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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R. I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$25.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and α , kappa and k , mu and μ , nu and ν , and v , eta and η .

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2b}}{\cos \frac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{(a + (b + cx)^n) \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (for books—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354-372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOK REVIEW SECTION—

Perturbation methods in fluid mechanics. By Milton Van Dyke. New, enlarged and annotated edition. Parabolic Press, P. O. Box 3032, Stanford, CA 94305. 274 pp. \$7.00.

It is hard to imagine any fluid dynamicist or for that matter any scientist using asymptotic methods who has not used Milton Van Dyke's book at some time. Although other fine books on similar and related topics have appeared since publication of *Perturbation methods* in 1964, Van Dyke's treatment remains unique and useful. The book being long out of print, Van Dyke himself has now re-issued it—and at the original price of \$7.00, which itself is an achievement.

The re-issue corrects errors in the original text. Also added are a set of interesting and sometimes provocative notes (34 additional pages) updating and discussing portions of the book; and a greatly expanded and more up-to-date bibliography. The last two items are themselves worth the price of admission.

L. SIROVICH (*Providence, R. I.*)

Highway traffic analysis and design. By R. J. Salter. Addison-Wesley Publishing Co., Reading, Mass., 1974. vi + 373 pp. \$17.50.

This is a text on the fundamentals of highway traffic analysis and design. It includes sections on principles of land use, common transportation planning methods and the economic valuation of alternative highway designs. The fundamentals of traffic flow analysis are described and followed by sections which discuss noise generation and pollution, congestion restraint and road pricing. Particular attention is devoted to highway intersection problems: traffic signal control, signal-controlled intersections and area-wide signal control.

This text is designed for self-study and includes, at the end of each chapter, a section containing questions and answers. The mathematical background required of the reader is very modest.

E. F. INFANTE (*Providence, R. I.*)

Introduction to mathematical control theory. By Stephen Barnett. Clarendon Press, Oxford, 1975. viii + 264 pp. \$15.00.

This book is based on lectures given to British undergraduates in mathematics and to postgraduate engineers. It is a good introduction to the basic and fundamental concepts of control theory, emphasizing state space methods (differential equations). It can be recommended as a text for a one-semester course in applied mathematics following an introductory course in differential equations and linear (matrix) algebra, as a good book for mathematicians who teach courses in differential equations and linear algebra who do not know these applications, and as a good introduction for engineers and scientists to the mathematical aspects of modern control and stability theory. The author has striven for, and achieved, both conciseness and range and hopes "that it will help to promote interest in this exciting field, thus providing at least a small counterbalance to the deadweight of British classical applied mathematics."

J. P. LASALLE (*Providence, R. I.*)

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Review Section.

Electric probes in stationary and flowing plasmas—theory and applications. By D. M. Chung, L. Talbot and K. J. Touryan. Springer-Verlag, 1975. xi + 150 pp. \$19.80.

This is vol. 11 in the series *Applied Physics and Engineering*. It provides a comprehensive account of the latest information available on probes, particularly in the areas of transitional and continuum flow phenomena. It offers, also, a critical appraisal of the more significant probe theories and experimental investigations for all aspects of probe application.

Tensors, differential forms, and variational principles. By David Lovelock and Hanno Rund. John Wiley and Sons, 1975. xi + 364 pp. \$21.95.

This is a book in the series *Pure and Applied Mathematics*. Chapters 1–5 (which constitute a one-semester undergraduate or junior graduate course) comprise: affine tensor algebra in Euclidean geometry, tensor analysis on manifolds, and the calculus of differential forms. Chapters 6–8 (which constitute a one-semester graduate course) comprise: invariant problems in the calculus of variations, Riemannian geometry, invariant variational principles and physical field theories.

The theory of tensors and forms is developed by a process of successive abstraction, starting with physical examples. The treatment of the calculus of variations of single and multiple integrals is based on Carathéodory's method of equivalent integrals. The effects of variance postulates on variational principles is explored, focussing on relativistic field theories. A wide variety of problems is included.

Theoretical statistics. By D. R. Cox and D. V. Hickley. Chapman and Hall, London; distributed in the U. S. A. by Halsted Press, a division of John Wiley & Sons, Inc., 1974. xii + 511 pp. \$18.00.

This is an advanced text on modern statistical inference, emphasizing general concepts rather than mathematical rigor. It is assumed that the reader is familiar with the main elementary statistical techniques and how they are used in practice. Knowledge of the elements of probability theory, the standard special distributions and the theory of the linear model is also assumed.

The book is unique in its thoroughly modern treatment of quite advanced matters and in yet keeping the mathematical level as elementary as possible. Since it deals primarily with the theory of statistical methods for the interpretation of scientific and technological data, it is thus of interest both to students of statistics and students of applications.

Continuum mechanics of viscoelastic liquids. By R. R. Huilgol. Halsted Press, a division of John Wiley & Sons, 1975. xix + 367 pp. \$27.00.

This book is a systematic outline of nonlinear continuum mechanics, especially of the issues involved in the study of non-Newtonian liquids. It is concerned with developments during the last thirty years associated with the names of Ericksen, A. E. Green, Gurtin, Noll, Oldroyd, Pipkin, Rivlin, Tanner, Truesdell, and others.

Chapter headings: 1. Kinematics of fluid flow; 2. Kinematics of motions with constant stretch history; 3. Objectivity and changes of local configuration; 4. Balance equations; 5. Formulation of constitutive equations—simple fluids; 6. Entropy and the equations of motion; 7. Dynamics of motion with constant stretch history; 8. Other constitutive equations and responses in some flows; 9. Perturbation about an arbitrary ground state: nearly viscometric flows; 10. Perturbations for simple constitutive equations; 11. Experimental results: theoretical basis.

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Elastodynamics. By A. Cemal Eringen and Erdogan S. Suhubi, Academic Press. Volume I: *Finite motions*, 1974, xv + 341 pp., \$38.50; Volume II: *Linear theory*, 1975, xiii + 662 pp., \$65.00.

These volumes constitute a rigorous study of the mathematical theory of elastodynamics. Volume I consists of four chapters and two appendices: 1. Basic theory—the development of the fundamental equations of the dynamical theory of thermoelasticity; 2. Propagation of singular surfaces; 3. Finite motions of elastic bodies—radial oscillations of cylinders and spheres; 4. Small motions superimposed on large static deformations; appendices on tensor analysis and the theory of characteristics. Volume II on the mathematical problems relevant to the linear theory contains five chapters: 5. Fundamental theorems—uniqueness, reciprocal, integral representation, variational principles, etc.; 6–8. Exact solutions of significant problems in one, two and three dimensions, respectively; 9. Diffraction of elastic waves.

Optimization theory—the finite-dimensional case. By Magnus R. Hestenes. John Wiley & Sons, 1975. x + 447 pp. \$24.95.

This book is devoted to the mathematical theory of optimization; it does not discuss computational procedures or applications. It covers such topics as unconstrained and linear constrained extrema, the Lagrangian multiplier rules for extrema subject to equality and inequality constraints, augmentability, convex programming, connections between linear and nonlinear programming, generalized multiplier rule, theory of derived sets, applications to matrices and quadratic forms, and the Raleigh quotient theory for determination of eigenvalues and eigenvectors.

The author is well known for his contributions to the infinite-dimensional optimization case (the calculus of variations) and it is consequently natural that this book can be viewed as an introduction to variational theory as well as an exposition of finite-dimensional optimization theory. There are many exercises and an extensive bibliography.

Lie groups and their representations. Edited by I. M. Gelfand. Halsted Press, a division of John Wiley & Co., 1975. 726 pp. \$79.00.

This volume contains the lectures delivered at the Summer School of the Bolyai Janos Mathematical Society, Budapest, Hungary, in 1971. It contains several complete monographs on the structure and representation of Lie groups, such as: "On certain series of representations", by B. Konstant, "On arithmetic varieties", by D. A. Kajdan, "On discrete subgroups", by G. A. Margulis. The 24 papers illustrate the ties between representation theory and number theory and the relation of representation theory to algebraic and differential geometry, and to quantum mechanics, field theory and harmonic analysis.

The structure of turbulent shear flow. Second edition. By A. A. Townsend. Cambridge University Press, 1975. xi + 429 pp. \$42.50.

This is a completely revised edition of the work first published in 1956. It provides a systematic and comprehensive discussion of turbulent motion, and applies the theory and associated concepts to: free turbulent flows (such as wakes and jets), wall flows in pipes and boundary layers, flows affected by buoyancy forces such as heat plumes and the atmospheric boundary layer, and flows with curved streamlines.

Mathematical programming. Second edition. By Claude McMillan. John Wiley & Sons, 1975. xi + 650 pp. \$18.95.

This is a revised edition of the book first published in 1970. It is intended as an introduction for the non-mathematician and thus includes no formal convergence proofs, only intuitively appealing arguments. It presents algorithms for linear programming, nonlinear programming, integer programming and network optimization, and goal programming (multiple objectives).

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Prospects of fracture mechanics. Edited by G. C. Sih, H. C. van Elst and D. Broek. Noordhoff International Publishing, Leyden, 1974. xviii + 643 pp.

These are the proceedings of an international conference held at Delft University of Technology, June 1974. They are arranged in the following sections: inclusion and void growth, energy criteria and path-independent integrals, fatigue crack propagation, dynamics and arrest, structural analysis, stress analysis and analytical modeling, fracture experiments and testing.

Mathematical techniques in chemistry. By Joseph B. Dence. John Wiley & Sons, 1975. xv + 442 pp. \$14.50.

This is an introduction to calculus, series expansions, differential equations, vectors and tensors with examples drawn from different branches of chemistry.

The mechanics of the contact between deformable bodies. Edited by A. D. de Pater and J. J. Kalker, Delft University Press, 1975. xiii + 414 pp. Dfl. 150.

These are the proceedings of the symposium of the International Union of Theoretical and Applied Mechanics, held at Enschede, Netherlands, 20-23 August 1974. There are papers by J. J. Kalker, K. L. Johnson, M. Boucher, J. Dundues, D. A. Spence, A. Ju. Islinskij, B. L. Abramjan, G. M. L. Gladwell, J. B. Ablas, L. E. Goodman and L. M. Keen, and J. Christoffersen.

Modern introduction to classical mechanics and control. By David N. Burghes and Angela M. Downs. Halsted Press, a division of John Wiley & Sons, 1975. 320 pp. \$18.50.

This undergraduate book contains chapters on: kinematics, Newtonian mechanics, rocket flight performance, fields of force, conservation theorems, space dynamics, stability, oscillations and vibrations, variational calculus, and optimal control.