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—BOOK REVIEW SECTION—

Nonlinear system theory. By J. M. Holtzman. Prentice–Hall, Englewood Cliffs, N. J., 1970. ix + 213 pp. $10.50.

This book is an important addtion to the growing library of texts on nonlinear systems. The book is characterized by the functional analysis approach, particularly the application of the contraction mapping principle for operator equations is a Banach space. From a systems analysis viewpoint, the major problems considered in the book are: (i) the existence and uniqueness of solutions (including periodic solutions) in nonlinear feedback systems, (ii) the development of periodic solutions in nonlinear systems containing a small parameter, (iii) the accuracy of linearization in using the equation of first variation, and (iv) the stability of nonlinear feedback systems. Following an introductory chapter on mathematical background, there are two chapters on the contraction mapping theorem and its application to a general class of nonlinear feedback systems. The important feature of the second and third chapters is the treatment of the practical problem of finding the best contraction constant for an operator equation representing a nonlinear feedback system and its interpretation in the light of the familiar Nyquist criterion for linear systems. In addition, there is a lucid discussion on the problem of finding a suitable region about an approximate solution to a given operator equation within which the operator is a contraction. These chapters form the basis for the succeeding chapters, which consider problems (i)—(iv) mentioned above. As the author admits, the book is strongly oriented towards the application of functional analysis to various nonlinear system problems which are amenable to the techniques developed in the early chapters. This is in contrast to the practical problem faced by an engineer in which one seeks the best approach to a given problem irrespective to the origin of that approach. Nevertheless, the book should be of interest to a number of practicing engineers, as well as to researchers in the area of systems theory.

A. E. PEARSON (*Providence, R. I.*)

Praktische Funktionenlehre. By Friedrich Tölke. Band 4. *Elliptische Integralgruppen und Jacobische elliptische Funktionen im Komplexen.* 1967. viii + 191 pp. $21.75. Band 6. *Tafeln aus dem Gebiet der Theta-Funktionen und der elliptischen Funktionen.* Erster Teil 1969, lxxxii + 450 pp, Zweiter Teil 1970, iv + 597 pp. $51.70.

Volumes 2 and 3 of this work were reviewed in Quarterly of Applied Mathematics, vol. 26, no. 3, and volume 5 in vol. 27, no. 4.

Volume 4 consists of two chapters. The first of these, slightly less than half of the volume, contains formulae for numerous integrals whose integrands are rational combinations of Jacobian elliptic functions and their derivatives, and formulae for numerous elliptic integrals both in algebraic and trigonometric and hyperbolic form. The second chapter is devoted to Jacobian elliptic functions in the complex domain and contains formulae, diagrammatic representations of poles, zeros, general behaviour, diagrams drawn to scale of curves along which the real or imaginary parts of a variety of functions is constant, discussion of the inversion problem, conformal mapping by means of elliptic functions.

Volume 6 contains six numerical tables and 120 examples illustrating the use of these tables. Part 1 contains the examples; Table 1 (20 pp., mostly 5S): k, k', the modular angle α, k^2, k'^2, K, K' as functions of the parameter $\kappa = K'/K$; Table 2 (24 pp.): further 57 functions of κ including complete elliptic integrals, invariants of and other quantities related to Weierstrass' functions, and combinations of theta functions and their derivatives (for $z = 0$); Table 3 (first half, 403 pp.): theta functions and their logarithmic derivatives, Jacobian elliptic functions, zeta functions, Weierstrass' elliptic functions for $\kappa \leq 1$. Part 2 contains Table 3 (second half, 362 pp.) for $\kappa \geq 1$; Table 4 (92 pp, 9D): Legendre's elliptic integrals of the first and second kinds, Jacobi's zeta function, and Heuman's lambda function Λ_0 as functions of φ and the modular angle; Table 5 (42 pp, 6D): integrals of theta functions; Table 6 (101 pp., 6D): complete elliptic integrals as functions of k^2.

The printing is good and the tables are legible. Unfortunately, in the reviewer's copy a few pages were multilated by faulty cutting, folding, and binding.

A. ERDÉLYI (*Edinburgh*)

Celestial mechanics. Volume 1: dynamical principles and transformation theory. By Yusuke Hagihara. The M.I.T. Press, Cambridge, Massachusetts, 1970. xiii + 689 pp. $25.00.

Volume 1 is the first in a series of five volumes on celestial mechanics. The author in his preface states that "the rather ambitious aim of the present series is to recapitulate the results of the whole field of celestial mechanics and the associated branches of science during the last hundred years." The patience and scholarship displayed in Volume 1 are impressive. The author has incorporated something on the order of a thousand references in the text, which should make it valuable to those interested in the history of celestial mechanics. Researchers in the subject will find Volume 1 an enlightening reference and guide to the literature.

R. W. EASTON (*Providence, R. I.*)

Theory of automata. By Arto Salomaa. Pergamon Press, Oxford, 1969. xii + 263 pp. $12.00.

The book under review is a well-written introduction to a rather narrow segment of automata theory. It is divided into four chapters, of which the first and the fourth are completely standard, while the second and the third are among the best treatments of their subject matter available in the textbook literature, and are illuminated by the author's own contributions.

The first chapter, "Finite deterministic automata", and the fourth chapter, "Formal Languages and generalized automata", contain standard material in a standard exposition. In fact, the latter chapter does not even contain any thorough treatment of Turing machines, and was unfortunately written too early to include Salomaa's recent work on programmed grammars, and his other new and interesting contributions to language theory.

Turning from these chapters about which one cannot be enthusiastic, we may look at the intervening two chapters in which Salomaa's own work illuminates the presentation. Chapter 2, on "Finite nondeterministic and probabilistic automata", treats the former problem rather briefly, and then goes into a full and careful treatment of probabilistic automata, including Salomaa's own extensions of work by such authors as Rabin and Paz on finite state machines in which each input keys a Markov chain of state transitions. Let us say that a set of input sequences is stochastically acceptable if there exists a finite state stochastic automaton, an initial state, a set of designated final states, and a threshold such that the given set of input sequences is precisely the set of sequences that will carry us from the initial state to one of the designated final states with a probability that exceeds the given threshold. A typical result is that if a set is stochastically acceptable, then it is acceptable by some deterministic machine—unless we make the rather implausible (but mathematically interesting) assumption that we can distinguish, for a given stochastic machine, probabilities that exceed some irrational number, but do not exceed arbitrarily close rational numbers. To actually estimate the probability of acceptance of an input sequence using a stochastic "black box" requires repeated trials—other results, then, show that this is somewhat compensated for by a reduced number of states in the stochastic machine that does the job that could be done by a larger deterministic machine. These sentences give some of the flavor of the material which constitutes chapter 2.

One of the first results in automata theory was that due to Kleene: that what he called a "regular expression" provided precisely the correct device for describing the set of input sequences that would drive a given deterministic finite state machine from its initial state to one of a designated set of final states. One of Salomaa's most notable contributions has been to devise axiom systems in which the theorems are precisely the regular expressions. The core of his third chapter, "Algebra of regular expressions", is then an exposition of this work and the complementary work of the Ukrainian, Redko, with hitherto unpublished refinements, augmented by the work of Eggan and others on what is called "star height". I found this by far the most useful and interesting chapter.

In summarizing the drawbacks of the book, I found the two most outstanding that it contains virtually no motivation as to why the topics it treats are of interest, and that it gives no hint of the fact that its subject matter—despite the book's encompassing title—represents only a very small fraction of current automata theory. However, for teachers wishing to present regular expression theory and stochastic automata, this book will certainly be on the short list of possible texts. Not only is the treatment full and thorough for these two topics, but the selection of exercises has been made with great care, and the book has the unusual feature, for one published in the English language, of having extensive guides not only to the western literature, but also to Russian publications, and other eastern publications as well.

M. A. Arbib (*Amherst, Mass.*)

Continuous univariate distributions. (Two volumes.) By Norman L. Johnson and Samuel Kotz. Houghton Mifflin Co., New York, 1970. Vol. 1, xiv + 300 pp. Vol. 2, xiii + 306 pp. $13.95 each.

Several years ago I was told by an eminent non-parametric statistician that the study of special distributions was wasted effort, since inference was sure to become exclusively distribution-free in the near future. Nowadays, on the contrary, the particular distributions seem more and more to provide a convenient format for collections of statistical information. The present volumes are excellent examples of the *genre:* throughtfully organized, comprehensive without being tedious and attractively set out.

The normal distribution presents the most serious problem in a work of this sort, since, without great restraint, it can grow irresistably until it has swallowed whole branches of statistics. The present authors provide a compact chapter on the normal, followed by a completely new chapter on quadratic forms in normal variables. Their other chapter headings are: lognormal, inverse Gaussian, Cauchy, gamma, exponential, Pareto, Weibull, extreme value, logistic, Laplace, beta, uniform, "F", "t", non central chi-square, noncentral "F", noncentral "t", correlation coefficient, miscellaneous. They introduce a useful distinction in terminology between "sampling" (e.g. Wishart) and "modelling" (e.g. Weibull) distributions.

The tables are well chosen and the diagrams uniformly informative. I was especially impressed by the good modern references provided in the chapter bibliographies. The index, to be sure, is skimpy and the rationale for reproducing sheets of graph paper (normal, Weibull) baffling.

Frank A. Haight (*State College, Penna.*)