

QUARTERLY
OF
APPLIED MATHEMATICS

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QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in engineering. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e(a^2 + b^2)^{1/2}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{(a + (b + cx)^n) \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (for books—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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BOOK REVIEW SECTION

Trends in elasticity and thermoelasticity: Witold Nowacki anniversary volume. Editorial board: R. E. Czarnota-Bojarski, M. Sokolowski, H. Zorski. Wolters-Noordhoff Publishing, Groningen, 1971. xx + 307 pp.

This book is a collection of papers by a number of different authors—articles dealing with problems in elasticity and thermoelasticity and dedicated to Witold Nowacki on the occasion of his 60th birthday. The editors have attempted to select the contributing authors and articles so that the volume might present a reasonable survey of a well-defined branch of modern mechanics. With a few exceptions the papers deal with special classes of problems in the linear theory, and in general they focus on recent advances in the handling of these certain classes of problems.

The book starts with a list of the publications of Nowacki—a combined total of research papers, text books and monographs numbering almost 200. From this list it is clear that the articles in the book are meant to reflect the principal research interests of Nowacki.

The list of articles and authors is as follows:

1. Some contact problems for a semi-plane with elastic stiffeners—N. K. Arutunyan and S. M. Mkhitaryan.
2. On a melting problem with temperature-dependent properties—B. A. Boley.
3. Second-order thermoelasticity theory for isotropic and transversely isotropic materials—P. Chadwick and L. T. C. Seet.
4. Self-stresses in doubly connected solid—K. Herrmann and M. Hieke.
5. Tensorial equations of motion for elastic materials with microstructure—J. Ignaczak.
6. Influence of constant stresses on velocities of propagation of elastic waves—Z. Kaczkowski.
7. On the possibility of generation of self-excited increasing thermal waves—S. Kaliski.
8. On dynamic problems of the theory of elasticity—V. D. Kupradze and T. V. Burchuladse.
9. On thermal stresses in plane elasticity—A. I. Lurie.
10. Generalized theory of thermoelasticity associated with boundary problems of classical fields—M. Misicu.
11. Contribution to heated punch problem—Z. Olesiak.
12. Constitutive equations of the linear viscoelastic dielectric—H. Parkus.
13. On the decomposition of the Hilbert space of the fields in a linear continuum into two orthogonal spaces—G. Rieder.
14. Diffraction of elastic waves in non-symmetric elasticity in the case of multiply-connected regions—G. N. Savin, A. N. Guz and V. T. Golovchan.
15. Almansi analogue of Lamé's equations—B. R. Seth.
16. The problem of the unsymmetrical cruciform crack—I. N. Sneddon and S. C. Das.
17. A non-linear theory of thermoelasticity with couple-stresses—R. Stojanovic.
18. Some problems of the torsion theory solved by means of the Mehler-Fock transform—Y. S. Ufland.
19. Piecewise linear isotropic elastic material—Z. Wesolowski.
20. Composite systems and discrete models—C. Wozniak.

The layout and printing of the volume are well done with one notable exception (which may apply only to the reviewer's copy of the book): pages 142, 143, 146, 147, 151, 154, and 155 are completely blank. Affected are the papers of Kupradze and Burchuladse as well as the paper of Lurie.

This book will perhaps be useful as a reference book for engineers interested in the specific types of problems treated in the volume. It is clearly not intended as a textbook, although it might be used as a basis for a seminar course on certain classes of problems in mechanics.

L. E. Payne (*Ithaca, N. Y.*)

Group theory. By Rudolf Kochendörffer. McGraw-Hill, London, 1970. vii + 297 pp. \$17.50.

Group representation theory. Part A: Ordinary representation theory. By Larry Dornhoff. Marcel Dekker, Inc., New York, 1971. vii + 254 pp. \$13.50.

These two books are very different. The first is intended to provide an introduction to the basic facts of abstract group theory with a brief account of representation theory in the last chapter. The second concentrates on representation theory with deriving abstract results as its goal; this latter approach is necessarily directed to a much more sophisticated mathematical audience. To this extent they are complementary, but the comparison raises some real philosophical questions as to the future of the subject.

Difficulties abound in any worthwhile area of research and many of those inherent in the work of Burnside, Frobenius and Schur have been overcome in recent years by Brauer and his followers. Dornhoff has set himself the formidable task of relating and co-ordinating this work on ordinary representation theory in Part A, the book under review, reserving the modular theory for Part B, to appear in 1972. The field covered is so vast that only a few comments are possible. Each of the two books provides a valuable list of references.

Kochendörffer concentrates his efforts on clarity of exposition in a modern context. The style is readable and the book should be useful as an undergraduate text as well as at the postgraduate level. The role of group theory in geometry is recognized by the approach could have been further developed. More especially is this true in Dornhoff's book, which could have introduced the notion of the fundamental region of a representation to advantage and led up to Coxeter's results on generators and relations, to which no reference is made.

This reviewer was pleased with the emphasis given by Dornhoff to induced representations, Clifford's theorem, the work of Blichfeldt and the Frobenius-Schur count of involutions. All these are highly significant results which are sometimes overlooked. He would, however, plead for an index and a list of symbols referring to their definitions in a second edition.

At the risk of being accused of prejudice, he would like to register his complaint that there is no reference in either book to the work of Alfred Young who, perhaps more than anyone else, made possible the really significant applications of group theory to theoretical physics by Weyl, carried on by Wigner and many others. It might be argued that these applications have been covered elsewhere, but this reviewer suggests that the pattern of past history may be repeated and practical applications may influence the course of future abstract research. Like early incarnations of invariant theory, the search for simple groups may have run its course and be awaiting resurrection in some new context. It is this reviewer's hope that some of the ideas so significant for geometry and theoretical physics will stimulate renewed interest in the actual form of a representation of a finite group $G \subset S_n$, in relation to the representations of S_n .

It has been a pleasure to read these two volumes. They speak to different audiences but—each in its own way—with vigour and authority, and provide valuable surveys of the ever-growing literature.

G. DE B. ROBINSON (*Toronto*)

Group representation theory. Part B: Modular representation theory. By Larry Dornhoff. Marcel Dekker, Inc., New York, 1972. vi + 256 pp. \$15.50.

In this reviewer's experience a course in ordinary representation theory is much less sophisticated than one in modular theory. It is the author's aim in this second volume to make readily available pertinent research material in an organized form. From this point of view the book is most valuable, but the uninitiated or those in process of initiation will find it difficult reading. The style is concise and convincing but an overall view with suitable illustrations is lacking.

Again, this reviewer can be accused of prejudice, but the explicitness found in the modular representation theory of S_n gives meaning to the deep abstractions so concisely described by Dornhoff. E. G. Brauer's formula $\nu_p(\chi_s(1)) = a - d + \epsilon$, where ϵ is not known in general, becomes explicit for S_n (cf. *Representation theory of S_n* , Toronto, 1961, Chapter V). One further question—why has all reference to Nakayama's generalization of Frobenius' reciprocity theorem been omitted?

If the reader or lecturer using this book will develop the necessary illustrative examples to clarify and interpret the general theory, then Nordhoff's second volume will provide a most valuable source of material. The index at the end of Vol. II covers Vol. I also, so the publisher should encourage readers to buy both volumes!

G. DE B. ROBINSON (*Toronto*)