

QUARTERLY
OF
APPLIED MATHEMATICS

EDITED BY

H. W. BODE
P. J. DAVIS
E. T. ONAT
J. L. SYNGE

G. F. CARRIER
D. C. DRUCKER
I. S. SOKOLNIKOFF

HIRSH COHEN
U. GRENANDER
P. S. SYMONDS

W. F. FREIBERGER, *Managing Editor*

EDITED WITH THE COLLABORATION OF

M. A. BIOT
C. FERRARI
G. E. HAY
E. REISSNER
J. J. STOKER

J. M. BURGERS
P. GERMAIN
P. LE CORBEILLER
S. A. SCHELKUNOFF
SIR GEOFFREY TAYLOR

H. W. EMMONS
J. A. GOFF
F. D. MURNAGHAN
W. R. SEARS

FOUNDER, AND
MANAGING EDITOR 1943-1965
W. PRAGER

VOLUME XXXII

JANUARY • 1975

NUMBER 4

QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R. I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$25.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

The subscription price for the QUARTERLY is \$20.00 per volume (April-January). Single issues can be purchased at \$8.00, and back volumes at \$20.00 per volume as far as they are available. Subscriptions and orders for back volumes must be addressed to: American Mathematical Society, P. O. Box 6248, Providence, R. I. 02904.

Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

QUARTERLY
OF
APPLIED MATHEMATICS

EDITED BY

H. W. BODE
P. J. DAVIS
E. T. ONAT
J. L. SYNGE

G. F. CARRIER
D. C. DRUCKER
I. S. SOKOLNIKOFF

HIRSH COHEN
U. GRENANDER
P. S. SYMONDS

W. F. FREIBERGER, *Managing Editor*

EDITED WITH THE COLLABORATION OF

M. A. BIOT
C. FERRARI
G. E. HAY
E. REISSNER
J. J. STOKER

J. M. BURGERS
P. GERMAIN
P. LE CORBEILLER
S. A. SCHELKUNOFF
SIR GEOFFREY TAYLOR

H. W. EMMONS
J. A. GOFF
F. D. MURNAGHAN
W. R. SEARS

FOUNDER, AND
MANAGING EDITOR 1943-1965

W. PRAGER

Printed by the
WILLIAM BYRD PRESS, INC.
Richmond, Virginia

INDEX

J. D. Achenbach and V. K. Varatharajulu: Skew crack propagation at the diffraction of a transient wave.....	123
R. Adenstedt and Bennett Eisenberg: Linear estimation of regression coefficients	317
S. M. Ahn (<i>see E. I. Jury</i>)	
Stuart S. Antman: Kirchhoff's problem for nonlinearly elastic rods.....	221
C. A. Borges, L. Cesari and D. A. Sánchez: Functional analysis and the method of harmonic balance.....	457
S. B. Castell and C. Rogers: Applications of invariant transformations in one-dimensional non-steady gas dynamics.....	241
R. G. Casten, H. Cohen and P. A. Lagerstrom: Perturbation analysis of an approximation to the Hodgkin-Huxley theory.....	365
L. C. Cerny (<i>see D. M. Stasiw</i>)	
L. Cesari (<i>see C. A. Borges</i>)	
Il. G. Chambers: Some functional differential equations.....	445
C. Y. Chan: Maximal and minimal positive solutions for nonlinear radiation processes with nonlinear sinks.....	475
O. P. Chandna (<i>see V. I. Nath</i>)	
D. L. Clements (<i>see C. Rogers</i>)	
Hirsh Cohen (<i>see Richard G. Casten</i>)	
F. B. Cook (<i>see D. M. Stasiw</i>)	
L. Y. Cooper: A singular perturbation solution to a problem of extreme temperatures imposed at the surface of a variable-conductivity halfspace: small surface conductivity.....	427
M. C. DeTraglia (<i>see D. M. Stasiw</i>)	
Bennett Eisenberg (<i>see Rolf K. Adenstedt</i>)	
Ray Engelke (<i>see W. C. Rivard</i>)	
Bernard Epstein: A formal expansion procedure for the solitary wave problem	89
J. L. Ericksen: Plane waves and stability of elastic plates.....	343
H. O. Fattorini and D. L. Russell: Uniform bounds on biorthogonal functions for real exponentials with an application to the control theory of parabolic equations	45
Wildon Fickett: Motion of an explosive-induced plane shock wave.....	71
Isaac Fried: Finite-element method: accuracy.....	149
M. Gurtin: Some mathematical models for population dynamics that lead to segregation.....	1
J. E. Hartka: Temperature of a semi-infinite rod which radiates both linearly and nonlinearly.....	101
M. A. Hayes and R. S. Rivlin: Plane waves in linear viscoelastic materials.....	113
R. P. Herrmann and R. R. Nachlinger: A theorem on the uniqueness of solutions in nonlinear heat conduction.....	329
K. Huseyin and R. H. Plaut: On extremum properties of the generalized Rayleigh quotient associated with flutter instability.....	189
E. Infeld: On the stability of solutions of a second-order differential equation.....	465
E. I. Jury and S. M. Ahn: Remarks on the root-clustering of a polynomial in a certain region in the complex plane.....	203
James P. Keener: Buckling imperfection sensitivity of columns and spherical caps	173
Steen Krenk: On the use of the interpolation polynomial for solutions of singular integral equations.....	479

Paco A. Lagerstrom (<i>see</i> Richard G. Casten)	
E. V. Laitone and Wen-Fan Lin : Application of the Sonin-Polya oscillation theorem	285
W. D. Lakin : On the equation of a rapidly rotating slender rod	11
M. Lakshmanan (<i>see</i> P. M. Mathews)	
Wen-Fan Lin (<i>see</i> E. V. Laitone)	
P. M. Mathews and M. Lakshmanan : On a unique nonlinear oscillator	215
B. J. Matkowsky and L. J. Putnick : Transitions and stability in the nonlinear buckling of elastic plates	137
Herman Medwin (<i>see</i> Peter C. C. Wang)	
V. I. Nath and O. P. Chandna : On plane viscous MHD flows, II	337
R. Ray Nachlinger (<i>see</i> Robert P. Herrmann)	
D. R. Owen : On the non-uniqueness of elastic rotations for deformations of materials with elastic range	355
A. C. Popkin : Generalized plane deformations of ideal fiber-reinforced materials	253
R. H. Plaut (<i>see</i> K. Huseyin)	
L. J. Putnick (<i>see</i> B. J. Matkowsky)	
W. C. Rivard and R. Engelke : Motion of a compressible plate driven by shock or detonation	29
R. S. Rivlin : Stability of pure homogeneous deformation of an elastic cube under dead loading	265
R. S. Rivlin (<i>see</i> M. A. Hayes)	
C. Rogers and D. L. Clements : On the reduction of the hodograph equations for one-dimensional elastic-plastic wave propagation	469
C. H. Rogers (<i>see</i> S. P. Castell)	
L. A. Rubinfeld : Nonlinear dynamic buckling of a compressed elastic column	163
D. L. Russell (<i>see</i> H. O. Fattorini)	
D. A. Sánchez (<i>see</i> C. A. Borges)	
Michael R. Sheets (<i>see</i> Robert L. Sternberg)	
Abraham Shigematsu (<i>see</i> Robert L. Sternberg)	
D. M. Stasiw, F. B. Cook, M. C. DeTraglia and L. C. Cerny : The drag and sphericity index of a spindle	351
Alice L. Sternberg (<i>see</i> Robert L. Sternberg)	
Helen M. Sternberg (<i>see</i> Robert L. Sternberg)	
Robert L. Sternberg, Michael R. Sheets, Helen M. Sternberg, Abraham Shigematsu, and Alice Sternberg : Multiple Fourier analysis in rectifier problems II	293
J. W. Thomas : A numerical study of the relationship between the dimensionless parameters in the problem of periodic waves of permanent type in a liquid of finite depth	403
V. K. Varatharajulu (<i>see</i> J. D. Achenbach)	
J. A. Walker : Necessary conditions for divergence in linear elastic systems	85
J. A. Walker : On state transformation and stability analysis of distributed parameter systems	333
C.-Y. Wang : Axisymmetric stagnation flow on a cylinder	207
Peter C. C. Wang and H. Medwin : Stochastic models of the scattering of sound by bubbles in the upper ocean	411
A. D. Wiggins : Statistical estimation of a crack damage parameter in manufacturing processes	347
C. H. Wu : Infinitely stretched Mooney surfaces of revolution are uniformly stressed catenoids	273

BOOK REVIEWS

M. J. Beckmann: <i>Input-output techniques</i> , edited by A. Brody and A. P. Carter . . .	220
J. L. Doob: <i>Markov processes: structure and asymptotic behavior</i> , by Murray Rosenblatt	97
Robert W. Easton: <i>Celestial mechanics II: perturbation theory</i> , by Yusuke Hagihara	219
Editors: <i>Oscar Zariski: collected papers. Vol. II: Holomorphic functions and linear systems</i> , edited by M. Artin and D. Mumford	364
——— <i>Cambridge summer school in mathematical logic</i> , edited by A. R. D. Mathias	485
——— <i>Classical Banach spaces</i> , by J. Lindenstauss and L. Tzafriri	485
——— <i>Ordinary differential equations</i> , by V. I. Arnold	485
——— <i>Stochastic processes and applications in biology and medicine</i> , by M. Iosifescu and P. Tautu	485
Jack K. Hale: <i>Theory of partial differential equations</i> , by H. Melvin Lieberstein . . .	219
Harold Kushner: <i>Stochastic differential equations</i> , by I. I. Gihman and A. V. Skorohod	362
A. M. Ostrowski: <i>Iterationsverfahren, Numerische Mathematik, Approximations- theorie</i> , edited by L. Collatz, G. Meinardus, H. Unger and H. Werner	97
Emanuel Parzen: <i>A short course in computational probability and statistics</i> , by W. Freiberger and U. Grenander	361
Allen C. Pipkin: <i>Handbuch der Physik. Vol. VIa/2: Mechanics of solids II</i> , edited by S. Flügge	361
Ian N. Sneddon: <i>Transformations de Laplace et de Mellin</i> , by S. Colombo and J. Lavoine	219
George Veronis: <i>Buoyancy effects in fluids</i> , by J. S. Turner	363
A. S. Wightman: <i>Algebraic methods in statistical mechanics and quantum field theory</i> , by Gerald G. Emch	98

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used. Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{(a + (b + cx)^n) \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

RICHARD G. CASTEN, HIRSH COHEN AND PACO A. LAGERSTROM: Perturbation analysis of an approximation to the Hodgkin-Huxley theory. . . .	365
J. W. THOMAS: A numerical study of the relationship between the dimensionless parameters in the problem of periodic waves of permanent type in a liquid of finite depth.	403
PETER C. C. WANG AND HERMAN MEDWIN: Stochastic models of the scattering of sound by bubbles in the upper ocean.	411
LEONARD Y. COOPER: A singular perturbation solution to the problem of extreme temperatures imposed at the surface of a variable-conductivity halfspace: small surface conductivity.	427
LL. G. CHAMBERS: Some functional differential equations.	445
C. A. BORGES, L. CESARI AND D. A. SÁNCHEZ: Functional analysis and the method of harmonic balance.	457
NOTES:	
E. INFELD: On the stability of solutions of a second-order differential equation.	465
C. ROGERS AND D. L. CLEMENTS: On the reduction of the hodograph equations for one-dimensional elastic-plastic wave propagation.	469
C. Y. CHAN: Maximal and minimal positive solutions for nonlinear radiation processes with nonlinear sinks.	475
STEEN KRENK: On the use of the interpolation polynomial for solutions of singular integral equations.	479
BOOK REVIEW SECTION:	
A. R. D. MATHIAS, editor: <i>Cambridge summer school in mathematical logic</i>	485
J. LINDENSTRAUSS AND L. TZAFRIRI: <i>Classical Banach spaces</i>	485
V. I. ARNOLD: <i>Ordinary differential equations</i>	485
M. IOSIFESCU AND P. TAUTU: <i>Stochastic processes and applications in biology and medicine</i>	485

—BOOK REVIEW SECTION—

Cambridge summer school in mathematical logic. Edited by A. R. D. Mathias. Lecture Notes in Mathematics, Vol. 337, Springer-Verlag, Berlin, Heidelberg, New York, 1973. ix + 660 pp. \$17.30.

These proceedings are divided into the following parts: intuitionism (one introductory and survey paper by Divan Dalen—95 pages—and 7 other papers), the priority method (2 papers), automata (1 paper), set theory and combinatorics (7 papers), model theory (5 papers).

Classical Banach spaces. By J. Lindenstrauss and L. Tzafriri. Lecture Notes in Mathematics, Vol. 338, Springer-Verlag, Berlin, Heidelberg, New York, 1973. ix + 243 pp. \$9.10.

The purpose of these notes is to describe some of the main known results on some classes of special Banach spaces, especially classical spaces (i.e., the space of continuous functions and L_p -spaces with $1 \leq p \leq \infty$) and more generally, Banach spaces having a symmetric structure. The notes are divided into two main parts, sequence spaces and function spaces, and assume that the reader is familiar with basic facts in functional analysis (at most chapters I–VI of Dunford and Schwartz, *Linear Operators*). The first chapter in each of the two parts is introductory in nature. There is an extensive bibliography (214 items).

Ordinary differential equations. By V. I. Arnold. Translated from the Russian by Richard A. Silverman. M.I.T. Press, Cambridge, Mass., 1973. vii + 280 pp. \$16.50.

This book—on a subject in which there is no dearth of textbooks—can be highly recommended for three reasons: its approach makes full use of modern mathematical concepts and terminology of considerable sophistication and generality (including aspects of qualitative theory of differential equations); there is constant appeal to geometrical and physical considerations, amplified by numerous problems and exercises; and it is brought to us through the medium of a translation which is of the highest standard (to be expected from this experienced translator) and a production which it is a pleasure to peruse.

Stochastic processes and applications in biology and medicine: Volume I (theory), Volume II (models). By M. Iosifescu and P. Tautu. Springer-Verlag, Berlin-Heidelberg-New York, 1973. I: 331 pp.; II 337 pp. \$21.80 each volume.

Volume I is a revised and enlarged version of chapters 1 and 2, and Volume II of chapter 3, of a book with the same title published in Romanian in 1968. Volume I contains two long chapters, discussing

discrete-parameter and continuous-parameter stochastic processes respectively. The first chapter presents the theory of Markov chains with denumerable and arbitrary state spaces, and the second processes with independent increments and Markov processes. The models discussed in Volume II include population growth models, population dynamics processes, evolutionary processes, and stochastic models in physiology and pathology.

Being the product of collaboration between a probabilist and a physician, the work covers both the mathematical and the practical aspects of the subject authoritatively. Particularly the second volume contains some material not easily accessible elsewhere. The production and translation are of the usual high standard associated with this publisher.