QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXXIII JANUARY • 1976 NUMBER 4

QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXXIII 1975/6

Printed by the
William Byrd Press, Inc.
Richmond, Virginia

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BOOK REVIEWS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

–BOOK REVIEW SECTION–

Fundamentals of mathematics. Edited by H. Behnke, F. Bachmann, K. Fladt and W. Süss. Volume I: *Foundations of mathematics: the real number system and algebra.* vii + 549 pp. Volume II: *Geometry,* vii + 685 pp. Volume III: *Analysis,* xiii + 541 pp. Translated from the German by S. H. Gould. The MIT Press, Cambridge, Mass., 1974. $15.95 each volume, $40.00 the set.

This remarkable work is the product of about 150 authors and coordinators. Each chapter is, in general, written by two authors: one a university professor active in research and the other a teacher of long experience in the German educational system. The authors met frequently to coordinate their progress. The product is an exceedingly well written (and excellently translated) work, thoroughly modern in spirit and emphasizing interrelations between the topics treated. It is abundantly supplied with references.

The first volume opens with a section on mathematical foundations. It covers such topics as axiomatization, the concept of an algorithm, proofs, the theory of sets, the theory of relations, Boolean algebra and antinomies. The closing section, on the real number system and algebra, takes up rational numbers, groups, linear algebra, polynomials, rings and ideals, number theory, algebraic extensions of a field, complex numbers and quarternions, lattices, the theory of structure, and Zorn's lemma.

Volume II begins with eight chapters on the foundations of geometry, followed by eight others on its analytic treatment. The latter include discussions of affine and Euclidean geometry, algebraic geometry, the Erlanger program and higher geometry, group theory approaches, differential geometry, convex figures, and aspects of topology.

The final volume, on analysis, covers convergence, functions, integral and measure, fundamental concepts of probability theory, alternating differential forms, complex numbers and variables, points at infinity, ordinary and partial differential equations, difference equations and definite integrals, functional analysis, real functions, and analytic number theory. A concluding chapter examines "The changing structure of modern mathematics".

W. FREIBERGER (*Providence*)

Theory of branching of solutions of nonlinear equations. By M. M. Vainberg and V. A. Trenogin. Noordhoff International Publishers, Groningen, 1974. xxvi + 485 pp. Dfl. 112.

The theory of bifurcation or branching of solutions of nonlinear equations is certainly of utmost importance in the applications. Since the pioneering work of Liapunov and Schmidt at the turn of the century, there has been a tremendous amount of literature on this subject, and general methods have been developed for the solution of this problem. This book is devoted to a description of these methods for the local theory of analytic systems together with applications.

In an attempt to make the book accessible to a wide audience, the authors present in the first six chapters the detailed theory of implicit functions and Newton's polygon with applications to integral, differential and integro-differential equations. The abstract theory for Fredholm operators in a Banach space with applications to other integral equations, partial differential equations and the perturbation of eigenvalues is given in chapters 7, 8, 9. The last chapter is devoted to a brief discussion of other applications.

Analytical computation of the bifurcating solutions is emphasized throughout. Although they are very important, the reviewer feels this emphasis was carried to the extreme, especially in chapter 4 where about 50 pages are devoted to explicit general formulas for some coefficients.

In spite ot this, the book is an important contribution to the subject and should be required reading for students in bifurcation theory. It contains the basic methods and many interesting applications.

JACK K. HALE (*Providence*)

Introduction to control theory. By O. L. R. Jacobs. Clarendon Press, Oxford, 1974. xii + 365 pp. $21.75.

This book gives an excellent and comprehensive introduction to control theory for discrete and continuous, deterministic and stochastic systems. It covers both frequency response and state variable methods, and optimization and stability. It does this quite well, assuming a minimum introduction to elementary analysis, matrix algebra, and probability theory. The presentation is nontheoretical and straightforward. There are good illustrations and problems (with answers) and sufficient references. The book was designed for an undergraduate course in engineering science, but is not strictly an engineering text. The subject is treated as a branch of applied mathematics and is a good introduction for all scientists and science students. There are statements which are not completely correct, and some definitions are not adequate (for example, stability on p. 209). But this is excusable, and there is no pseudo-theory.

Part I (166 pages) covers deterministic linear systems. In the preface the author says, "Control theory is similar to other branches of applied mathematics in that the majority of solved theoretical problems are linear and the majority of real control systems non-linear." Part II (63 pages) deals with nonlinear systems and Part III (107 pages) with uncertain systems, including a brief review of the required probability theory. The dual treatment of both discrete and continuous systems is an outstanding feature of the book, as is its broad coverage of control theory.

Highly recommended to all scientists and students as an introduction.

J. P. LaSalle (*Providence*)

Mathematical problems in biology. Edited by Pauline van der Driessche. Springer-Verlag, Berlin, Heidelberg, and New York, 1974. iv + 280 pp. $11.50.

This is the second volume of a new series of Lecture Notes in Biomathematics. The aim of the series is "to report new developments in biomathematics research and teaching" and to do so "quickly, informally and at a high level". This volume is a partial collection of papers (about 50%) which were presented at the conference "Some Mathematical Problems in Biology" held at the University of Victoria, Victoria, B.C., on May 7–10, 1973. The contributors are mathematicians interested in biological problems and scientists engaged in developing mathematical models in biology and biomedicine. The volume should be of interest to the same people and to teachers of students in the biomedical sciences.

The fields of mathematics represented are differential and difference equations (stability, limit cycles and asymptotic behavior in general, and diffusion and wave propagation), stochastic models, optimization and optimal control, some game theory and numerical methods, and general discussions of mathematical modelling. Applications are to the dynamics of competitive populations, epidemics, mechanochemical systems, bioeconomics and ecology in general, and morphogenesis, among others.

The list of problems posed is disappointing (two on combinatorial optimization, one on statistics, and one on limit cycles in 3-space). The bibliographies following the papers and the one at the end of the volume are useful. The lack of titles and pages in some references is annoying.

The invited lectruers were: J. J. Blum, J. Lubiner, C. W. Clark, J. D. Cowan, C. S. Holling, N. D. Kazarinoff, D. Ludwig, Z. A. Melzak, E. C. Pielou, and A. T. Winfree.

J. P. LaSalle (*Providence*)

Nonlinear differential equations of higher order. By R. Ressig, G. Sansone, and R. Conti. Noordhoff International Publishers, Leyden, 1974. xiii + 669 pp. Dfl. 180.

In 1963 the authors published a book (*Quantitative Theorie nichtlinear Differentialgleichungen,* Edizione Cremonese, 1963) which was an excellent selection and presentation of results for second-order equations. They decided then to continue their collaboration and to write a similar book devoted primarily to equations of third and fourth order. This is a translation of that book (*Nichtlineare Differentialgleichungen höhrer Ordnung,* Edizione Cremonese, 1969), and considering the range of specialized results

is an amazingly fine and stimulating report on what is known. While useful, a survey or simply a collection of facts can be absolutely boring. The authors, all experts with a wide knowledge of their subject and access to the world's literature, succeed where most of us would fail.

Any scientist whose work involves differential equations should find the book useful and interesting. Anyone involved in research on differential equations will find something new and will find specialized results to ponder. Anyone teaching the subject will find good illustrative examples of general results and general methods. A serious student can learn a great deal from it. The translation is uniformly good.

The first three chapters are devoted to general qualitative methods (stability, boundedness, oscillations, comparison theorems, and behavior of solutions near singular points (equilibria), limit cycles and infinity). Chapters 4–6 are the study of specialized equations of third and fourth order and contain recent results not easily accessible elsewhere. Stability and boundedness of solutions of systems of arbitrary order with nonlinearities in separate variables are discussed in chapter 7. Chapter 8 is a concise treatment of the Lur'e control problem (absolute stability). At the end of each chapter there is a list of references. As the authors point out, "the lion's share of these research results belongs to the USA and USSR". Their presentations are their own and often are improvements of the results of others—improvements not only in content and proof but in exposition—and have a unity and uniformity that does not exist in the original papers.

J. P. LaSalle (*Providence*)