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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2b}}{\cos \frac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\textstyle\int u^{-1} \sin u \, du \text{ is preferable to } \displaystyle\int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEW SECTION—

Introduction to asymptotic analysis. By J. D. Murray. Oxford University Press, New York, 1974. $11.25.

This book gives an introductory account of basic methods for the asymptotic approximation of functions defined by integrals and ordinary differential equations. It contains numerous exercises and worked examples and is suitable for a senior or first-year-graduate course. Chapter headings are: Asymptotic Expansions, Laplace's Method for Integrals, Method of Steepest Descents, Method of Stationary Phase, Transform Integrals, Differential Equations.

LAWRENCE SIROVICH (*Providence*)

Essential mathematics for economists. By J. Black and J. F. Bradley. London, John Wiley & Sons, 1973. xx + 268 pp. $15.95 cloth, $7.95 paper.

This book is designed to give undergraduate students within the English university system some of the basic mathematical tools needed to understand the development of modern economic theory. It assumes that students are "interested in economics, and [are] concurrently following an economics course." Further, it assumes virtually no background in mathematics, starting the discussion with the simplest notions of linear functions. From there it covers the usual topics including simultaneous linear equations, differentiation, integration, optimization under a constraint, and difference and differential equations.

When the book is used for its stated purpose, it will probably be found adequate. Even here, however, some difficulties arise. In discussing the use of mathematical techniques in economics, relevant and important economic topics are used. For example, production theory is used to illustrate the use of partial derivatives and optimization under a constraint, and consumer surplus computations are used in the integration chapter. But the small amount of space used here for these illustrations cannot begin to give a comprehensive picture of the theory involved nor of its economic implications. Perhaps some of these discussions could have been forgone and more complete coverage of the mathematical techniques given. The student is supposed to be receiving his economics training in other courses.

Some examples are given within each chapter and a few exercises (three to seven in number) using economic topics are given at the end of each chapter. Each of the exercises is worked out in elaborate detail. The student can learn a good deal by following the working-out of the exercises, but then, of course, there are no exercises in the book for which answers are not given and which thus could be used for homework assignments. Many of the exercises seem forced and artificial, but this usually results from sticking to economic examples using a particular mathematical technique. Interesting economic theory and problems nearly always use combinations of several techniques and thus would be unsuitable to end-of-chapter exercises. It might have been better to give more exercises in just the mathematical technique involved so the students will at least get practice in using the mathematics.

There are no references in the book which could lead the student to places where further development of either the mathematics or the economics takes place. The student is denied any guide as to how to proceed further and the instructor is denied guidance to the literature which would allow him to elaborate on certain topics covered in the book. This is a serious deficiency for a book which will be used in a class which some students may never go beyond but which other students may be using to start building a base towards future graduate work.

Finally, some question must be raised about the book's purpose. It is clearly too limited in its coverage to be useful for graduate students or as a reference work for researchers. For that purpose, one must turn to a book like Alpha C. Chiang, *Fundamental Methods of Mathematical Economics*, 2nd ed. (New York: McGraw-Hill, 1974). But even for undergraduate students in the United States context, the book will be of limited use. The best and most quantitatively-oriented students will generally take several mathematics courses along with their economics, either as electives or as part of a regular mathematics-economics concentration pattern. This program will usually consist of four to six mathematics courses including calculus, differential equations, and linear algebra (which is not covered in this book at all) and may include some probability, statistics, and advanced analysis. This book is clearly not useful for these people.

For the non-mathematical undergraduate student, a varied and rich economics program seems quite viable using virtually none of the mathematical techniques covered in this book. While explanations of some economic theory concepts are certainly easier using even simple calculus, most U.S. colleges and universities have not so far judged that the gain is worth the time taken and the mental struggles of the students which would be encountered in teaching mathematical techniques at the level of this book.

M. B. SCHUPACK (*Providence*)

Integral geometry and inverse problems for hyperbolic equations. By V. G. Romanov. Springer Tracts in Natural Philosophy, Volume 26, Springer-Verlag, Berlin, 1974. 151 pp. $23.80.

The practical motivation behind the topic studied in this book comes from geology. Assuming that the propagation velocity of seismic waves is a monotonic function of depth below the surface of the earth, it is possible to infer the velocity distribution from surface measurements. In other words, given a hyperbolic equation where the elliptic part is only partially specified, we draw conclusions about the tensor from data on the boundary.

This type of problem can be viewed as one from integral geometry. Indeed, the surface measurements are integrals along the the geodesics belonging to the Riemannian geometry generated by the tensor. Having observed these integrals one wants to draw conclusions about the integrand, and this is classical integral geometry.

The author gives a very clear presentation of the theoretical aspects of this problem area. A short chapter at the end of the book contains some numerical results and a discussion of the applicability of the methodology that has been presented.

This is an elegant little book, highly technical but also quite readable.

U. GRENANDER (*Providence*)

Numerical solution of integral equations. Edited by L. M. Delves and J. Walsh. Clarendon Press, Oxford, 1974. x + 339 pp. $14.50.

This collection of chapters is based partly on material presented in a Joint Summer School, July 1973 organized by the Department of Mathematics, University of Manchester and the Department of Computational and Statistical Science, University of Liverpool. The writers include C. T. H. Baker, I. Barrodale, A. Burton, L. M. Delves, I. Gladwell, D. F. Mayers, G. F. Miller, L. P. Rall, G. T. Symm, F. Ursell, R. Wait, and J. Walsh.

The purpose of the book is to present the main problems and methods for the numerical solution of integral equations. Some theoretical background material is given as well as a number of applications. While this book is by no means a self-contained presentation, since it requires a considerable degree of familiarity with integral equations and the general techniques of numerical analysis, it could very well form the basis for a one-semester course or be a useful and important addition to the practicing numerical analyst's reference shelf. Numerous references to the recent literature put the reader in touch with the latest thinking and experience. One will learn (among other things) of quadrature methods, product integration methods, collocation methods, expansion methods, projection methods, linear programming and other optimization methods, Rayleigh-Ritz-Galerkin methods, multi-step methods, Runge-Kutta methods, block methods, generalized Newton's, finite-element methods, conversion methods, methods for treating singular kernels, iterative methods, probabilistic methods, etc.

Among the types of equations considered are the Volterra equation of first and second kind, the Fredholm equation of first and second kind, the associated eigenvalue problems, nonlinear integral equations, and integro-differential equations, ordinary and partial.

Among the applications are problems in two- and three-dimensional potential theory, water wave theory, scalar diffraction theory and scattering theory in quantum mechanics.

One feels a gap between the pure theory presented and the numerical practice—almost as though the theory is there largely for Comfort and Reassurance. This is not so much a fault of the book as it is a gap in the state of the art. On the other hand, despite a plethora of methods, the book suffers from a paucity of numerical results. Admittedly, it isn't always clear what one learns from a specific numerical result, but it can be a Great Comfort: look at all the trouble that fellow went to and what good answers he seems to be getting.

A chapter on library programs should be of interest to those faced with production computations.

P. J. DAVIS (*Providence*)

Theory of bifurcations of dynamical systems on a plane. By A. A. Andronov, E. A. Leontovich, I. I. Gordon and A. G. Maier. Halstead Press, John Wiley & Sons, New York, 1973. xiv + 482 pp. $42.50.

The basic problem in differential equations is to classify systems according to the behavior of their trajectories. More specifically, with an appropriate topology on the vector fields, one says two vector fields X, Y are equivalent if it is possible to map the trajectories of X onto the trajectories of Y by means of a homeomorphism. A vector field X is said to be structurally stable if it is equivalent to every other vector field Y sufficiently close to X.

If the vector field X_λ depends on a real scalar parameter λ, then it is possible there is a λ_0 such that X_λ belongs to one equivalence class for $\lambda < \lambda_0$ and to a completely different class for $\lambda > \lambda_0$. Such a point $\lambda = \lambda_0$ is a bifurcation point for X_λ. The bifurcation points represent vector fields which have elements from at least two equivalence classes in every neighborhood.

The present book discusses these questions in great detail for planar vector fields. It is the culmination of many years of effort on a subject begun by Andronov, Pontryagin and Leontovich in 1938-1940. The first half of the book is devoted to the classification and the open and denseness of structurally stable systems in the plane. The last part classifies those systems X which satisfy first-order instability; i.e. there is a neighborhood of X such that each Y in this neighborhood is either structurally stable or equivalent to X. The systems which satisfy first-order instability are the simplest bifurcation points.

It is unfortunate that the publication of the book in English translation was delayed for so long and that the English translation does not contain a supplementary chapter on recent developments in the plane by Sotomayor [Proc. Am. Math. Soc. **74** (1968), 722-726] as well as higher order systems [see Smale, Bull. Am. Math. Soc. 73 (1967), 747-817, and the book *Dynamical systems*, ed. by M. Peixoto, Academic Press, 1973].

In spite of this remark, the book is a welcome addition to the subject of differential equations. The topics are extremely important and are presented in a manner to be accessible to students and research workers in both mathematics and the applied sciences.

J. K. HALE (*Providence*)