# QUARTERLY

# OF

# APPLIED MATHEMATICS



VOLUME XXIII JULY · 1965 NUMBER 2

# QUARTERLY OF APPLIED MATHEMATICS

This periodical is published quarterly by Brown University, Providence, R. I. 02912. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following companies are gratefully acknowledged:

AVCO RESEARCH LABORATORY; EVERETT, MASS.,
BELL TELEPHONE LABORATORIES, INC.; NEW YORK, N. Y.,
THE BRISTOL COMPANY; WATERBURY, CONN.,
CALIFORNIA RESEARCH CORPORATION; SAN FRANCISCO, CAL.,
CURTISS WRIGHT CORPORATION; AIRPLANE DIVISION; BUFFALO, N. Y.,
EASTMAN KODAK COMPANY; ROCHESTER, N. Y.,
GENERAL ELECTRIC COMPANY; SCHENECTADY, N. Y.,
GULF RESEARCH AND DEVELOPMENT COMPANY; PITTSBURGH, PA.,
HUGHES AIRCRAFT COMPANY; CULVER CITY, CAL.,
INTERNATIONAL BUSINESS MACHINES CORPORATION; NEW YORK, N. Y.,
LEEDS & NORTHRUP COMPANY; PHILADELPHIA, PA.,
LOCKHEED AIRCRAFT CORPORATION; BURBANK, CAL.,
PRATT & WHITNEY DIVISION, NILES-BEMENT-POND COMPANY; WEST HARTFORD, CONN.,
THE RAMO-WOOLDRIDGE CORPORATION; LOS ANGELES, CAL.,
THE RAND CORPORATION; SANTA MONICA, CAL.,
REPUBLIC AVIATION CORPORATION; FARMINGDALE, LONG ISLAND, N. Y.,
UNITED AIRCRAFT CORPORATION; EAST HARTFORD, CONN.,
WESTINGHOUSE ELECTRIC AND MANUFACTURING COMPANY; PITTSBURGH, PA.

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (one copy only) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to Professor E. T. Onat, Managing Editor, or Miss Marjorie B. Luther, Editorial Assistant, Editorial Office, Box F, Brown University, Providence, R. I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of $5.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

The subscription price for the QUARTERLY is $8.00 per volume (April-January). Single issues can be purchased at $2.50, and back volumes at $8.00 per volume as far as they are available. Subscriptions and orders for single issues must be addressed to: Brown University Press, Providence, R. I. 02912.

Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA

## CONTENTS



## ANNOUNCEMENT

### FIFTH U. S. NATIONAL CONGRESS OF APPLIED MECHANICS

University of Minnesota, Minneapolis, Minnesota
June 14-17, 1966

The U. S. National Congresses of Applied Mechanics are held every fourth year and are planned to supplement the International Congresses of Applied Mechanics.

The Congress concerns itself with experimental and theoretical Applied Mechanics, including mechanics of rigid bodies and deformable solids, mechanics of fluids and gases, thermodynamics and heat transfer. There will be invited survey lectures in selected fields; in addition, research workers are encouraged to submit 200-word abstracts covering current work. Based upon the abstract, each accepted paper will be allotted time for presentation and discussion at an appropriate session of the Congress.

In order to encourage new contributors in the field, the Editorial Committee will select a group of research papers from among the submitted abstracts for presentation and publication in full. These research papers, the invited survey lectures and all accepted abstracts will be reproduced in a Proceedings to be available during registration at the Congress. Leading technical publications have agreed that the abstracts will not constitute official publication. It is hoped that the emphasis will be on current work and work in progress. All abstracts must be submitted by January 15, 1966, on special forms supplied by the Secretary of the Congress.

Inquiries regarding the Congress should be addressed to Professor Robert Plunkett, Secretary of Applied Mechanics Congress, 107 Aero Building, University of Minnesota, Minneapolis, Minnesota 55455.

# BOOK REVIEWS

*An introduction to waves, rays and radiation in plasma media.* By J. J. Brandstatter. McGraw-Hill Book Co., Inc., New York, 1963. xiv + 690 pp. $15.00.

This comprehensive text is written at the level of a good senior or beginning graduate student. The material is arranged in nine chapters as follows: 1. Basic mathematical concepts of waves and rays in plasmas; 2. The homogeneous electron plasma in a uniform magnetic field; 3. A résumé of the hydrodynamical equations and shock conditions for gases; 4. The homogeneous ionic plasma in a magnetic field; 5. Energy considerations and electrodynamics; 6. The structure of waves in non-homogeneous media; 7. The theory of propagation of rays in an inhomogeneous, anisotropic, dispersive and absorbing medium; 8. Propagation phenomena based on the Boltzmann equation Microscopic considerations; 9. Radiation in a plasma.

The presentation is clear and easy to follow. Some chapters are self-contained and provide good introductions to the respective subjects. There are nearly 100 pages of problems, none of them trivial, and some truly challenging.

W. Prager

*Stress waves in anelastic solids.* Editors: Herbert Kolsky and William Prager. IUTAM Symposium April 3-5, 1963. Springer-Verlag, Berlin, Gottingen, Heidelberg, 1964. xi + 342 pp. $17.50.

This collection of excellent papers covers a wide range of inelastic behavior and types of loading. Materials described in the 23 papers include homogeneous and inhomogeneous metals, polymers, rocks, and soils subjected to vibration, pulse, and shock loading of high and of low intensity. Analytical approaches are presented on the continuum and dislocation level along with considerable experimental information.

D. C. Drucker

*Physical acoustics—principles and methods.* Volume 1, Part A. Edited by Warren P. Mason. Academic Press, New York and London, 1964. xii + 515 pp. $18.00.

This is the first volume of a series of books planned to survey the techniques, applications and results obtainable in the field of physical acoustics. Physical acoustics has become a powerful tool and the field of application is now so broad that eight separate books (four volumes, each with parts A and B) have already been planned by Dr. Warren P. Mason, the Editor.

In Volume 1, Part A, the contents are as follows: "Wave Propagation in Fluids and Normal Solids" by R. N. Thurston, Bell Telephone Laboratories; "Guided Wave Propagation in Elongated Cylinders and Plates" by T. R. Meeker and A. H. Meitzler, Bell Telephone Laboratories; "Piezoelectric and Piezomagnetic Materials and Their Function in Transducers" by Don A. Berlincourt, Daniel R. Curran and Hans Jaffe, Clevite Corporation; "Ultrasonic Methods for Measuring the Mechanical Properties of Liquids and Solids" by H. J. McSkimin, Bell Telephone Laboratories; "Use of Piezoelectric Crystals and Mechanical Resonators in Filter and Oscillators" by Warren P. Mason; "Guided Wave Ultrasonic Delay Lines" by John E. May, Bell Telephone Laboratories; "Multiple Reflection Ultrasonic Delay Lines" by Warren P. Mason, Bell Telephone Laboratories.

This collected work on physical acoustics is well done by the best talent in the field and it will certainly be important to people in the many areas in which the use of high frequency stress waves has become a working tool.

Rohn Truell

*Stability of motion.* Applications of Lyapunov's second method to differential systems and equations with delay. By N. N. Krasovskii. Translated by J. L. Brenner. Stanford University Press, 1963. vi + 188 pp. $6.00.

The Russian original appeared in 1959. The present translation is competently done; it incorporates some minor alterations and additions suggested by the author.

*(Continued on p. 120)*

# BOOK REVIEWS

*(Continued from p. 108)*

*Asymptotic solutions of differential equations and their applications.* Edited by C. H. Wilcox. John Wiley & Sons, Inc., New York, 1964. x + 249 pp. $4.95.

The volume, which is dedicated to Professor Rudolph E. Langer, contains the following papers presented at a Symposium conducted by the Mathematics Research Center, United States Army, at the University of Wisconsin in May 1964:

Asymptotic Expansions for Ordinary Differential Equations: Trends and Problems, by W. Wasow; Solvable Related Equations Pertaining to Turning Point, by H. L. Turritin; Asymptotic Methods for the Solution of Dispersive Hyperbolic Equations, by R. M. Lewis; Asymptotic Solutions and Indefinite Boundary Value Problems, by R. W. McKelvey; Some Examples of Asymptotic Problems in Mathematical Physics, by C. C. Lin; On the Problem of Turning Points for Systems of Linear Ordinary Differential Equations of Higher Orders, by Y. Sibuya: Error Bounds for Asymptotic Expansions, with an Application to Cylinder Functions of Large Argument, by F. W. J. Olver; Asymptotic Solutions of Elastic Shell Problems, by R. A. Clark; The Integral Equations of Asymptotic Theory, by A. Erdélyi; Application of Langer's Theory of Turning Points to Diffraction Problems, by N. D. Kazarinoff.

*Linear system theory: the state space approach.* By L. A. Zadeh and C. A. Desoer. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1963. xxi + 628 pp. $15.00.

The concept of "control," and of "feedback control" in particular, is paramount in contemporary science, economics, and engineering. Mathematical investigations in this domain grew out of the technology of the nineteenth century, from such sources as the steam engine. Through a series of logical steps, initiated by Maxwell, cf. the account in

H. Bateman, "The Control of an Elastic Fluid," *Bull. Amer. Math. Soc.*, Vol. 51, 1945, pp. 601–646,

the fundamental physical idea of feedback control was replaced by the mathematical study of linear systems of differential equations with constant coefficients. Using transform techniques, this, in turn, was replaced by the study of particular classes of transfer functions. Eventually, "control theory" became virtually synonymous with tracing loops in the complex plane, and thus became a branch of elementary complex variable theory.

Clearly, elegant as much of this is, it furnishes little basis for the mathematical study of the demands of modern technology with its nonlinearities, constraints, and stochastic and adaptive features, and, simultaneously, its symbiotic relation with the digital computer. To take account of these features of realism and, for example, to provide the algorithms for on-line computing, it is essential to return to the original idea of feedback control in the time domain, one founded on the concept of "state space."

This concept arises in the following fashion. Given a system $S$, we describe it at any time $t$ by a vector $x(t)$. This is called the *state* of the system. The time history of the system is often determined by means of a differential equation of the form $dx/dt = g(x, t)$, or a difference equation, $x(t + 1) = g(x(t), t)$. It is now important, for all further work in this area, to make precise what one means, in relation to equations of this and more general structure, by such common terms as "linearity," "casuality," "input-output," "controllability," "equivalent system," and so forth. This, and much more, the authors do in careful detail, with numerous discussions and illustrative examples. Furthermore, this is systematically combined with the classical concepts, such as matrix methods, stability theory, complex variable techniques, Laplace transform theory, and even the Bochner-Friedrichs-Schwartz theory of distributions, to provide the reader with a firm basis for his further studies in modern control theory of the type presented, for example, in

L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko, *Mathematical Theory of Optimal Processes*, Interscience, New York, 1962.

R. Bellman, I. Glicksberg, and O. Gross, *Some Aspects of the Mathematical Theory of Control Processes*, The RAND Corporation, R-313, 1958.

*(Continued on p. 170)*

# BOOK REVIEWS

*(Continued from p. 120)*

This brief discussion actually does little justice to the wide scope of the book and to the erudition the authors have brought to bear on their task of exposition. The format of the book is most attractive and the presentation is leisurely and lucid. In all ways, it is highly recommended for a classroom text, and for individual reading by students and teachers alike.

RICHARD BELLMAN

*Methods of quantum field theory in statistical physics.* By A. A. Abrikosov, L. P. Gorkov, I. E. Dzyaloshinski. Translated from the Russian by R. A. Silverman. Prentice-Hall, Inc., New Jersey, 1963. xv + 352 pp. $12.00.

In recent years, methods developed in quantum field theory have been applied with remarkable success to many-particle problems in statistical physics. The present book, whose authors are among the most prominent contributors in this field, gives a systematic account of the subject. The treatment is confined to the approach based on the use of Green's functions and Feynman diagrams, which has been developed mainly by Russian authors. Other methods such as those developed by Hugenholtz, and by Bloch and de Dominicis, are not covered. The representation is further limited to systems in thermal equilibrium; transport processes are not included. The reader will appreciate the emphasis on the physical content behind the rather complicated mathematics involved.

Thus, the book starts with a phenomenological discussion of the quasi-particle concept in the theories of liquid helium and of the Fermi liquid (chapter 1), before the mathematical apparatus of field theoretical methods is introduced in full detail in chapters 2 (for absolute zero) and 3 (for finite temperatures). The practical character of these general methods is illustrated in the following chapters by applying them to a variety of specific problems: Theory of the Fermi liquid (chapter 4); systems of interacting bosons (chapter 5); electromagnetic radiation in an absorbing medium, with application to the calculation of molecular interaction forces (chapter 6); and theory of superconductivity (chapter 7). The extensive bibliography gives references and titles of the important papers in the field.

According to the preface, the present English edition is the product of the closest cooperation between the authors and the translator. Drastical changes of two sections and a variety of smaller additions and corrections make it a revised edition rather than a mere translation.

On the whole, the book can certainly be recommended as a competent introduction into a fast growing field. The reader should be familiar with the elements of statistical physics and quantum mechanics. The method of second quantization and all information needed to derive the field theoretical methods are included in chapter 1.

H. THOMAS

*Variational principles in the theory of collisions.* By Yu. N. Demkov. Translated by N. Kemmer. The Macmillan Co., New York, 1963. x + 157 pp. $6.50.

Variational methods are widely used in quantum mechanics to determine the stationary states of atomic systems, which are given by the *discrete* spectrum of a self-adjoint operator. Scattering problems, i.e. problems connected with the *continuous* spectrum of such operators, on the other hand, are usually treated by perturbation theory, and variational principles have been formulated only fairly recently for these problems. In the present book, an account is given of the application of variational methods to collision problems, and the relation between various approaches is discussed. Emphasis is put on questions of principle rather than on practical results of specific problems. Nevertheless, the methods are displayed for concrete problems which are successively generalized, instead of developing them for an abstract, most general form of scattering theory. Mathematical proofs are given with "physical" degree of accuracy. Numerical examples serve to illustrate general statements and methods. In the selection of problems, attention is confined to stationary problems in non-relativistic theory, although the extension to non-stationary problems is discussed.

*(Continued on p. 180)*

# BOOK REVIEWS

*(Continued from p. 170)*

Chapter headings: I. Formulation of the Variational Principle; II. The Connection between Various formulations of Variational Principles and their Application in the Theory of Collisions; III. The Symmetry of the Functionals, the Principle of Detailed Balance, and the Unitarity of the Scattering Operator; IV. Variation of Scale and the Virial Theorem for Scattering Problems. Each of the chapters II, III and IV is self-contained and can be read directly after chapter I.

The book can be recommended as a competent introduction into a field of new applications of old principles. Familiarity with non-relativistic quantum mechanics, including the principles of the theory of collisions, is required.

H. Thomas

*Elements of quantum electrodynamics*. By A. I. Akhiezer and V. B. Berestetskii. Oldbourne Press, London, and Program For Scientific Translations, Jerusalem, 1962. 301 pp. $12.00.

The Russian original of the present translation (2nd edition: Moscow 1959) is one of the classical treatises of the theory of electromagnetic interactions. Starting from a description of the electromagnetic field and the electron-positron field, and their quantization, the main part of the book is devoted to the interaction between these two fields. The representation is based on the invariant formulation of perturbation theory, which gives the results (power series in the fine structure constant $\alpha = e^2/\hbar c$) in closed and relativistically invariant form, and allows a rigorous formulation of the renormalization rules which are required to remove the divergences. A good deal of space is devoted to general principles, as symmetry properties, etc.

The present translation contains a full description of the general principles of quantum electrodynamics (essentially chapters I, II, III, IV and VII of the Russian original). The applications to specific problems, i.e., the calculation of actual electrodynamic processes (essentially chapters V and VI: first order interaction processes, and IX: higher order radiation corrections, of the Russian original) have mostly been omitted. A few applications, namely the scattering of a photon by a free electron, the Bremsstrahlung, and the emission of long-wavelength phonons, have been included as an appendix.

Chapter headings: I. Quantum Mechanics of the Photon; II. Relativistic Theory of the Electron; III. The Quantized Electromagnetic and Electron-Positron Fields; IV. Fundamental Equations of Quantum Electrodynamics; V. The Scattering Matrix; Appendix.

The shift of emphasis from specific problems to the basic theoretical principles makes the book especially useful to those readers who are primarily interested in an introduction into the theory of quantized fields. The reader should be familiar with the principles of quantum mechanics; previous knowledge of relativistic quantum theory is not required.

It should be noted that a German translation of the complete book has been published (B. G. Teubner, Verlagsgesellschaft, Leipzig 1962).

H. Thomas

*Differential equations*. Fourth Edition. By Max Morris and Orley E. Brown. Prentice-Hall, Inc., New Jersey, 1964. vi + 366 pp. $8.50.

This revised and enlarged edition covers the standard topics in an elementary course in differential equations. Some new material has been added in this edition, especially the Laplace transform and some new topics on Fourier series and the solution of partial differential equations by separation of variables. Also included are chapters on the numerical solution of both ordinary and partial differential equations.

Exercises are numerous and well chosen. The authors do not emphasize any of the geometric aspects of differential equations; for example, the concept of state space or phase space is not discussed. There is also not much discussion about forced linear systems and no discussion of nonlinear systems.

Jack K. Hale

*(Continued on p. 191)*

# BOOK REVIEWS

*(Continued from p. 180)*

*Tables of the principal unitary representations of Federov groups.* By D. K. Faddeyev. Translated by Prasenjit Basu. The Macmillan Co., New York, 1964. xxvi + 155 pp. $10.00.

The Mathematical Tables Series provides translations of works most of which were originally published in the U. S. S. R. This book is Volume 34 of the Series and is printed in Poland.

The fifteen page introduction gives a very well-written and concise formulation of the theorems and relations which allow the construction of all irreducible unitary representations of the space groups of crystallography (Fedorov groups). These include the Shubnikov groups which are important in crystals with magnetic moments.

While the mathematician may be delighted by the introduction, the physicist and crystallographer would have more use of a theoretical introduction followed by a separate section with rules for the use of the tables, which are by no means self-explanatory.

The book contains the so-called "principal representations," that is the unitary irreducible representations of the normalizers of the 73 series of Fedorov groups. It is then left to the reader to construct all unitary irreducible representations of the entire group by the method of regular continuation. The latter method, while simple in principle, is rather delicate to carry out in practice, and it is deplorable that no example is given to elucidate its use.

This book proves once more, that a literal translation of a Russian monograph will not meet the needs of the English-speaking scientist without competent editorial work. For example, it contains symbols which are defined only in hardly available Russian books. Also, most references inconveniently relate to Russian works (even though some of them are translated into English) or to Russian editions of Western books. That the translator is not entirely familiar with his subject may be seen from his consistent transliteration of the Bravais cell into Brave's parallelepiped.

These remarks, of course, should not impair the great accomplishment of the author and his collaborators in preparing the tables.

P. Erdös

*Introduction to mathematical sociology.* By James S. Coleman. Collier-Macmillan Lts., London, 1964. xiv + 554 pp. $9.95.

The application of theory to empirical sociological data is still infrequent. Armchair-theorists tend to weave theories which are ignored by empiricists, who in turn are often content with fitting an occasional curve or obtaining a statistically significant coefficient of relationship. The use of mathematics to express a sociological theory is still a novelty, and a book devoted to the topic is welcome to the small band of mathematically-oriented sociologist and their graduate students. Coleman makes some effort to have his book live up to its general title, by including several chapters devoted to a general discussion of mathematics and theory construction, problems of quantitative measurement in sociology, measures of structural characteristics, and tactics and strategies in the use of mathematics. The central core of the book, however, represents Coleman's own prodigious personal efforts in the field of mathematical models, rather than a compendium of useful available mathematical techniques of laborers in the same vineyard.

In an early chapter Coleman points out the advantage of using mathematics to express theoretical relationships. He claims that by using differential equations to express relationships analytic solutions to problems can be found. He notes, however, that a true metric is nonexistant in sociology and limits himself generally to categorical data obtained through counting procedures. He ignores the statistician's handling of continuous scales in such techniques as multiple regression, analysis of variance and factor analysis. He also insists upon application of continuous time process rather than discrete probabilistic ones to his discrete data. These positions provide the book with a unifying point of view. The models which he covers are largely represented by $2 \times 2$ contingency tables at two or more discrete periods of time.

Even within the confines of his own restrictions Coleman covers a wide range of topics. The most useful models for students of sociology are probably found in the chapters on the Poisson process and its contagious relatives. Since the Poisson process deals with frequency counts and can be assumed to be a continuous time function, Coleman's selected approach is suited to the type of problems covered by it. He applies it to such social processes as the number of sociometric choices perceived by a girl, a model of voting in small groups, the size of free-forming small groups.

Some of the other models considered by Coleman can be handled more easily by other methods, which are more familiar and more easily understood. For example, the continuous time process version of the familiar Markov chain process leads to the need for lengthy estimation of transition probabilities, for which the use of a computer program is advised. The multivariate model proposed by Coleman for frequency counts in $2 \times 2 \times 2$ contingency table is more easily handled by use of analysis of variance. His urn model of organization of social attitudes to explain the consistency of attitudes measured at two points in time is ingenious but artificial. Many sociologists undoubtedly will settle for a correlational measure of degree of consistency. Coleman offers an alternative null hypothesis to the "distance-interaction hypothesis," but its adherents probably will prefer to stick with the original version.

The continuous time process approach using differential equations would be more suitable to sociology if it were applied to continuous variables, if social data were accumulated over a continuous time period rather than at a few discrete points in time, and if social processes were deterministic and nonstatistical in nature. The mathematical models offered by Coleman are applicable to relatively simple segments of social processes, and cannot be expected to portray the complex social processes studied by sociologists generally. These simple models are useful in the analysis of limited amount of quantitative data. For more complex situations other approaches such as multivariate statistical techniques and computer simulation will be needed.

James M. Sakoda

*Fourier series*. By G. P. Tolstov. Translated by R. A. Silverman. Prentice-Hall, Inc., New Jersey, 1962. x + 336 pp. $13.00.

This is an English translation of Tolstov's famous book on Fourier Series. The contents is indicated by the following list of chapter headings: Trigonometric Fourier series, orthogonal systems, convergence of trigonometric Fourier series, trigonometric series with decreasing coefficients, operations on Fourier series, summation of trigonometric Fourier series, double Fourier series and the Fourier integral, Bessel functions and Fourier-Bessel series, the eigenfunction method and its applications to mathematical physics.

Morton E. Gurtin

*Discrete dynamic programming: an introduction to the optimization of staged processes.* By Rutherford Aris. Blaisdell Publishing Co., New York, 1964. x + 148 pp. $5.00.

This is a well-written introduction to dynamic programming as it applies to the optimization of processes with discrete stages. Various methods of optimization are discussed in a qualitative manner in Chapter 1, and a number of closely related problems from chemical engineering are formulated in Chapter 2 for subsequent use as examples. Bellman's optimality principle is carefully discussed in Chapter 3. Chapter 4 is concerned with graphical methods and Chapter 5 with duality and Lagrangian multipliers. The next three chapters treat applications in economics (Chapter 6) communication and information theory (Chapter 7), and miscellaneous fields (Chapter 8: curve fitting, reliability theory, pest control, and Jacobi matrices). The relations between continuous and discrete models are discussed in Chapter 9. Chapter 10 is concerned with extensions and limitations of the method. Chapter 11, finally, presents associated mathematical ideas such as a discrete form of Pontryagin's principle and the maximum transform.

The presentation is on an elementary level and easy to follow. With the exception of Chapters 1 and 10, each chapter contains problems, which will enable the student to test his understanding of the subject matter.

W. Prager