MATHEMATICS

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QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R.I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$25.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime ([']), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol \exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{a^2 + b^2^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOK REVIEW SECTION—

Viscoelasticity. By Wilhelm Flügge. Springer-Verlag, New York Heidelberg, Berlin, 1976. \$15.60.

This is the second edition of Professor Flügge's book, first published in 1967. The first edition was an outstanding text on the engineering analysis of viscoelastic structures. The present edition is enlarged by the addition of a chapter on beams on a continuous support and by a great deal of additional material on wave propagation.

The presentation relies on the differential operator formalism too much for my taste, but this approach has the advantage that problems can be worked out in all detail. A very wide range of significant engineering problems is treated. For this reason, Professor Flügge's book remains the standard text on the subject.

A. C. PIPKIN (*Providence*)

Nonlinear operators and differential equations in a Banach space. By R. H. Martin, Jr. John Wiley & Sons, New York, 1976. \$24.00.

The purpose of this book is to introduce the reader to some of the basic elements of nonlinear functional analysis which have proved to be applicable to differential equations in a Banach space—especially to the existence theory.

Approximately the first third of the book is devoted to introductory material from analysis, Lipschitz operators, resolvent operators, selfadjoint operators, the contraction principle, Schauder's fixed-point theorem, fixed-point theorems for α -contractions as well as in cones. Except for one chapter on some classes of integral operators, the remainder of the book is devoted to the theory of differential equations in a Banach space.

The differential equations considered are equations where either the vector field is bounded or is the sum of an unbounded linear operator and a bounded operator. The linear part is assumed to generate a C_0 -semigroup of linear transformations which allows the existence theory to be developed from the variation of constants formula. Some applications are made to partial differential equations in one space variable. More sophisticated applications to show the implication of the theory would have been desirable since the reader now must go elsewhere for this information.

The book is carefully written and presents clearly the topics discussed.

JACK HALE (*Providence*)

Continuum mechanics—concise theory and problems. By P. Chadwick. Halstead Press, New York, 1975. \$12.50.

This volume consists of a sequence of problems and examples on the mathematical foundations of continuum mechanics, some of them worked out in detail and others left for the student to solve. Since the book is intended for use in connection with a lecture course, the explanatory text is very terse. It appears that the book should be excellent in its intended function as a problem book to accompany a lecture course. Indeed, a student willing to work through the problems could learn continuum mechanics very well from the book alone, without benefit of lectures.

A. C. PIPKIN (*Providence*)

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Review Section.

Statistics and experimental design in engineering and the physical sciences, 2nd ed. By Norman L. Johnson and Fred C. Leone. John Wiley & Sons, New York, 1977. Vol. I, xiv + 592 pp. \$24.95. Vol. II, xiv + 489 pp. \$24.95.

These volumes in the Wiley Series in Probability and Mathematical Statistics were first published in 1964. The stated features of the work are: it is not meant to be encyclopedic or academically advanced; there is no interest in mathematical rigor or elegance for its own sake; it has the main aim of providing a sound and thorough training in useful applications of statistical techniques. A major feature of the new edition is the greatly extended coverage of solutions to exercises.

Introduction to decision theory. By J. Morgan Jones. Richard D. Irwin, Inc., Illinois, 1977. xiv + 369 pp. \$15.50.

This new volume in the Irwin Series in Quantitative Analyses for Business is designed to give the mathematically naive student an introduction to decision theory. It includes three aspects of decision theory: sampling strategies; practical applications; and modern computational tools, through APL and BASIC programs, which are used to help the conceptual learning process. An instructor's manual describes these programs and their use.

Quantitative methods for administration. By Clin E. Bell. Richard D. Irwin, Inc., Illinois, 1977. xvii + 596 pp. \$15.50.

This new volume in the Irwin Series in Quantitative Analyses for Business introduces mathematical—and particularly probabilistic—models of business situations and their statistical analysis. It requires no background in calculus and all the techniques are developed in detail. Some of the topics covered are simulation, linear programming, network models, dynamic programming, inventory and equipment replacement models.

Statistical methods for business and economics. By Roger C. Pfaffenberger and James H. Patterson. Richard D. Irwin, Inc., Illinois, 1977. xvii + 750 pp. \$16.95.

This new volume in the Irwin Series in Quantitative Analyses for Business covers the usual material with modest mathematical prerequisites. An Instructor's Manual with complete solutions to all exercises is available, as are computer programs in Fortran for the statistical procedures presented in the text.

Combinatorial connectivities in social systems. By R. H. Atkin. Birkäuser Verlag, Basel und Stuttgart, 1977. ii + 241 pp. sFr. 38.

This volume 34 of 'Interdisciplinary Systems Research' is subtitled: "An application of simplicial complex structures to the study of large organizations". An introduction to the approach can be found in the author's 'Mathematical Structure in Human Affairs' (Heinemann, London, 1974).

Introduction to probability and statistics. By Henry L. Alder and Edward B. Roessler. W. H. Freeman and Co., 1977. xii + 426 p. \$12.95.

This is the sixth edition of a work first published in 1960. Prerequisites are only two years of high-school algebra.

Statistical methods for digital computers. Edited by Kurt Enslein, Anthony Ralston and Herbert S. Wilf. John Wiley & Sons, New York, 1977. viii + 454 pp. \$24.95.

This is Volume III of the well-known 'Mathematical Methods for Digital Computers' by Ralston and Wilf, of which Volume I came out in 1960 and Volume II in 1967. Its 15 chapters, by various authors, are divided into 5 parts: Introduction, regression and discriminant analysis, principal components and factor analysis, cluster analysis and pattern recognition, time series.

Stochastic integration and generalized martingales. By A. U. Kussmaul. Pitman Publishing Co., London-San Francisco-Melbourne, 1977. iv + 163 pp. \$15.00.

This book is volume 11 in the 'Research Notes in Mathematics' series and develops the theory of the stochastic integral together with the necessary prerequisites from the theory of stochastic processes, for readers familiar with Banach spaces and with measure theory.

Zeta-functions: An introduction to algebraic geometry. By A. D. Thomas. Pitman Publishing Co., London-San Francisco-Melbourne, 1977. vii + 230 pp. \$15.00.

This book is volume 12 in the 'Research Notes in Mathematics' series and aims to explain the development of the statements and proofs of the Weyl conjectures from the analogous results for the Riemann zeta function, at the same time describing the transformation of classical algebraic number theory and algebraic geometry into the language of schemes.

Problems in the theory of functions of a complex variable. By L. Volkovysky, G. Lunts and I. Aramanovich. Mir Publishers, Moscow, 2nd 1977. 332 pp. \$5.40.

A collection of problems with answers or solutions, somewhat in the style of Polya and Szegő, and providing a wealth of interesting information.

Convolution integral equations with special function kernels. By H. M. Srivastava and R. G. Buschman. Halsted Press, New York, 1977. 164 pp. \$9.75.

The book attempts to collect all information available in the literature on the topic of the title. It also develops alternative methods for Volterra integral equations of the first kind where the kernels are given special functions; it presents tables of equations and their solutions, and an up-to-date bibliography.

An introduction to ordinary differential equations. By Etgen J. Garret and William L. Morris. Harper & Row, New York, 1977. x + 517 pp. \$14.95.

This is an elementary introduction, presupposing only beginning calculus, and includes difference equations, numerical methods, and applications.

Nonlinear networks. By Vaclav Dolezal. Elsevier Scientific Publishing Co., Amsterdam and New York, 1977. ix + 156 pp. \$29.95.

The aim of this book is to develop a general model of a nonlinear network governed by Kirchhoff laws, which encompasses all electrical, mechanical and hydraulic networks encountered in engineering sciences. The book is concerned mainly with the Hilbert network, i.e. an interconnection of finitely or countably many lumped elements, whose variables belong to a Hilbert space and obey Kirchhoff laws. The network elements are assumed to be described by, in general, nonlinear multivalued operators.

The main results of the book relate to solvability of networks, some qualitative properties, causality, driving-point-set-impedance and certain linear networks containing finitely many nonlinear elements. Only knowledge of basic functional analysis and classical network theory is presupposed.

Calculus. By Jack G. Ceder, David L. Outcalt. Allyn and Bacon, Inc., Boston, London, Sydney, Toronto, 1977. xiii + 1050 pp. \$20.95.

A standard calculus course for students in the physical sciences, engineering, biology, economics and mathematics. It features an intuitive, geometrically motivated approach. Only especially enlightening proofs are included. Numerous applied examples and exercises from various fields.

Parameter estimation in engineering and science. By James V. Beck and Kenneth J. Arnold. John Wiley & Sons, New York, 1977. xix + 501 pp. \$24.95.

This book, in the Wiley Series in Probability and Mathematical Statistics, features clear descriptions of methods for non-linear parameter estimation; new estimation methods such as sequential estimation and the use of prior information; extensive discussion of design of nonlinear experiments, of design of experiments for model-building, and of design in cases including the partial differential equation of heat conduction; careful treatment of statistical assumptions and of correlated measurement errors.

Statistical concepts and methods. By Gouri K. Bhattacharyya and Richard A. Johnson. John Wiley & Sons, New York 1977. xv + 639 pp. \$14.95.

A non-mathematical introductory statistics text, with illustrative data analyses from diverse fields. Aims at the instillation of correct attitudes in approaching statistical problems.

Advanced engineering mathematics. By A. C. Bajpai, L. R. Mustoe, D. Walker. John Wiley & Sons, New York, 1977. x + 578 pp. \$24.95 cloth; \$11.95 paper.

The book is aimed at second year students in universities, colleges and institutes of technology, in particular those studying for the Council of Engineering Institutions, Part II examination in the United Kingdom.

Computer methods in image analysis. Edited by J. K. Aggarwal, Richard O. Duda, A. Rosenfeld. IEEE Press. John Wiley & Sons, New York, 1977. vi + 466 pp. \$24.95 cloth; \$14.95 paper.

This collection of 31 landmark papers focuses on the use of computers to extract useful information from digitized pictures. The seven major parts of the volume are: correlation and matching, segmentation: regions, region representation and description, segmentation: boundaries, boundary representation and description, scene analysis: the blocks world, scene analysis: the natural world.

A theory of programming language semantics. Part a & b. By Robert Milne and Christopher Strachey. Halsted Press, London, 1976. 858 pp. \$35.00.

This work consists of two volumes: part a treats fundamental concepts and mathematical foundations, and part b standard, store and stack semantics. Detailed indices, and appendices, are also included in part a. It is an advanced and difficult work which will be of great interests to specialists in the field.

Linear programming and network flows. By Mokhtar S. Bazaraa and John J. Jarvis. John Wiley & Sons, 1977. x + 565 pp. \$21.00.

The simplex method of linear programming is specialized to take advantage of particular program structures. Techniques are illustrated by numerical examples and given detailed mathematical analyses showing convergence to an optimal solution.

Elementary differential equations. By William E. Boyce and Richard C. DiPrima. John Wiley & Sons, New York, 1977 3rd Ed. xiv + 451 pp. \$15.95.

This new edition of a well-known text is supplemented by: applications of the qualitative theory of differential equations to biology and ecology, a reorganized chapter on systems of linear equations, a revised discussion of multistep numerical methods. Other revisions aim to clarify the book and increase its usefulness.

Matrix computation for engineers and scientists. By Alan Jennings. John Wiley & Sons, New York, 1977. xv + 330 pp. \$23.95.

An elementary introduction, with chapters on sparsity and on computer implementations.

Computer power and human reason (From Judgement to Calculation). By Joseph Weizenbaum. W. H. Freeman and Co., San Francisco, 1976. xii + 300 pp. \$9.95.

A philosophical—and controversial—discussion of the limits to the power of computers in simulating human reasoning.

Fate of pollutants in the air and water environments. Part 1: Mechanism of interaction between environments and mathematical modeling and the physical fate of pollutants. Edited by L. H. Suffet. John Wiley & Sons, New York. xx + 484 pp. \$24.95.

This is volume 8 in the Wiley Series in Advances in Environmental Science and Technology and contains papers presented at a symposium at the 165th National American Chemical Society Meeting in April 1975 in Philadelphia.

Completeness and basis properties of sets of special functions. By J. R. Higgins. Cambridge University Press, New York, 1977. x + 134 pp. \$19.95.

This is volume 72 of the 'Cambridge Tracts in Mathematics' and presents an exposition of methods for testing sets of special functions for completeness and basis properties, mostly in L^p and L^2 spaces.

Problems of analytical statistics. By Yu. V. Linnik. Statistical Publishing Society, Calcutta, 1975. ix + 115 pp. \$15.00.

Academician Linnik (1915-1972) was one of the great mathematicians of this century. This book presents some of his outstanding work on the Behrens-Fisher problem, sufficient statistics, the Fisher-Welch-Wald test, signal detection, and other topics. It is based on lectures given at the University of Paris in 1966, and is a continuation of Linnik's 'Statistical problems with nuisance parameters' of which the American Mathematical Society published an English translation in 1968.

Mathematical ecology. By E. C. Pielou. John Wiley & Sons, New York, 1976. x + 385 pp. \$19.50.

This is the second edition of a well-received book first published in 1969, and presents a modernized and expanded treatment of the same topics. The chapter headings are: 1. Population dynamics, 2. Spatial patterns in one-species populations, 3. Spatial relations of two or more species, 4. Many-species populations.

Mathematical models for surface water hydrology. Edited by Tito A. Ciriani, Ugo Maione and James R. Wallis. John Wiley & Sons, New York. xiii + 423 pp. \$27.50.

These are the Proceedings of a Workshop held at the IBM Scientific Center, Pisa, Italy, December 9-12, 1974. Problems relative to synthetic hydrology and flood modelling are dealt with in Parts 1 and 2 to give the theoretical basis for the applications covered in Part 3 on computer programs.