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The Quarterly prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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BOOK REVIEWS

Mathematical optimization techniques. Edited by Richard Bellman. University of California Press, Berkeley and Los Angles, 1963. xii + 346 pp. $8.50.

The papers in this volume were presented at a Symposium on Mathematical Optimization Techniques in Santa Monica, California October 18–20, 1960. The table of contents reads as follows:

Part 1: Aircraft, rockets, and guidance

1. A survey of the problem of optimizing flight paths of aircraft and Missiles, by A. Miele
2. Estimating performance capabilities of boost rockets, by P. Dergarabedian and R. P. Ten Dyke
3. The Optimum Spacing of Corrective Thrusts in interplanetary navigation, by J. V. Breakwell
4. The analysis and solution of optimum trajectory problems, by S. E. Dreyfus

Part 2: Communication, prediction, and decision

5. A new approach to the synthesis of optimal smoothing and prediction systems, by E. Parzen
6. Adaptive matched filters, by T. Kailath
7. Optimization Problems in Statistical communication theory, by D. Middleton
8. Estimators with minimum bias, by W. J. Hall
9. On optimal replacement rules when changes of state are Markovian, by C. Derman

Part 3: Programming, combinatorics, and design

10. Simplex method and theory, by A. W. Tucker
11. The present status of nonlinear programming, by P. Wolfe
12. The number of simplices in a complex, by J. B. Kruskal
13. Optimization in structural design, by W. Prager
14. Geometric and game-theoretic methods in experimental design, by G. Elfving

Part 4: Models, automation, and control

15. Automation and control in the Soviet Union, by J. P. LaSalle
16. The theory of optimal control and the calculus of variations, by R. E. Kalman
17. Mathematical model making as an adaptive process, by R. Bellman

The first three papers consider optimization problems arising in the determination of optimal trajectories for manned and unmanned flights. They approach these problems by the conventional calculus of variations. The fourth paper also applies the method of successive approximations and the theory of dynamic programming to these questions. The fifth and sixth papers are concerned with prediction and filtering while the seventh is a study of optimization problems arising in communication theory. The eighth paper discusses optimal allocation of effort in testing and experimentation. The last paper in part 2 applies dynamic programming to the study of certain replacement processes. Papers 10 and 12 examine certain questions in linear programming, while the eleventh paper deals with some of the basic results in nonlinear programming. Paper 14 discusses some new developments in the theory of experimental design and paper 13 considers organization problems in the design of structures. The first paper in part 4 is a survey of work on the theory of control processes in the Soviet Union. Paper 16 develops some aspects of dynamic programming along the lines of Hamilton—Jacobi theory. The last chapter by the editor is devoted to a formulation of mathematical model making as an adaptive control process. The volume contains an interesting collection of papers on a variety of problems concerned with optimization.

M. ROSENBLATT

Psychological statistics. By Quinn McNemar. Third Edition. John Wiley & Sons, Inc., New York and London, 1962. vii + 451 pp. $7.75.

Like the two earlier editions, the third edition is an elementary introduction to statistics from a nonmathematical point of view, written primarily for the psychologist or the educator. It is too difficult to be a beginning text in statistics unless a fair degree of mathematical sophistication is assumed. The reason for this is that the author develops statistics from a theoretical point of view, and shows symbolically the relationships between different statistical procedures rather than emphasizing the intuitive meaning of basic concepts. In some respects, the level is uneven since parts of the text could

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BOOK REVIEWS

(*Continued from p. 132*)

easily be handled in a first course in statistics, while other parts would be difficult even for the relatively sophisticated. My own feeling is that it serves best as a review book for someone with one or two semesters of applied statistics.

The scope of the material is that traditionally covered in elementary texts: graphical and descriptive statistics, hypothesis-testing including Chi-Square, analysis of variance, nonparametric statistics, and tests of trends. Correlational methods and analysis of variance are emphasized with the relationship of reliability estimation to correlation well developed.

Perhaps the chief recommendation for the book is that McNemar knows his statistics and presents the material clearly and correctly. He gives sound conclusions on a number of recent controversies; for example, the uses of nonparametric versus parametric statistics, the strictness of the homogeneity of variance assumption in analysis of variance, the relationship between measurement scales and statistics, and the proper procedure for testing selected contrasts. Other good points include the thought exercises at the end of the book, the discussion of the theoretical relationships among the normal, t, Chi-Square, and F distributions, and the section on trend analysis—especially useful for experimental psychologists. (However, see below for some reservations concerning these points.)

The major criticism is that the referencing procedure is very bad. There is no bibliography or list of references but only footnotes where necessary. For almost all topics considered, no general list of other material, primary sources, or secondary sources is given. Because some of the topics are not covered thoroughly, the student needs references in order to pursue the topic further. For example, the topics of discriminant functions and trend tests are not fully developed. McNemar describes a variant of Alexander's test for trend and verbally describes Grant's "Analysis-of-Variance Tests in the Analysis and Comparison of Curves," but does not cite either of these two sources. There is no discussion of decision theory, factor analysis, or the use of computers, all of which should be mentioned in a modern textbook on statistics. The author nowhere discusses the case of unequal cell frequencies in factorial analyses of variance and the rule for proportional cell frequencies. This omission could lead to some confusion on the part of the student. Finally, the concept of expectation is introduced only in an indirect way. Since this is a basic concept of statistics and one that McNemar uses a great deal in deriving the expectations of sample statistics, it would seem expedient to introduce it early in the book.

When one considers this book from the point of view of what was attempted, it is very well done. However, from my own point of view concerning the needs of statistics in psychology, it is not the right approach. I feel that a more mathematically orientated approach is needed, with emphasis on mathematical statistical concepts. The use of statistics in psychology is growing rapidly, but it is moving in the direction of mathematical models for learning and psychophysics, methods for scaling, new statistics based on new experimental procedures, decision theory, and Markhov chains. The use of these types of statistics does not require cookbook knowledge of statistical tests, but rather the ability of the researcher to master and apply new statistical techniques as needed. This ability can best be fostered by teaching rigorous mathematics and not specific tests. In some respects, McNemar's *Psychological Statistics* is better than many psychological statistics texts in that he does emphasize the theoretical relationships between statistical procedures.

R. B. MILLWARD

Probability theory and mathematical statistics. By Marek Fisz. John Wiley & Sons, Inc., New York and London—published in collaboration with PWN, Polish Scientific Publishers, 1963. xvi + 677 pp. $15.75.

This book is a revised and enlarged edition of an earlier work by the author printed in Polish. There are two parts, the first on Probability Theory and the second on Mathematical Statistics. The chapters in the first part read as follows: 1. Random Events, 2. Random Variables, 3. Parameters of the Distribution of a Random Variable, 4. Characteristic Functions, 5. Some Probability Distributions, 6. Limit Theorems, 7. Markov Chains, 8. Stochastic Processes.

A system of axioms for the theory of probability (following Kolmogorov) is introduced. Conditional probabilities and independent events are then discussed. Problems of historical importance and interest like the Bertrand paradox and Buffon's problem are given in the exercises. Random variables and distribution functions are defined and treated at some length. Mean values, moments, and the Chebyshev inequality follow. The properties of characteristic functions are then given. At times the proof of a basic theorem, such as the inversion formula for characteristic functions, is too difficult to be given. In that case a heuristic discussion or the proof in a restricted context may be given .Basic probability distributions such as the Bernoulli, Poisson, uniform, normal, and gamma distribution are presented. Various versions of the law of large numbers and the central limit theorem follow. The first part concludes with a short chapter on Markov chains and a discussion of specific random processes like the Poisson process, Wiener process, and Polya process. Brief remarks on classes of processes, for example, birth and death processes, stationary processes and such are given together with references to relevant literature.

The second part contains the following chapters: 9. Sample Moments and their Functions, 10. Order statistics, 11. An Outline of the Theory of Runs, 12. Significance Tests, 13. The Theory of Estimation, 14. Methods and Schemes of Sampling, 15. An Outline of Analysis of Variance, 16. Theory of Hypotheses Testing, 17. Elements of Sequential Analysis.

The theory of sampling from a normal population is first given. This leads to a discussion of the X^2 distribution, Student's t-distribution, and Fisher's z-distribution. In one chapter on order statistics, limit distributions for sample quantities and empirical distribution functions (Kolmogorov, Smirnov theorems) are treated. Parametric tests and nonparametric tests are examined. Among the nonparametric tests, the X^2 test, tests of the Kolmogorov and Smirnov type, as well as the Mann-Whitney are given. Unbiasedness, sufficiency and efficiency of estimates are dealt with in one chapter on estimates. One-way and multiple classifications are given in the chapter on analysis of variance. The power function, the concepts of most powerful tests and unbiased tests are given in Chapter 16. The final chapter on sequential analysis considers the sequential probability ratio test.

The book can be recommended as a text suitable for many courses in the theory of probability and statistics. The reader and prospective student is brought face to face with many of the modern techniques in the theory of probability and statistics.

M. ROSENBLATT

Programming and coding digital computers. By Philip M. Sherman. John Wiley & Sons, Inc., New York and London, 1963. xiv + 444 pp. $11.00.

This is a beginner's text, very simple and detailed, yet it describes many important computer programming techniques, such as switches, push down lists, indirect addressing, list structures, monitors, macros, and interpretive programming. This, in itself, is sufficient to place this book at the top of the list of similar texts seen by this reviewer.

An imaginary computer (a one-address, single accumulator, binary machine called DELTA 63) is used to describe the techniques of programming in machine language (called "coding" by the author, who restricts the term "programming" to the flow charting stage of problem analysis). Fortran and Algol are introduced towards the end of the book.

This book can be strongly recommended for self-study, or for a formal course for which an actual computer is not available. However, it would probably be confusing for a student to study DELTA 63 and a real computer simultaneously.

B. A. CHARTRES

Mécanique des milieux continus. By P. Germain. Masson et Cie, Paris VI, 1962. x + 412 pp. $15.50.

Solid mechanics and fluid dynamics have a good deal of their mathematical apparatus, and a little of their basic laws, in common. A desirable economy therefore results from teaching this common part only once. The text under review has developed from courses reflecting that fact. It was developed in France, and so is naturally distinguished by clarity and a certain degree of formality—the latter enhanced by the economy sought in the 'commonality' of the apparatus.

The author is a fluid dynamicist, and a necessary condition for a favorable review is satisfied by the choice of a similar specialist for reviewer. It is indeed striking how the felicity of touch increases as the author turns from classical Elasticity (90 pp.) to near-classical Fluid Mechanics (131 pp.), and some sections on topics close to the author's research are little gems. Of course, every other professor will deplore how little space was left for topics close to his own heart.

However, good mathematical texts on solid *or* fluid mechanics are scarce, and most professors preparing courses on either subject will find this text useful in several ways. It also contains a collection of 208 problems.

R. E. MEYER

Mathematics for the physical sciences. By Herbert S. Wilf. John Wiley & Sons, Inc., New York and London, 1962. xii + 284 pp. $7.95.

This book assumes some knowledge of the theory of functions of real and complex variables and is based on a two semester course in mathematical methods at the graduate level. The appeal of a book such as this depends upon the reader's estimate of the relative importance of the subjects treated in comparison with those omitted and also upon the manner in which the topics are presented. The selection of topics here is (in order) vector spaces and matrices, orthogonal functions, roots of polynomial equations, asymptotic expansions, ordinary differential equations, conformal mapping, and extremum problems. Numerical methods are discussed in some of the sections and about one hundred exercises with solutions are provided. The book is carefully written and the style is pleasing.

R. T. SHIELD

Introduction to partial differential equations. By Donald Greenspan. McGraw-Hill Book Co., Inc., 1961. viii + 195 pp. $7.50.

The Preface of this book asks the reader to note that "Chapters 1 and 2 merely develop those aspects of ordinary differential equations, complex variables, and Fourier series which are essential for the study of partial differential equations, which begins properly in Chapter 3." Since Chapters 1 and 2 occupy only sixty-five pages (four of these being devoted to ordinary differential equations), the reviewer cannot help but question the reasonableness of this request. The remaining one-hundred and thirty pages contain a poorly written and uninspired introduction to the theory of second order partial differential equations.

R. T. SHIELD

An introduction to the calculus of variations. By L. A. Pars. John Wiley & Sons, Inc., New York and London, 1962. xi + 350 pp. $8.50.

The author's aim is to present an introduction to classical topics in the calculus of variations. What sets this book off from other recent texts on the subject by Funk, Akhiezer, Elsgolc, and Cicala is the liberal use of carefully worked examples. Students will appreciate this feature. The treatment of the problem of Lagrange is noteworthy. The book clearly shows the influence of R. Courant.

To gain an adequate view of this area, though, ome additional reading would be necessary. The realization that engineering design can be viewed as an optimization problem and that the desire to carry out various processes in optimal fashion leads directly to variational problems has led to great activity in the last decades. In particular, there has been emphasis on the introduction of stochastic elements into the processes considered and on the computational solution of the Euler equations subject to two-point boundary conditions. These topics are discussed in R. Bellman, *Adaptive Control Processes*, Princeton Univ. Press, 1961, and L. S. Pontryagin et al., *The Mathematical Theory of Optimal Processes*, Interscience Publishers, 1962. A comparison of various approaches to modern problems in automatic control is given in the Russian paper O. M. Avdeyev et al., "A Comparison of New Methods of Applied Mathematics in Automatic Control Theory," *Akademiya Nauk SSSR. Izvestiya. Otdeleniye Tekhnicheskikh Nauk. Energetika i Avtomatika,* no. 6, 1962, pp. 178–200.

R. KALABA

Magnetohydrodynamic shock waves. By J. Edward Anderson. The M. I. T. Press, Cambridge, 1963. xiv + 226 pp. $6.50.

This book is a Research Monograph—a short book on a single special research topic. The first three chapters, some 90 pages, are devoted almost entirely to "setting the stage" by reviewing in a coherent fashion most of the work establishing the nature of the kinds of magnetohydrodynamic shock waves that are possible under the conservation laws and their relative stability on interaction with impinging MHD wave trains in an arbitrary direction.

The types of discontinuities which can exist according to the conservation laws and second law of thermodynamics are discussed in graphical terms. The three surfaces in temperature, specific volume and tangential magnetic field 3 space are examined for number and ordering of possible shock points. This reviewer finds this 3D treatment more difficult to visualize than the 2D equations obtained by elimination of the tangential field. However, the 3D visualization effort is worth the effort in view of its use in Chapter 5 on the internal shock structures in the MHD shocks where the basic system is 5D.

The stability of the MHD shocks to perturbation by various impinging, refracting, and reflecting wave trains, reminds one of the corresponding discussions of the stability of strong and weak oblique shock solutions in ordinary compressible flow. In MHD there are ranges of values of shock strengths and velocities for which wave interactions are found and other ranges where no solutions exist. These latter are called "non-evolutionary." The exact fate of such waves is left uncertain. In particular, intermediate, switch-on, and switch-off shocks are non-evolutionary.

After a chapter devoted to a brief review of the kinetic theory derivation of the continuum magnetohydrodynamic equations, we come to a long chapter (5) on "General Qualitative Study of the Shock Layer." This is a general discussion of the existence of shock structures capable of producing the various types of shocks described by the steady discontinuity theory. While much progress is made in this task, it is left unfinished because of its complexity. The structure equations are five in number, describing the rates of change of density, temperature, tangential velocity, tangential magnetic field, and tangential current. Thus, for complete insight, one should be able to visualize solution trajectories in 5D space. The nature of solutions near the end points is examined in careful detail and the intermediate region is pieced together as best can be done.

In this way, fast shocks get a clean bill of health, slow shocks are left incomplete because of the "impossibility of visualizing 5D space in toto," while intermediate shocks appear sometimes to exist and sometimes not to exist.

To supplement the unfinished arguments of Chapter 5, Chapter 6 considers various special cases in which the 5D space reduces to 3D. Thus, various combinations of negligible bulk viscosity, shear viscosity, thermal conductivity, electrical conductivity and current inertia are examined to firm conclusions.

This 220-page book concludes with a brief (10 page) appendix by W. H. Heiser on an attempt to experimentally produce MHD shock waves in a Hydrogen plasma. In spite of the very considerable data scatter, the existence of some of the theoretically predicted shocks seem to be suggested by the experimental results.

HOWARD W. EMMONS

Handbook of nonparametric statistics. By John E. Walsh. D. Van Nostrand Co., Inc., Princeton, Toronto, New York, London, 1962. xxvi + 549 pp. $15.00.

This book is in a literal sense a handbook for the material on nonparametric analysis developed up to the year 1958. It is organized in chapters and some idea of the broad scope of the work can be given by listing them as follows:

1. Outline and scope of handbook, 2. Standardized notation, 3. Description and use of format, 4. Discussion of concepts and special terminology, 5. Tests of randomness, 6. Tchebycheff type inequalities, 7. Estimates and tests for expected values, 8. Estimates and tests for population percentiles, 9. Distribution—free tolerance regions, 10. Nonsequential results for distributions from ungrouped data, 11. Sequential, decision and categorical data results for distributions.

A specific format is generally followed. The name of the procedure is given first. The type of data used is then indicated. The description of the procedure discusses the context in which it can be applied as well as properties of the population. A statement of the assumptions and results then follow. The characteristics of the procedure and a list of relevant bibliography complete the discussion.

Assuming a careful reading of the sections on notation, format, and terminology by the reader, this book offers a helpful and useful survey of the literature in nonparametric prior to 1958. The author should be congratulated on the completion of what must have been an exceedingly arduous task.

M. Rosenblatt

An introduction to digital computing. By B. W. Arden. Addison-Wesley Publishing Co., Reading, Mass. ix + 389 pp. $8.75.

This text is based on a one-semester undergraduate course which the author has taught for several years at the University of Michigan. This course consists of one hour of lecture and one hour of recitation per week and has elementary calculus as a prerequisite.

While the text is primarily concerned with programming in MAD, nearly all examples deal with numerical methods, for which a modicum of mathematical background is developed where necessary. The scope of the book is best summarized by the following chapter headings: Language and notation—Elements of a practical language—Statements and flow charts—Functions and example programs—The design of a practical machine.—Machine language and components—Number systems and arithmetic—Computational error—Taylor's series and divided differences—The solution of equations—Additional programming topics—Interpolation—Numerical integration—Simultaneous linear equations—Approximation—The numerical solution of ordinary differential equations—Nonnumerical problems—A simple compiler. Appendices are devoted to the MAD programming language and the operating system. There are 19 pages of exercises at the end of the book. Throughout the book the presentation is clear and easy to follow. Flow charts are given for all example programs and this feature will be welcomed by the reader who plans to program in another language.

W. Prager

Oscillations in nonlinear systems. By Jack K. Hale. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1963. ix + 180 pp. $9.00.

This is an excellent presentation of results recently obtained concerning the existence and construction of periodic and almost-periodic solutions of linear and nonlinear vector differential equations of the form $dx/dt = Ax + \epsilon f(x, \epsilon, t)$. Many of the results stem from earlier techniques of Poincare, Krylov-Bogoliubov, and others, but a number of the most important are the results of research due to Cesari, Gambill, and Hale himself.

There is a useful introduction to the more complex theory of integral manifolds, where the theories of Diliberto and others are sketched.

The discussion is clear and the notation is easily followed. The book is heartily recommended to all those concerned with the theory and application of differential equations.

Richard Bellman

Differential-difference equations. By Richard Bellman. Academic Press, New York, London, 1963. xvi + 462 pp. $13.75.

This book is primarily concerned with the stability of the difference-differential equation

$$u'(t) + au'(t - \omega) + bu(t) + cu(t - \omega) = f(t), \qquad \omega > 0, \tag{1}$$

and its generalizations. Equation (1) is said to be of *retarded* type if $a = 0$, and of *neutral* type if $a \neq 0$. Its characteristic equation is $h(s) = s + ase^{-\omega s} + b + ce^{-\omega s}$.

After a brief two-chapter review of some basic facts about the Laplace transform and first-order systems of linear ordinary differential equations, the author devotes three chapters to the equation (1). He formulates well-set "initial value" problems for it and for the related *advanced* type equation $u'(t - \omega) + bu(t) + cu(t - \omega) = f(t)$. He then obtains representations of the solutions of (1) by definite integrals of fundamental solutions.

What is especially interesting, he proves that every continuous solution of the reduced equation obtained from (1) by setting $f(t) = 0$ approaches zero as $t \to \infty$, if and only if the real parts of the roots of the characteristic equation of (1) have a negative upper bound. In the next chapter, this result (which is essentially the condition that the input-output problem be well-set) is extended to systems of the form

$$(2) \qquad \sum_i A_i u'(t - \omega_i) + \sum_i B_i u(t - \omega_i) = f(t),$$

where the A_i and B_i are matrices with det $A_0 = 0$, $u(t)$ is a vector-valued function, and $0 = \omega_0 < \omega_1 < \cdots < \omega_m$. In this generalization, the characteristic roots are the numbers s such that the matrix $H(s) = \sum_{i=1}^{m} (sA_i + B_i)e^{-\omega_i s}$ is singular.

In Chapters VII and VIII, the author obtains related results for the renewal equation, and for systems of renewal equations whose coefficient-matrix is *positive*. Bounds on the rate of exponential growth are given.

Chapters IX and X treat the asymptotic behavior (as $t \to \infty$) of systems having asymptotically constant variable coefficients:

$$(3) \qquad \sum_{i=1}^{m} A_i(t)u'(t - \omega_i) + \sum_{i=1}^{m} B_i(t)u(t - \omega_i) = f(t),$$

where $A_i(t) = A_0 + A_1/t + A_2/t^2 + \cdots$ and det $A_0 \neq 0$. In Chapter XI, the Poincaré-Liapounov Theorem is extended to the quasilinear equation $u' + bu + cu(t - \omega) = f(u(t), u(t - \omega))$, and the exponential rate of growth of solutions is bounded.

The last two chapters analyze the location and "stability" of exponential polynomials of the form $\sum p_j(z)\, e^{\omega_j z}$; the results are relevant to the earlier part of the book.

GARRETT BIRKHOFF

General stochastic processes in the theory of queues. By Vaclav E. Benes. Addison-Wesley Publishing Co., Inc., Reading, Mass., Palo Alto, London, 1963. viii + 88 pp. $5.75.

This short monograph deals mostly with the evaluation of the virtual delay for a single server queue but it is unlike any previous treatments of queues in that very few assumptions are made regarding the stochastic structure of the arrivals and service times. It is essentially a review of the author's own recent work on the subject. Although this work is mathematically very elegant, it will be of interest mostly to specialists in the mathematical aspects of queueing theory.

G. F. NEWELL

Principles of the statistical theory of communication. By Willis W. Harman. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1963. xi + 291 pp. $10.50.

Professor Harman has condensed much of statistical communication theory into a 291 page textbook for first-year engineering graduate students. The first 85 pages cover mathematical preliminaries (Fourier analysis, probability). This leaves 81 pages for information theory, 48 pages for analysis of noise in networks, 70 pages for statistical decision theory, and 7 pages for bibliography and index.

The book is an introduction to statistical communication theory, not a survey of it. Professor Harman offers a well selected assortment of topics to illustrate some of the basic ideas in an elementary way. Other topics which would require too much space or too advanced mathematics are simply omitted. In spite of its small size and wide range, the book does not seem hurried. It can serve as textbook for a valuable course for students who want statistical communication theory as part of their general education but who don't intend to specialize in it.

The mathematical treatment is convincing rather than rigorous. An intriguing comment on page 13 describes the δ-function: "... (at one time considered rather disreputable)." There seems to be an impression in engineering circles that the theory of distributions has legalized every imaginable manipulation with δ-functions. Nevertheless, Professor Harman does get correct answers by short direct arguments which will satisfy most engineers.

Although primarily theoretical, Professor Harman's book contains many illustrative examples to illustrate practical applications or hardware realizations of the theory. Other examples appear among the exercises for the reader.

E. N. Gilbert

Recent Soviet contributions to mathematics. Edited by J. P. LaSalle and S. Lefschetz. The Macmillan Co., New York, 1962. viii + 324 pp. $8.75.

This is a critical appraisal of Soviet contributions in mathematics based upon papers published mainly during the period 1957–1960. The fields covered are algebra, control and stability theory, functional analysis, numerical analysis, partial differential equations, probability and statistics, and topology. The book is not intended as an exhaustive reference but only to give a general indication of the direction and intensity of recent work. Various topics are described by different authors, not always in the same style, but generally for mathematicians who are not specialists in any particular field. The book is well written and very interesting.

G. F. Newell

An introduction to electronic analogue computers. By M. G. Hartley. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1962. vii + 155 pp. $4.50.

This monograph is intended for the engineer or scientist who uses analog computing equipment and feels, as he indeed should, a need to understand the operation of the electronic devices he employs. The book, in conjunction with the usual manual describing applications of his particular machine, should accomplish that end effectively and relatively painlessly. The reader is presumed to have nothing more sophisticated than a knowledge of Laplace transforms and a slight acquaintance with electric circuits.

After a brief introduction, including a discussion of the relative merits and capabilities of analog and digital machines, the reader is introduced to the fundamental unit of the electronic analog machine, the operational amplifier, its interconnections in the machine, and its design considerations in both vacuum tube and transistor form. The treatment is very concise, as it must be since in these few pages all of the relevant electronic circuit analysis is developed, but still quite readable. The electronic jargon used is British rather than American, but this should cause little difficulty to American readers; jargon is equally annoying in any language.

This book seems eminently suited to the purpose for which it was written, and can be recommended to all those users of analog machines still applying the cookbook technique.

Edward T. Kornhauser

Probability theory. By Michel Loève (Third Edition). D. Van Nostrand Co., Inc., Princeton, New York, Toronto, London, 1963. xvi + 685 pp. $14.75.

This third edition of a popular text and reference on probability theory for mathematicians differs very little from the second edition (1960). Except for correction of errors etc. the only change is a revision of a section on martingales.

G. F. Newell

Ordinary differential equations. By Morris Tenenbaum and Harry Pollard. Harper & Row, New York, Evanston and London, 1963. xvii + 808 pp. $10.75.

This is a very carefully written introduction to the theory of differential equations with many excellent examples and numerous discussions of problems in engineering and physics.

A basic objection to this book as a text is the fact that it is 808 pages long and costs $10.75. This is certainly a bit expensive for an undergraduate text. For those who wish to study the theory on their own, or for instructors of courses in differential equations, this book will be most valuable.

RICHARD BELLMAN

Introduction to the theory of statistics. By Alexander M. Mood and Franklin A. Graybill. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1963. xv + 443 pp. $8.95.

This is a new edition of a book on statistics published by A. M. Mood in 1950. One of the very good features of the original edition was an extensive and interesting set of problems. This has been retained in the revision. The book has been partially rewritten so as to include topics of current interest. The chapter on probability has been expanded. A much more extensive discussion of different criteria for estimates is given in the chapter on point estimation. The format of the chapter on regression and linear hypotheses has been changed while that on experimental designs has been reduced. The book is well worthwhile considering as a text for an introductory course in mathematical statistics.

M. ROSENBLATT

Mathematical theory of elastic equilibrium (recent results). By Giuseppe Grioli. Academic Press Inc., New York, and Springer-Verlag, Berlin, Gottingen, Heidelberg, 1962. viii + 167 pp. $7.25.

This book appears as one of the series "Ergebnisse der Angewandten Mathematik." Its main purpose is to give a mathematical discussion of certain topics in linear and nonlinear elasticity, including the properties of and forms for the thermodynamic potential, existence and uniqueness of solution, and various inequalities for special cases; a discussion of a polynomial approximation method perhaps suitable for machine computation is also given. The author includes a chapter dealing with non-symmetrical stress tensors. There are many topics in the book which workers in appropriate areas of mathematical elasticity will find useful; particularly interesting is the rather thorough survey of recent Italian work—including that of the author himself. There are some minor objections that could be made—such as to the use of unconventional tensor notation, exemplified by $e_r - e_{rr}$ (Eq. 1–12), to the apparently unnecessary preoccupation with the failure of certain equations to hold on set of measure zero, to a lack of appreciation of the generality of the function-space methods of Prager and Synge, and perhaps to a lack of clarity in the thermodynamic discussions—but on the whole the material seems well presented and easy to read.

CARL E. PEARSON

Mathematical methods in physics and engineering. By John W. Dettman. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1962. xii + 323 pp. $9.75.

Recently a number of textbooks have been published which cover more or less the same area as Volume I of Courant and Hilbert's Methods of Mathematical Physics. Since the latter is more a reference than a textbook, these recent texts fill a real need. Dettman's book is among the better ones of this type

In the contest between rigorous proofs and illustrative examples, Dettman favors proofs. The book is not a handbook for engineers. Without being overly abstract, it emphasizes the underlying concepts more than the details of applications. The point of view of linear functional analysis connects the various topics which are covered, and the book has additional coherence in comparison to Courant and Hilbert's work because, being shorter, peripheral topics are omitted. Contents: Algebraic Preliminaries. Calculus of Variations. Boundary-value Problems; Separation of Variables, Green's Functions. Integral Equations. Integral Transform Methods.

A. C. Pipkin

Elements of complex variables. By Louis L. Pennisi with the collaboration of Louis I. Gordon and Sim Lasher. Holt, Rinehart and Winston, New York, Chicago, San Francisco, Toronto, London, 1963. x + 459 pp. $7.50.

This book is designed as a text for a one year first course in complex analysis. The first three chapters treat background material: the complex numbers, point sets, mappings, limits, complex functions, continuity, analyticity. Chapter 4 is concerned with elementary functions, while Chapters 5, 6, and 7 deal with integration, power series, and the calculus of residues. There is a logical gap in the proof of Cauchy's integral theorem as the author tacitly assumes part of what he sets out to prove—namely that the derivative of an analytic function is integrable over a piecewise smooth curve (Lemma 5.8.1). Chapter 8 contains a discussion of the mapping properties of analytic functions with examples of conformal mapping. Finally, Chapter 9 deals with the application of analytic functions to the theory of two-dimensional flows. The book contains a wealth of examples which nicely illustrate the theory.

M. E. Gurtin

An introduction to transport theory. By G. Milton Wing. John Wiley & Sons, New York, London, 1962. xix + 169 pp. $7.95.

The "transport theory" in question is mainly transport in the sense of neutron transport in nuclear reactors rather than in the thermodynamic sense. The aim of the book is to present rather detailed exact analysis of the equations describing simplified physical models. It starts with quite elementary one-dimensional models and goes on to successively more realistic and more complicated ones. The method of attack alternates between the so-called Boltzmann and invariant imbedding approaches. The book is written for applied mathematicians and its purpose is to attract their attention to the types of mathematical problems that arise in this field. It suffers somewhat from frequent repetition of ideas which may be annoying to the specialist but to a reader unfamiliar with the subject it affords easy and leisurely bed time reading. The main ideas are presented in simple and elegant form so one can enjoy reading the book without feeling that one is fighting it. Although the mathematics is kept at a fairly elementary level throughout most of the book there are ample indications of where the more difficult problems lie.

G. F. Newell

Antiplane elastic systems. By L. M. Milne-Thomson. Academic Press Inc., New York, and Springer-Verlag, Berlin, Gottingen, Heidelberg, 1962. viii + 265 pp. $11.00.

This monograph is a sequel to the author's earlier volume on the plane problem in classical elastostatics [*Plane Elastic Systems*, Springer, Berlin-Göttingen-Heidelberg, 1960]. Modifying a notion originally introduced by Filon, the author calls an elastostatic field "antiplane" if there exists a choice of rectangular cartesian coordinates (x_1, x_2, x_3) such that (a) all components of stress, save the normal stress σ_{33}, are independent of x_3, and (b) the displacement component u_3 fails to vanish identically. The most significant class of antiplane problems is thus furnished by the Saint-Venant extension, torsion, and flexure of cylindrical and prismatic bodies, which constitute the main objective of the present volume.

Following a sketch of tensor algebra in terms of continued dyadic products, the basic ingredients of the linear equilibrium theory for anisotropic elastic solids are assembled in Chapter 1. The theory of integration appropriate to antiplane problems is then developed in Chapter 2 with the aid of complex stress and displacement combinations, as well as complex-valued stress functions. These general conclusions are particularized to the isotropic medium and applied to Saint-Venant extension and pure bending in Chapter 3, which also contains some antiplane solutions corresponding to non-vanishing tractions on the lateral boundary. Chapter 4 and Chapter 5 deal with pure torsion, and bending under transverse terminal loads, of isotropic beams. Results concerning the torsion and flexure of anisotropic beams with special elastic symmetries are taken up in Chapter 6, while Chapter 7 includes applications to materials with general linear, or with cylindrical curvilinear, anisotropy.

In selecting specific illustrations the author has evidently favored problems capable of an exact solution in elementary form. Although some experienced readers may find the present treatment of the subject under consideration somewhat eccentric, none are apt to be immune to the author's engaging expository style. As announced in the Introduction, no attempt is made here to supply even a semblance of a guide to the vast literature on the various topics covered; this resolve is reflected in the very sparse and rather whimsical bibliographical references that accompany the text.

Each chapter concludes with a set of useful exercises. The production of the book lives up to the uncommonly high standards that one has come to expect from its publisher.

E. Sternberg

Theory of ground water movement. By P. Ya. Polubarinova-Kochina. Translated from the Russian by J. M. Roger De Wiest. Princeton University Press, New Jersey, 1962. xix + 613 pp. $10.00.

This book, according to its author's statement in the preface, is based upon lectures which she gave at the State University of Moscow to students of hydromechanics, and it deals principally with problems of engineering interest such as the flow of ground water associated with hydraulic structures, and irrigation.

The literature references comprise 312 citations, of which only 20 are to non-Russian literature, and these mostly to treatises of a general nature. It appears, therefore, that the Russians are about as poorly informed about work in this field that is being done outside the U.S.S.R. as most of the rest of us are of work that has been done by the Russians. It may be, therefore, that one of the most useful aspects of this study is its extensive review of the Russian literature.

The present edition is reproduced from typewritten copy, which, on the whole, is clear but less desirable than reproduction from type. The book is marred by faults of translation into poor or incorrect English, some of which are found on nearly every page.

The theory of ground-water movement, as developed in the book, is principally a kinematical theory in which the important dynamical aspects have largely been suppressed. The principal device used is to relate the fluid-flow vector to a velocity potential which satisfies Laplace's equation for the special case of a fluid of constant density in a space homogeneous with respect to fluid permeability. This renders such problems amenable to treatment by the standard boundary-value procedures, and the principal content of the book is devoted to ponderous solutions of such problems, many of which are trivial.

The theory is not valid for a fluid of variable density or for a space of which the permeability varies with position. Since, in reality, underground fluids do have variable densities and the space also is inhomogeneous with respect to permeability, the theory developed here is not readily applicable to any but the simplest kinds of problems.

If, on the other hand, the behavior of underground fluids is approached as a dynamical rather than a purely kinematical phenomenon, the flow equations obtained are both physically and mathematically closely analogous to those for the steady-state conduction of electricity in three-dimensional space. Both systems have two superposed physical fields, a field of flow and a field of force. In both, the force field is irrotational and has a potential, whereas the flow yields in all but special cases are rotational and not derivable from a velocity potential. The coupling of the flow field to the force field in both cases is by a linear law—Ohm's law in the electrical case; Darcy's law in the fluid case—whereby the flow vector is proportional to the force vector, with the coefficient of proportionality a variable function of position.

Were electrical problems to be treated by the method of this book, one would discard the present electrical potential function, which is the potential of the force field, and substitute an electrical "velocity potential" related to the current field. While many problems of current flow could be solved by this method, as the present author demonstrates, many others would be almost intractable, and the treatment would be almost wholly lacking in physical insight.

On the other hand, when the flow of underground fluids is approached from the same dynamical viewpoint that has been fruitfully used in electricity, the same kind of simplification results. The theory is applicable to water, oil, or gas in inhomogeneous space, and the physical insight afforded is such that approximate solutions to most problems can be obtained almost by inspection.

In his address of acceptance of the Timoshenko Medal from the American Society of Mechanical Engineers in New York on November 27, 1962, M. A. Biot, commenting upon the importance of physical insight, remarked:

"Furthermore, deeper physical insight combined with theoretical simplicity provides the short-cuts leading to the core of extremely complex problems and to straight-forward solutions. This cannot be achieved by methods which are sophisticated and ponderous even in simple cases."

This passage aptly describes the book under review. It is largely devoid of physical insight and is pre-eminently characterized "by methods which are sophisticated and ponderous even in simple cases."

M. King Hubbert

John von Neumann—Collected Works. Edited by A. H. Taub. Volume 5: Design of computers—Theory of automata—Numerical analysis. Pergamon Press Ltd., Oxford, 1963 (distributed by the Macmillan Co., New York). ix + 784 pp. $14.00.

In addition to some not previously published papers of John von Neumann, this volume contains many papers that were originally submitted as reports to the U. S. Army Ordnance Department, the Navy Bureau of Ordnance, or the Los Alamos Scientific Laboratory and reprintings of contributions to periodicals and other publications. The first category consists of the following papers: "On the principles of large scale computing machines" (in collaboration with H. H. Goldstine), "On the numerical calculation of flow problems" (1st and 2nd Report), "Non-linear capacitance or induction switching, amplifying and memory devices," and a brief review (by J. Bardeen) of notes on the photon-disequilibrium-amplification scheme, which contained the basic principle of the MASER. The second category contains reports on logical design of an electronic computing instrument (in collaboration with A. W. Burks and H. H. Goldstine), planning and coding of problems for an electronic computing instrument (in collaboration with H. H. Goldstine), solution of linear systems of high order (in collaboration with V. Bargmann and D. Montgomery), numerical solution of partial differential equations of parabolic type (in collaboration with R. D. Richmyer), and statistical methods in neutron diffusion (in collaboration with R. D. Richmyer). Many of these reports, which have profoundly influenced the development of automatic computation, had become collectors' items; it is most welcome that they are now generally available. The third category includes, among others, papers on numerical inversion of high-order matrices (in collaboration with H. H. Goldstine), the Jacobi method for real symmetric matrices (in collaboration with H. H. Goldstine and F. J. Murray), and various techniques used in connection with random digits. A bibliography and a table of contents of the six volumes of these collected works concludes the present volume.

W. Prager

The foundations of statistical inference. By L. J. Savage and other contributors. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1962. 112 pp. $3.25.

This monograph arose from a two-day seminar held in London in July, 1959. Part I is a somewhat expanded form of the opening lecture given by L. J. Savage. Part II consists of previously prepared contributions by M. S. Bartlett, G. A. Barnard, D. R. Cox, E. S. Pearson, and C. A. B. Smith. An unprepared discussion among the foregoing and others, slightly edited, comprises Part III. Little of the monograph is highly technical, but neither is it expository. It is a discussion among professionals about the basic philosophy of their field. It is certainly the best such discussion available, providing interesting and unusually cogent presentations of diverse points of view. Byways are sometimes explored, but seldom mistaken for main roads, and the essential issues are faced more clearly than in most such discussions. Anyone interested in statistics can read the book with profit. Those already concerned about the foundations of inference will understand it more fully, but those not yet concerned should find it more eye-opening.

Savage sets the stage deftly and with his own personal flavor. Some of his topics are the problem of vagueness in subjective probabilities; the ununified, opportunistic structure of non-Bayesian theories; the likelihood principle, optional stopping, conditional inference, and precision obtained vs precision anticipated; the automatic way these matters are handled by Bayesian theory; and "precise measurement," more recently called "stable estimation," which deals quantitatively with the extent to which evidence brings diverse opinions together. The ensuing discussion centers on some of these and some other topics. It would be useless to attempt to summarize it here, to mention the many interesting points made, or even to reflect the discussants' many points of view. Instead I shall outline from my own point of view what emerge as the principal issues, except fiducial methods which are not much discussed in the monograph and are not always applicable.

Everyone wishes there were terminology distinguishing various kinds of probability and satisfactory to all schools, though it is debatable who should have the right to use the word "probability" by itself. Communication at this meeting was good, however, and the basic disagreement was not and is not terminological.

The choice of an action or a sample size can be viewed in both Bayesian and orthodox theories as a choice among operating characteristics. The Bayesian method of choice depends on weighted averages of operating characteristics, where the weights reflect costs and judgmental prior probabilities. Orthodox theory does not specify a method of choice and to this extent is compatible with Bayesian theory. Little was said at this meeting about choosing an action or sample size, but it cannot be concluded there is agreement in this area. For one thing orthodox statisticians could hardly maintain their views on inference and still make Bayesian decisions. For another, Bayesian theory permits by-passing computation of the operating characteristic, which orthodox statisticians object to doing. And in connection with a two-action problem introduced by Savage here, E. S. Pearson has argued elsewhere (Ann. Math. Stat., Vol. 33, pp. 394–403) in favor of determining the sample size and critical value together (not the latter alone) by specifying the operating characteristic at two points, keeping costs and judgments roughly in mind, rather than by settling on some quantitative utilities and prior probabilities and optimizing with respect thereto. In case of serious discrepancy, which method should yield? (For me, the latter is more fundamental, and if I had chosen values of the operating characteristic at two points which turned out seriously discrepant with any utilities and prior probabilities I would consider using, I would revise my choice of operating characteristic.)

The discussion in this book pertains mainly to the inference problem: what light do the data shed on matters of interest? Everyone wants inference statements to be reasonably simple and to be interpretable as inferences. "Objectivity" is also generally desired. Part of objectivity is honesty, and both orthodox and Bayesian inference statements are honest provided the orthodox statement specifies the procedure used and the Bayesian statement specifies the prior used. Part of objectivity is unanimity: different people analyzing the same data should make the same report insofar as it concerns inference from the data alone. Orthodox and Bayesian methods are essentially equally arbitrary in theory, each permitting roughly any admissible procedure. In practice, orthodox statisticians have achieved unanimity in many situations by adding convenient or pleasant but by no means necessary requirements and by gradual formation of conventions. Bayesian statisticians could do the same, and probably will in due course.

Orthodox statisticians feel that meaningful inferences can and should be expressed solely in terms of limiting-relative-frequency probabilities, which alone are "objective" in the sense of empirical verifiability. This immediately gives rise to the references sequence problem: a given orthodox inference statement belongs to many infinite sequences and the limiting relative frequencies differ from sequence to sequence and may depend on the state of nature. It is natural to consider a sequence of occasions no which the same data occur, but for such a sequence the limiting relative frequency may be anything between 0 and 1 or even fail to exist, depending on the state of nature on each occasion. Accordingly, orthodox statisticians must use reference sequences obliquely related to the natural one, and the probabilities they quote pertain directly to what particular states of nature would say about various possible data rather than what the given data say about various possible states of nature. Furthermore, several sequences with different limiting relative frequencies may seem relevant in the same problem. This gives rise to a number of unpleasant symptoms of orthodox methods, several of which are mentioned by Savage.

Bayesians feel that what the data tell is how to transform prior distributions into posterior distributions. Equivalently, the data provide the likelihood function, which is, of course, interpretable as a limiting relative frequency, but is not in itself an inference statement. A convenient way of reporting the likelihood function and simultaneously making an inference statement is to give the posterior probability distribution that the data would yield in conjunction with some specified prior. From this, the posterior distribution corresponding to any other prior can be obtained. Since posterior distributions depend on prior distributions, Bayesians feel the data alone cannot provide a meaningful inference, nor are limiting relative frequencies sufficient in themselves, whatever special status they may have. (Orthodox inference statements mean something to me *as inferences* only if I misinterpret them as posterior probability statements corresponding to who knows what prior, if any, and I believe they are widely so misinterpreted. This alone is enough to make me prefer the Bayesian method of reporting an avowedly posterior distribution arising from a stated prior.)

These remarks should indicate the main issues discussed in the monograph, but let me repeat that they are written entirely from my own point of view. The monograph itself is an excellent place to find other points of view.

John W. Pratt

Nonlinear theory of continuous media. By A. Cemal Eringen. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1962. xii + 477 pp. $14.50.

A good deal of research has been done within the last fifteen years on the mechanics of materials which satisfy non-linear constitutive equations. The various theories share a common foundation in the analysis of stress and deformation, and in certain basic rules governing the forms of constitutive equations. The present book attempts to explain this common foundation, and to give a connected account of some of the recent work in finite elasticity, non-Newtonian flow, hypoelasticity, plasticity, viscoelasticity, and electroelasticity. There is also an apparent effort to simplify and interpret the research work, in order to bring to toward an engineering level of application.

Since the purpose of the book is highly ambitious and its scope is very wide, it is not remarkable that its aim is only partially achieved. One might complain that in dealing with particular theories, the book often degenerates into uncritical paraphrases of the original papers. However, it is useful to have these papers organized under one cover. A more serious complaint is that in trying to simplify the results of research, the author quite regularly commits fundamental errors in logic and interpretation.

The errors in classical areas are easy to spot and thus perhaps unimportant. For example, Theorem 1 on p. 69 states that "The streamlines and the path lines coincide if *and only if* the motion is steady" (Italics mine). On pp. 141, 159, and 210 there are statements to the effect that the number of elastic constants for a general anisotropic linear elastic material is 21 rather than 36 if and only if the strain energy is positive definite; this is tantamount to asserting that every quadratic form is positive definite.

Mistakes in the newer areas are likely to cause more difficulty. Thus, the claims on pp. 177 and 283, that finite simple extension cannot be supported by tensile forces alone, might lead the reader to conclude that finite elasticity theory is nonsense. However, the mistakes are more often in derivations than in final results. The "quick derivation" on p. 255, for example, would be correct only if matrix multiplication were commutative, although the stated result is correct. A theorem on p. 261 is proved by using the simplifying assumption that displacement gradients (i.e. rotations as well as strains) can be expressed uniquely in terms of stress components.

None of the numerous blunders such as these are serious enough to affect greatly the purpose of the book, but they seem to be symptoms of a larger confusion. One of the fundamental principles common to all the theories is what Noll calls "the principle of objectivity." This idea is discussed extensively and used repeatedly, as it should be. However, it is used incorrectly so often that it is made to appear to be a vague and probably false principle. For example, it is used to obtain equation (46.8), which violates objectivity, and the correct result (46.12) is then arrived at by a spurious argument. Possibly the most unfortunate misuse of the principle occurs in the chapter on viscoelasticity, where the constitutive equations (94.30) and (94.31) are derived and attributed to Green and Rivlin and to Noll, respectively.

Both of these equations violate objectivity, and the cited authors have proposed no such impossible relations.

Similar confusion reigns in matters pertaining to material symmetry, which is also of basic importance. For example, when the correct result (46.12) is to be specialized to the case of isotropic materials, the false relations (46.24) and (46.28) are used to obtain the true but incomplete relation (46.31); further argument then leads to the final result (46.32), which is absurd.

Although the book is not what it purports to be, it has its uses nevertheless. The bibliography is particularly valuable, running to some 350 entries, about half of which are references to papers published since 1952. From the book, the reader can quickly obtain a rough idea of any of the subjects covered, and can then turn to the cited references for a better understanding.

A. C. Pipkin

Statistical mechanics. By Kerson Huang. John Wiley & Sons, Inc., New York, 1963. xiii + 470 pp. $10.75.

Kerson Huang's book STATISTICAL MECHANICS for graduate students in physics is divided into three parts. The first is a review of thermodynamics and an introduction to the kinetic theory of gases. The second is a presentation of the essential theory of statistical mechanics; and the third is a discussion of some special topics in statistical mechanics currently of interest to physicists, the author being among them.

In the section on the basic theory Huang treats Liouville's theorem, the canonical and grand canonical ensemble, quantum statistical mechanics, and the evaluation of partition functions. As examples there are given discussions of the ideal Bose-Einstein and Fermi-Dirac gas, imperfect gases by the method of pseudo potentials, and the modern theory of phase transition; the presentation is clear and good. The authors aim of conveying essential understanding rather than ultimate rigor is justified and fulfilled. The emphasis is not on the foundations of the theory.

The special topics considered are: the Ising model and order-disorder phenomena with simple solutions and Onsager's solution of the two dimensional case; liquid helium, including super-fluidity; and the hard sphere Bose gas. It is good to see the last topic brought so soon from the research literature to a testbook even though much more work remains to be done. This up-to-date book on statistical mechanics will be of very good use to graduate students in physics.

In the first section, Huang presents a brief review of thermodynamics and kinetic theory which I did not enjoy. The ideal gas is defined only by use of an equation of state. "Since temperature and volume may be taken to be the independent parameters, and since U is a state function, we conclude that for an ideal gas U is a function of the temperature alone. This conclusion can also be reached theoretically, without reference to a specific experiment, with the help of the second law of thermodynamics." But then it is circuitous reasoning to prove that the ideal gas temperature scale equals the absolute thermodynamics scale as Huang attempts to do on page 15. The ideal gas scale is not defined according to the recommendation of the international commission. The discussion on osmotic pressure is very misleading. Huang states "this indicates that the sugar solution has a pressure pgh higher than that of pure water at the same temperature. This pressure must be due to the presence of sugar, and is called the *osmotic pressure* exerted by the sugar in solution." This of course is just not so.

In the derivation of the Boltzmann equation, Huang states that the time differential dt may be considered a truly infinitesimal quantity. I believe this not to be correct since time intervals considered by the Boltzmann equation must be larger than times necessary for completed collisions to occur.

John Ross

Introductory statistical mechanics for physicists. By D. K. C. MacDonald. John Wiley & Sons, Inc., New York, London, 1963. ix + 176 pp. $6.75.

MacDonald's book is intended for undergraduate physicists and, as cited by the author, designed as a counterpart to Rushbrooke's "Introduction To Statistical Mechanics" for physical chemists. The presentation is short and to the point; students will find it a good book for a study of statistical mechanics.

After a short introduction of the purpose of statistical mechanics and its relation to thermodynamics and mechanics the expression for the entropy in a microcanomical ensemble is stated, made reasonable, and applied to a simple model. Then partition functions are introduced and used for the discussion of ideal crystals and gases. Elementary discussions of the resistibility of metals, liquid helium, and electromagnetic radiation are also presented. Brief mention is made of the problem of irreversibility but unfortunately the author chose to discuss the ergodic problem at the same time, with some resulting confusion, I believe.

John Ross

Matrix algebra for social scientists. By Paul Horst. Holt, Rinehart and Winston, Inc., New York, Chicago, San Francisco, Toronto, London, 1963. xxi + 517 pp. $10.00.

Social scientists make considerable use of matrix calculations on electronic digital computers, frequently with very little understanding of the mathematics employed. A book which sets out, as does this one, to teach the concepts and rules of matrix manipulation to "behavioural scientists with no more than a reasonably good working knowledge of high school freshman algebra . . . and an introductory course in statistics" is therefore very welcome. The treatment is very elementary, very detailed, and all concepts introduced are expounded and driven home with numerous examples. Yet some concepts (e. g. generalised inverse, rank reduction) are introduced that would be considered advanced in a customary course in "linear algebra." There is a summary and a set of exercises for the reader at the end of each chapter, making the book highly suitable for self-study. It can be enthusiastically recommended. It is, however, unfortunate, that the author has chosen not to explain how matrices are used to solve the problems of multiple regression, factor analysis, and the analysis of variance (though he mentions these topics in the preface, they are not to be found in the text), but goes no further than the calculation of a correlation matrix, and the least squares solution of over-determined linear equations.

B. A. Chartres

Multistage inventory models and techniques. Edited by H. E. Scarf, Dorothy M. Gilford, and M. W. Shelly. Stanford University Press, Stanford, Cal., 1963. vii + 225 pp. $7.50.

This is the first volume in a series of research monographs on mathematical methods in logistics sponsored by the Office of Naval Research. The way in which the manuscripts are obtained departs sufficiently from normal publishing practice to warrant reproduction of the following passage from the Preface. "In cooperation with an advisory committee of the Institute of Management Sciences, the Office of Naval Research announces a given topic and invites contributed papers. Manuscripts are judged by a committee appointed by the Institute of Management Sciences; the chairman of the committee is the senior editor of the current monograph. The author of the paper selected as the best receives an Office of Naval Research contract which will permit him to devote full time for one year to the research he has chosen. Several other contributed papers judged to be outstanding are selected for publication, and the members of the judging committee are also invited to submit papers".

The present volume contains the following contributions: "Dynamic programming and stationary analysis of inventory problems" by D. L. Iglehart; "A delivery-lag inventory model with emergency" by K. H. Daniel; "Centralized inventory control in multi-location supply systems" by D. Gross; "Optimal stockage policies with non-stationary stochastic demands" by A. F. Veinott, Jr.; "An inventory-transportation model with N locations" by G. Hadley and T. M. Whitin; "System analysis of linear models" by R. A. Howard; "A survey of analytic techniques in inventory theory" by H. E. Scarf.

W. Prager

Methods in analysis. By Jack Indritz. The Macmillan Co., New York, 1963. viii + 481 pp. $12.95.

This book begins with a study of linear transformations on a finite dimensional vector space. Next, the author deals with integral equations, orthonormal sequences, and Sturm-Liouville Theory. The book closes with an introduction to Fourier analysis on the real line. The thread, which unifies the entire treatise, is eigenvalue theory. This excellent book would make a welcome addition to any applied mathematician's bookshelf.

MORTON E. GURTIN

Distributions: an outline. By Jean-Paul Marchand. North-Holland Publishing Co., Amsterdam, and John Wiley & Sons, Inc. (Interscience Div.), New York, 1962. ix + 90 pp. $4.75.

This is a short book, the aim of which is "to expound in a simple but nevertheless coherent way the basis of" distribution theory. The book (which is divided into two parts: Schwartz's functions theory of distributions and Mikusinski's algebraic theory of distributions) definitely fills an open void in the existing mathematical literature. However, I feel that the author renders many otherwise simple ideas complicated through the use of cumbersome notation. Moreover, the author's cribbed style of writing makes the book very tiring to read.

M. E. GURTIN

Statistical theory of reliability. Edited by Marvin Zelen. The University of Wisconsin Press, Madison, 1963. xvii + 166 pp. $5.00.

This book reports the proceedings of an Advanced Seminar conducted by the Mathematics Research Center of the U. S. Army at the University of Wisconsin in May 1962. It contains papers by George H. Weiss, Frank Proschan, Richard E. Barlow, Larry C. Hunter, Joan R. Rosenblatt and William Wolman, together with discussion of these papers by invited discussants. The aim was for the papers to be expository in nature, and although the authors state that their articles were written stressing their own personal interests and prejudices, the book does provide a good survey of and introduction to recent work in this subject. The style of writing of the papers is easy to read and an adequate number of references is provided. George Weiss's article is entitled "A survey of some mathematical models in the theory of reliability." It is a very extensive paper—51 pages—most of which is devoted to renewal theory and allied topics, maintenance policies, etc. Topological reliability is also introduced. Frank Proschan's article (16 pages) discusses "Redundancy for reliability improvement." One example he considers is the design of systems such as systems of relays, arranged in subsystems in parallel, the subsystems to contain redundant elements, and be designed to maximise 'reliability' (suitably defined), or expected system life. Richard Barlow's paper (14) pages is entitled "Maintenance and replacement policies." His main theme is periodic replacement, and the determination of optimum replacement intervals, when the costs of replacement and of in-service failures are given. Larry Hunter discusses "Optimum checking procedures," (13 pages). The checking problem arises when failures are detected by checking, and the costs of undetected failures and of checking (with possibly other factors) enter into the model. Some theorems are presented on optimum checking procedures. Mrs. Rosenblatt presents a paper entitled "Confidence limits for the reliability of complex systems," (20 pages, plus 3 pages of references). It is assumed that the performance of a system can be described by a scalar quantity x_0, which depends on independent random variables $x_1, \cdots, x_n$ through a function $x_0 = f(x_1, \cdots, x_n)$. Reliability is defined as $R = Pr\{f(x_1, \cdots, x_n) \geq c\}$, and Mrs. Rosenblatt discusses point and interval estimation of R. Finally, William Wolman presents a paper "Problems in systems reliability analysis," (12 pages). He considers a model for reliability growth (for example, in test flights of a missile), and a problem involving the relationship between different modes of failure and complete failure of the system.

The reviewer's main criticisms of the book are that the examples are frequently rather artificial and the models unrealistic. There is little discussion relevant to problems of analysis and inference from observed results.

In the preface it is stated that the book can be used as a source of problems for further work. This claim is well justified but it is no doubt true that a detailed study of some actual case histories may yield more useful, if more complex, problems.

G. B. Wetherill

Quantum mechanics. Volume I and II. By Albert Messiah. North-Holland Publishing Co., Amsterdam, and John Wiley & Sons, Inc., New York, 1963. xv + 504 pp. $8.00 (Volume I), xv + 632 pp. $8.50 (Volume II).

Volume I is concerned with the formalism of quantum mechanics and its interpretation as well as simple systems such as scattering problems, Coulomb interaction, and the harmonic oscillator. Volume II is concerned with symmetries and invariance and with approximation methods of calculation well as with parts of relativistic quantum mechanics—the Dirac equations and quantization of the electromagnetic field.

Various Appendices deal with special functions, distribution theory and the continuous spectrum, rotation matrices and elements of group theory. The work seems to be clear and easy to read. It does not seem to have suffered through translation and students of quantum mechanics should find it valuable.

Rohn Truell

Principles of coding, filtering and information theory. By Leonard S. Schwartz. Spartan Books, Inc., Baltimore, 1963. xiii + 255 pp.

This book is aimed at the beginning engineering student and the practicing engineer and attempts a presentation which is sufficiently mathematical to be convincing and which, at the same time, can be accepted intuitively.

In the first chapter, entropy and noiseless coding are discussed, and the next three chapters treat discrete noisy channels, the fundamental theorem of information theory, and various coding techniques. This is followed by chapters on continuous information theory and generalized harmonic analysis and applications to linear filtering. The last two chapters discuss signal detection and communication feedback systems, respectively.

While the omission of the mathematical aspects of the subject makes the book quite readable, it sharply detracts from its substance. It might have been appropriate, for example, to have included in an appendix proofs of the noiseless coding theorem and the fundamental theorem. Further, the book contains a number of significant errors. In particular, the equivalence of the Shannon-Fano and Huffman codes is erroneously asserted (p. 16), and in the discussion of the sampling theorem the false statement is made that a band limited signal may be specified by a finite set of numbers.

The book, in the opinion of this reviewer, is suitable for an undergraduate survey course in information theory (if due caution is given to the errors) but is, of course, inadequate for the needs of those intending to pursue research in this area.

Aaron D. Wyner

Mechanik II—Dynamik der Starren Körper und Systeme. By Hans Ziegler. Birkhauser Verlag, Basel and Stuttgart, 1962. 213 pp. $7.65.

This clearly written revision of the undergraduate text by Meissner and Ziegler distinguishes carefully between applied and internal forces, loads and reactions, conservative and non-conservative, gyroscopic and nongyroscopic forces. Kinematics comes first followed by kinetics of mass points, rigid bodies, and systems through Lagrange's equations.

D. C. Drucker