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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2 + b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\textstyle\int u^{-1} \sin u \, du \text{ is preferable to } \displaystyle\int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Fow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.e.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEW SECTION—

A short course in computational probability and statistics. By W. Freiberger and U. Grenander. New York: Springer Verlag, Inc., 1971. xii + 155 pp. $6.50.

Impressively discussing topics drawn from almost all important fields of probability and statistics, the authors successfully demonstrate that mathematical scientists (and statistical scientists) must learn how to use the computer not only as a computing machine but also to suggest theorems and to provide laboratory experience in the empirical interpretation and meaning of theorems. The uneven level of prerequisites necessary for the different chapters makes the book most suitable as a supplementary test for courses (rather than as the text for a single course). All courses in probability and statistics can be improved by including the approach of this book. The problem assignments are particularly novel and valuable as course supplements since they are mainly of the research project type.

The programming language APL used in the book seems the right language for the authors' aims. But the book can be read with profit even if one does not know APL or have convenient access to its use. While there is a lamentably high level of typographical errors in the text, the computer programs we tried all worked, so that the computer printouts are without error.

The basic building blocks of Randomness and Simulation are excellently covered in Chapters 1 and 2. The mathematical prerequisites for fully appreciating these rewarding chapters are higher than these required for all later chapters. One question: on page 35 the reader is told to form a matrix square root but is not told how (should not the Cholesky decomposition be mentioned later?).

Chapter 3 on Limit Theorems, although brief, does successfully make the point that the computer has given us new insight into finite sample interpretations of limit theorems.

Chapters 4 and 5 on Stochastic Processes have many interesting examples including investment of capital, memory referencing in a computer, birth and death, random phases, and renewal. Unfortunately many of the concepts used are not adequately defined (for example, a Markov chain is not defined).

Chapter 6 on Decision Problems is a particularly rich collection of novel examples: stochastic approximation, an insurance game, design of experiments, and a search problem. One criticism: on page 106 the Gram-Schmidt orthorgonalization procedure is mentioned without pointing out its basic role in computational statistics.

Chapter 7, A Computational Approach to Statistics, is the least successful chapter; it seems to say little.

Chapter 8 on Time Series discusses spectral density estimation, Fast Fourier Transforms, regression analysis, and signal detection. Time series enjoys extensive interaction with computational probability and statistics, the flavor of which cannot be adequately conveyed by a brief chapter. But the authors have succeeded in providing a good taste.

The authors deserve our gratitude for a seminal book which extremely successfully whets one's appetite for more extensive books which will stimulate research and provide training in computational probability and statistics.

EMANUEL PARZEN, MARCELLO PAGANO AND
JAMES. T. MCCLAVE (*Buffalo*)

Handbuch der Physik. Vol. VIa/2: Mechanics of solids II. Edited by S. Flügge. Springer-Verlag, Berlin, Heidelberg, New York, 1972. x + 745 pp. $73.10.

This volume contains six separate articles on elasticity theory, each a book in itself: "The linear theory of elasticity," by M. E. Gurtin; "Linear thermoelasticity," by D. E. Carlson; "Existence theorems in elasticity," by G. Fichera; "Boundary value problems of elasticity with unilateral constraints," by G. Fichera; "The theory of plates and shells," by P. M. Naghdi; "The theory of rods," by S. S. Antman.

Of these articles I have read thoroughly only Gurtin's treatise on classical elasticity and its extension to thermoelasticity in Carlson's work. Gurtin's long article is by far the most complete account of linear elasticity theory in existence. Solutions of particular boundary-value problems are excluded in order to allow space for the statements and proofs of nearly every significant theorem of the mathematical theory. But, far from being merely a compendium of theorems, this treatise is also the clearest and genuinely most informative work on elasticity theory that I have ever read. Although it might seem impossible to write a book that is both completely rigorous and also a pleasure to read, Gurtin has done it. For those who wish to know the genesis of each theorem, Gurtin's work will also serve as the most complete and reliable account of the history of the subject; but that is merely one of the smaller virtues of this remarkable work.

D. E. Carlson's work is written to be used in conjunction with Gurtin's article, and is accordingly much shorter. Carlson shows how the basic theorems of isothermal elasticity theory can be extended and generalized to account for thermal effects. In completeness, clarity, and rigor, Carlson's work maintains the high standard set by the first article.

The articles by Gurtin and Carlson are incomplete with respects to existence theorems. This difficult subject is taken up in the two articles by Fichera. In the first of these, general existence theorems for strongly elliptic linear systems are developed, and the general theorems are then applied to propagation problems, diffusion problems, problems involving thin plates, and finally, elastic equilibrium in three-dimensional homogeneous and inhomogeneous bodies. Fichera's second article deals with the even more troublesome subject of existence in problems with one-sided constraints. Although I am not qualified to comment on any of this, it appears to me that the treatment is complete and self-contained, and so far as I know there exists no comparable account of the subject elsewhere.

The articles by Gurtin, Carlson, and Fichera provide a truly encyclopedic description of the mathematical foundations of linear elasticity theory. The two remaining articles are concerned with two special classes of problems in the nonlinear theory.

The long article by Naghdi on plates and shells provides a systematic development of the basic equations of the theory, from two points of view. Naghdi presents both the treatment of the shell as a limiting case of a three-dimensional body and also the treatment as a Cosserat surface. Comparisons of the two approaches are emphasized. Although I am not an expert in this field, it appears to me that much of the material presented is original.

Antman's article on rods is similar to Naghdi's in that various rational approaches to the subject are compared, and indeed Antman shows that all justifiable approaches lead to governing equations of exactly the same form, thus resolving some controversies about the subject. There is also a substantial amount of material on the solution of problems, which are selected for the insight into the structure of nonlinear mechanics that they can give. Antman's approach to nonlinear elasticity theory by way of one-dimensional bodies seems to be extremely promising, and I expect to study his article very carefully.

ALLEN C. PIPKIN (*Providence*)

Stochastic differential equations. By I. I. Gihman and A. V. Skorohod. Springer–Verlag, New York, 1972. viii + 354 pp. $27.90.

The book deals with a branch of probability theory that is of large and growing interest in applications. Although not dealing with applications at all, it does contain a fairly comprehensive account of stochastic differential equations of the Ito type.

Part I deals with equations of first order. There is an introduction to the Wiener process and to the basic ideas of the contruction and properties of solutions, Markov properties, etc. A rather thorough discussion of asymptotic properties is given, and there is a detailed development of the problem on a finite interval under the various types of boundary conditions. Girsanov's results on transformation of measures are discussed. The material is not as detailed or as deep as that in Ito and McKean [1] (it is less concerned with the details of local properties or of the various operators connected with the process) and does not take advantage of some of the relatively recent developments in martingale theory. It is, however, too detailed to be an introduction to the subject. The authors' previous book [2] does provide a nice introduction to both stochastic differential equations and probability theory, and once the relevant parts of [2] are read, the current book can be used very profitably.

Part II deals with equations of arbitrary finite order. The general Ito equation (with respect to a Wiener process) is developed via an interesting approach to stochastic line integrals, an approach which (while not common) is not noticeably more difficult than more standard approaches and does allow some unification of concepts. The equation with 'Poisson and Wiener input' is treated somewhat along the lines in [3] (integrals with respect to 'Poisson' random measures), but in a more readable manner. Again the emphasis is on the contruction of solution under Lipschitz conditions, bounds and estimates for the solution, Markov properties, continuity (in probability) with respect to parameters, the relevant form of Ito's Lemma, and similar questions.

There seem to be many more typographical errors than there should be, and in many sections the motivation is inadequate. The text does not seem to refer to the references. But the book does contain a great deal of very useful information and will no doubt be of substantial value to anyone concerned with applications of such equations who has had some previous exposure to the subject.

HAROLD KUSHNER (*Providence*)

[1] K. Ito, H. P. McKean, *Diffusion processes and their sample paths*, Springer-Verlag, Berlin-Heidelberg, 1965
[2] I. I. Gikhman, A. V. Skorohod, *An introduction to the theory of random processes*, Saunders, Philadelphia, 1969
[3] A. V. Skorohod, *Studies in the theory of random processes*, Addison-Wesley, Reading, Massachusetts, 1965

Buoyancy effects in fluids. By J. S. Turner. Cambridge University Press, Cambridge, 1973. vi + 367 pp. $29.50.

This well-written monograph contains many examples of laboratory and field studies of fluids in which stratification is important. As with homogeneous fluids, the most pertinent buoyancy-driven flows are turbulent. However, buoyancy effects are associated with stratification which, in turn, provides a measure of order that is lacking in homogeneous fluids but often proves useful in analyzing the behavior of stratified flows. This point is evident in a number of the excellent photographs which Turner presents to record the behavior of buoyancy-driven flows.

The topics in stably stratified fluids embrace linear and nonlinear waves, hydraulic jumps, shear flow instability and turbulence. The treatment of buoyancy convection includes buoyant plumes and thermals, parallel plate convection and double-diffusive convection. Although many laboratory examples are discussed, the orientation and emphasis of the book are toward broader awareness of the processes and phenomena in smaller-scale geophysical flows. Since these flows are nonlinear and generally turbulent, they are not subject to ordered, deductive analyses. Accordingly, Turner relies heavily on similarity arguments, often based on internal parameters, to develop a remarkably coherent account of a variety of flows in which buoyancy forces are important.

His unifying interpretation of the experimental, observational, analytical and numerical studies listed in the very extensive bibliography goes far beyond the normal reporting of allied research. This book is clearly the product of a dedicated professional.

The flyleaf states that the book is intended for readers having a background knowledge of fluid mechanics of homogeneous media. This is necessary but by no means sufficient preparation. The treatment is rather too condensed in some sections and more detail, even at the expense of scope, would have been helpful.

Turner's own research has provided him with a penetrating insight into the phenomena and he analyzes them most effectively by means of scaling and similarity arguments. This approach is deceptively simple. Proficient use of it requires considerable familiarity with the flows, either through past experience or careful observations, a fact that becomes distressingly apparent in some of the more exploratory studies. Applied mathematicians with a more analytical orientation will find the going rough in these sections.

Still, the maze of interrelationships among the physical processes of the more complicated flows is intricate indeed, and it is difficult to imagine that any other approach could have yielded as much useful information. In spite of uncertainties along the way, Turner takes the reader to unexplored territory in each of the areas covered. His treatment shows that there are many fascinating problems both at the frontiers and along the outlined paths. The book should prove to be very useful not only in providing an interpretive framework of buoyancy-driven flows but in stimulating additional research into this very important area.

GEORGE VERONIS (*New Haven*)

Oscar Zariski: collected papers. Vol. II: Holomorphic functions and linear systems. Edited by M. Artin and D. Mumford. M.I.T. Press, Cambridge, Mass., 1973. xxiii + 505 pp. $17.95.

This volume contains a preface by Oscar Zariski describing the progress of his intellectual development, a bibliography of his publications (90 items), an introduction by M. Artin putting some of the papers in their historical context, and photographic reprints of 19 of Zariski's papers; they are divided into two parts: 8 papers on holomorphic functions, and the others on linear systems, the Riemann-Roch Theorem and applications. Four papers are in French and the remainder in English. The collected papers are presently expected to occupy four volumes, and should be a valuable addition to the library of anyone interested in the development of algebraic geometry.

W. FREIBERGER (*Providence*)