# QUARTERLY

## OF

# APPLIED MATHEMATICS



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# BOOK REVIEWS

*Guide to tables in mathematical statistics.* By J. Arthur Greenwood and H. O. Hartley. Princeton University Press, New Jersey, 1962.lxii + 1014 pp. $8.50.

Statisticians and users of statistical methods owe a great debt to all who contributed to the creation of this valuable *Guide.* The project itself has been in the making for over twenty years; H. O. Hartley has been associated with it since 1949, and J. A. Greenwood since 1959. Many committees of the National Academy of Sciences—National Research Council have sponsored the work, the latest one being the Committee on Statistics of the Division of Mathematics, S. S. Wilks, Chairman. Beyond the starting funds given by the Rockefeller Foundation, financial support for the effort was provided by the Office of Naval Research and the U. S. Army Research Office. Some work was supported by the National Bureau of Standards and some by the Bell Telephone Laboratories, Inc. One cannot call the roll indefinitely, but when a gigantic contribution appears, some credits are due.

Mathematical tables of interest to statisticians are described and placed in the context of statistical problems to which they are germane. An abridged list of chapter headings (from page xxxxv) gives some idea of the scope:

1. The normal distribution—2. The chi-squared and Poisson distributions—3. The beta and binomial distributions—4. The t-, F- and z-distributions—5. Various discrete distributions—6.Likelihood-test statistics—7. Correlation, mostly product-moment—8. Rank correlation; asympototic theory of extreme values—9. Non-parametric tests—10. Frequency curves; symmetric functions—11. Regression and other curves—12. Variate transformations—13. Random numbers—14. Quality control—15. Design of experiments, etc—16. Sundry mathematical tables.

In introductions to chapters and sections the authors discuss briefly the specific statistical problems solved by tables, and they relate one table to another, as well as one problem to another. I take it that in the main body of the text the reviews and discussions are original with the authors, except possibly for some borrowing from Fletcher, Miller and Rosenhead, *An Index of Mathematical Tables* (London and New York, 1946), but in Appendix 1, which gives reviews added after the main body of the text was closed, a number of reviews from *Mathematical Tables and Other Aids to Computation* are reproduced. In quantity, the whole of the reviews of tables run to 738 pages. Beyond these, Appendix 2, about 45 pages long, gives the contents of important collections of tables.

The authors say, in part, of the scope: ". . . up to 1948 we made a comprehensive search; from 1949 to 1954 we think we have covered the leading American and English statistical journals . . .". The tables described thereafter were known personally to the authors, recommended by colleagues, or found in collections and lists. In the first 15 of the 162 pages of the Author Index, I found references to 20 documents from 1955 through 1957 and to 9 from 1958 to 1961, so there is no sharp cut-off.

Hereafter I shall often mention references in exemplifying my investigation of the accuracy and adequacy of the *Guide.* The references are merely labels to assist in describing the *Guide,* so there is no need here for their full citation.

To find a description in the *Guide,* one can use the Table of Contents (32 pages), the Author Index (162 pages), and the Subject Index (62 pages). To try it out, I looked for a table of variances of sample medians from non-normal populations and found one by Rider, for the discrete uniform distribution, in about three minutes. It was listed under "median", but I started with "variance of median" and was led to normal distributions first.

As my next test search, I recalled a table I had used, of which Cureton was an author. The Author Index sent me to Gordon, Loveland and Cureton, where I found the full reference with authors' initials, and was directed to page 143. By a coincidence, that page of the *Guide* is unnumbered, but none on either side are. Percentage points of chi-square with 2 degrees of freedom were tabled to four decimals, with arguments .001(.001).999. While there, I was informed of three other tables of percentage points for chi-square with special numbers of degrees of freedom.

Since there is a chapter on frequency curves, I asked myself: "What is a Type VIII Pearson curve?"; the Table of Contents led me to page 401, where I found out. Page 402 told me that the rectangular distribution was a special case of Type VIII.

For a book with 103 pages, the *Guide* opens and handles easily. The text has for the most part been typed. Several typewriters have been used, type sizes change, and the degree of blackness also changes.

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When reviews are reproduced from printed text, the type size is sometimes startlingly large; for example, see pp. 666-667. My view is that this heterogeneity is acceptable in a book of this kind, provided that it is made palatable by the price. When one considers the limited sale this specialized book will have, I feel that the price is reasonable in 1962.

To check a bit on the references themselves, as a statistician, I am inclined to consider first examining a random sample. But I decided that checking up on material I did not know well and that I had low motivation for studying would lead either to low quality assessment or to rather time-consuming research. Therefore, I abandoned the random sample in favor of looking at references to tables that I have shared in bringing to print, in the belief that knowledge plus personal interest would excite loving care. An unfortunate feature of this approach is that a reported error sounds like a reply to a personal attack, and nothing is further from my intentions.

As a result, I found a variety of problems faced by the authors. Sometimes others computed the tables I published, and when the computer is named in the original publication, the *Guide* also names the computer. But one table was computed by transforming values computed for us by the Harvard Compution Laboratory. The *Guide* attributes these tables to Bush and Mosteller, the authors of the publication, and does not give the fuller explanation. The Author Index leads you from my name to the Harvard Binomial Tables, but the *Guide* does not give the connection (I wrote part of the introductory material).

Errors and inconsistences are inevitable in a work of this magnitude. I was unable to discover a rule for including or excluding titles of articles in the body of the text. For my 1941 article the title was omitted in the body (p. 374), but included in the Author Index (with the word "on" omitted, p. 891). My 1946 paper had the full title in two different places in the text as well as in the Author Index (the final page number is given, incorrectly, as 406 in both text references, and correctly as 408 in the Index). In the *Guide's* description of the 1948 article, an ambiguity could be abated by replacing the word "population" by "sample." The description of lot sizes for the book *Sampling Inspection* omits "550,000 and over."

In Appendix 1 I find page references that have not been filled in, but there should be enough redundancy in the way of names and section numbers that these cross-references can be ferreted out.

I hope I have given some notion of the problems faced by the authors, and that I have made clear that the sorts of errors I found should not much impair the value of the references.

A truce to details. The great thing is that this fine reference work is at last available to us all. How the authors ever finished such a heroic work and left it so nearly up-to-date is a mystery to me. The material is broad in scope, well organized, easy to use, and thoughtfully assembled. Let us hope this effort will be kept fresh with periodic supplements.

FREDERICK MOSTELLER

*Science and information theory.* By Leon Brillouin. Second Edition. Academic Press, Inc., New York, 1962. xvii + 351 pp. $9.00.

This is a stimulating book, packed full of information (in the usual sense), one that will be of value both to the practicing scientist and to all of those interested in the philosophy of science. Perhaps one of the great values of the book is that it shows so clearly how essential it is to the engineer, physicist, and mathematician to understand very precisely what it is that he is doing as contrasted to what it is that he might want to do. It is rare that a man as talented and experienced in so many fields takes the time to write so carefully and lucidly. Consequently, this book which is devoted to the interpretation and use of data in all parts of modern science will be a strong influence in training a new generation. It is also recommended to all of those who are interested in such varied topics as the analysis of signals, thermodynamics, thermal noise in electric circuits, quantum mechanics, computing machines, and in the overall questions of problem formulation and solution.

It is to be expected that a reviewer can readily choose a number of ideas with which to take issue in such a comprehensive volume in which the author is so engagingly candid. Let us then begin with the title. It is soon seen that Brillouin himself is a bit unhappy about the term "information theory" for the present small collection of specialized results concerning particular types of transmissions of 0's and 1's.

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In the introduction, pp. x-xi, he voices some qualms about the absence of utility functions in the formulation of so-called "information theory;" again on pp. 9-10 and on pp. 268-9 there are caveats; finally, on p. 294 he ends his misgivings with a warning about the frequent misuse of the results of Szilard, Shannon and others. It is obviously inevitable that a term such as *information* which has such a broad semantic base and currently such a narrow mathematical definition will lead to misconceptions.

Brillouin seems to be under the impression that there is difficulty in attaching utilities to the transmission of data. Yet this is standard technique in mathematical economics and in modern decision theory. Indeed, J. Marschak has used this idea often, and it is the basis of the mathematical theory of organization due to himself and Radner. Furthermore, Kelley made some tentative attempts to do this for Shannon's concept of capacity (essentially the smile of the Cheshire cat); his results were completed and generalized by Kalaba and the reviewer; see R. Bellman and R. Kalaba, "on the Role of Dynamic Programming in Statistical Communication Theory," *IRE Trans. of Professional Group on Information Theory*, Vol. IT-3, 1957, pp. 197-203. The work of Wald, Blackwell, Girshick, S. Sherman and others provides still other techniques.

Secondly, it seems that it should be pointed out that such estimates as Heisenberg's "uncertainty principle," a tenet which is as determinedly held by many as any belief in an orthodox creed, should be viewed in perspective, merely as consequences of imperfect theories. It is quite conceivable that more sophisticated and knowledgeable theories of the universe will provide far more refined estimates. It is in keeping with the author's thesis that uncertainty in basic knowledge must reflect itself in uncertainty as to conclusions.

Finally, let us add although such topics as error-correcting codes are discussed, there is no treatment of sequential analysis and adaptive processes, modern mathematical techniques specifically designed to overcome various types of uncertainty. Once again, it must be stressed that the classical estimates in engineering and physics are based upon the most naive and primitive probabilistic techniques, and upon simple modess of the physical world.

The reader of this most elegant presentation of scientific theories will doubtless find other points to disagree with, and far more, of course, with which to agree. The point is that the reading of this book provokes thought and discussion. What better tribute to an author? For many hours of edifying and entertaining reading, we call this volume to your attention.

RICHARD BELLMAN

*Introduction to hypersonic flow.* By G. G. Chernyi. Translated and Edited by Ronald F. Probstein. Academic Press, New York and London, 1961. xiv + 262 pp. $8.00.

The present book provides an introduction to the theory of inviscid hypersonic flows and some of the useful methods for determining the flow-field properties. The main text is based on material from a university course designed for undergraduate and first-year graduate students in the U. S. S. R., and is augmented by comparatively recent contributions in this field (prior to 1958), mostly by the author himself. In spite of omissions in certain important areas, the book should be highly regarded for its relative simplicity, and for the discussions of a number of comparatively new results (which may not be found in Western books on the subject published to date). The many footnotes as well as an extensive reference list, brought up to date, meticulously prepared and added in the English text by the editor, should also prove to be valuable.

The book begins with a survey on the development of supersonic-flow theory, and an excellent introduction to the basic aerodynamic problems of hypersonic flight, including certain information on high-temperature gases as well as experimental methods. General knowledge on hypersonic flows over blunt and slender bodies are presented in the first two chapters, with a rather thorough discussion on the hypersonic equivalence principle. The various well-known methods for predicting surface pressure and other flow-field properties (Newton's formula, tangent-wedge and tangent-cone methods, the Busemann formula, etc.) are discussed in Chapter III. This is followed by a rather lengthy but thorough exposition of the shock-expansion method, which takes up Chapter IV entirely. The last chapter deals

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with the leading-edge bluntness effect to which the author has contributed much of his own research.

As a text book of Hypersonic Flow Theory, the book is remarkable for its simplicity and clarity which are retained throughout the present English translation. Although it was written before 1958, there is presented a number of useful applications which are not to be found or have not been adequately discussed in some of the more recent books. The account on the thin-shock-layer theory under the title Boundary-Layer Method in Chapter III is short but very informative. The reviewer is pleased to note that the assumptions on the shock strength used in the boundary-layer method and in the hypersonic small-disturbance theory of Chapter II are less restrictive than those given in most existing text books. The treatment of the leading-edge bluntness effect by the entropy-layer concept is perhaps the most outstanding feature of the book. The authorative account provided therein should be valuable to those who do not have ready access to the Russian literature.

As pointed out by the author, most parts of Chapters III to V are those theoretical developments in which the author took part in one way or another. For this reason, there are the following important areas which are not covered by the present book: (1) Detached shock wave, (2) Viscous and other transport effects, (3) High-temperature flow chemistry. As a whole the book seems to dwell more on methods, rather than ideas and concepts from which these various methods are developed. This is reflected in the subtitles listed in the Table of Contents. While the treatment of the bluntness effect is novel, the degree of approximation and the domain of its applicability require more attention than has been denoted in the book. Though masterfully written, and carefully edited, the book is not without minor defects. For instance, on page 98 there is the statement related to the Newtonian resistance law "the particles continue to move by inertia along the surface .... and along the geodesic lines." The generality of this statement is questionable. For, in Newton's model each particle collides with the body surface only once. To move along a surface geodesic would call for a constraining force on the particle, which is, however, inconceivable in Newton's flow.

The author and the translator have undoubtedly contributed an important source of information for hypersonic aerodynamics. The book should be particularly useful in a short course on hypersonic flow theory.

H. K. CHENG

*Introduction to thermodynamics of irreversible processes.* By I. Prigogine. Interscience Publishers (A Div. of John Wiley & Sons) New York, London, 1962. xi + 119 pp. $5.00.

The first edition of this Introduction appeared in 1955. The only modifications introduced in this second edition are the addition of a section entitled "Continuous and Discontinuous Formalism" to Chapter V, and the replacement of the Appendix of the first edition by Chapter VII entitled "Nonlinear Problems."

*Discrete variable methods in ordinary differential equations.* By Peter Henrici. John Wiley & Sons, Inc., New York, London, 1962. xi + 407 pp. $11.50.

This book concentrates on single and multiple step methods for the numerical solution of initial value problems in ordinary differential equations. There is also a brief chapter on boundary value problems. The work is unique in two ways: firstly, it is completely rigorous; secondly, it contains a thorough theoretical treatment, supported by much experimental evidence, of round-off error propagation—largely the author's own work and appearing here for the first time.

The first chapter presents the basic existence and uniqueness theorem for a first order initial value problem and Euler's method, the second the Runge-Kutta class of methods, the third deals with systems of first order equations and the fourth with those of higher order—all four chapters being concerned with single step methods. Chapters five and six treat multiple step methods for equations of first and certain

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of second order, respectively, partially influenced by the work of Dahlquist. Chapter seven presents direct methods for monotonic boundary value problems for equations of the form $y'' = f(x, y)$. The case of $f(x, y)$ linear in $y$ is first disposed of, and for the non-linear case, the author adapts Kantorovich's treatment of Newton's method for his needs. In each chapter, questions of truncation and round-off error, amongst others, are thoroughly explored and appropriate problems for the reader appended.

There is little explicit discussion of the relative practical utility of the various methods presented, but the reader can draw his own conclusions on the basis of the detailed error estimates given. The book requires no previous knowledge of numerical analysis and much of the relevant background of differential equation and matrix theory, and statistics, is summarized. The argument is, however, on a level of considerable mathematical sophistication, as required by the high degree of rigour aimed at and achieved. The book deserves the careful study it requires for detailed understanding and sets a standard for future works in the field.

WALTER FREIBERGER

*An index of mathematical tables.* By A. Fletcher, J. C. P. Miller, L. Rosenhead, L. J. Comrie. Volumes I and II. Second edition. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962. xi + 608 pp. and iv + 386 pp. $42.00 per set.

The second edition of Fletcher-Miller-Rosenhead has been eagerly awaited for fifteen years. The authors' names are now joined with that of the late L. J. Comrie in a monumental work of reference, which will be appreciated by all users of mathematical tables.

There are four parts in this greatly expanded two-volume edition: Introduction; I. Index according to functions; II. Bibliography, III. Errors; IV. Index to introduction and Part I. Since the first edition was published, automatic computers have come into widespread use, causing a vast increase in the publication rate of tables. The authors have been able to take account of much of this new material, and were aided in their coverage, particularly of the Russian literature, by having available the index by Lebedev and Fedorova. The work is well complemented by the recent publication of Greenwood and Hartley's guide to tables in mathematical statistics.

The printing and production are excellent and much thought seems to have gone into the problem of making the index easy to use.

WALTER FREIBERGER

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## FIFTH U. S. NATIONAL CONGRESS OF APPLIED MECHANICS

### *PRELIMINARY ANNOUNCEMENT*

The Fifth U. S. National Congress of Applied Mechanics will be held at the University of Minnesota, Minneapolis 14, Minnesota during June 14–17, 1966. It is hoped that the scheduling of conflicting meetings can be avoided by this early announcement of the date chosen for the Congress. Further announcements concerning the preparation of papers will be made as the Congress draws nearer.

Inquiries regarding the Congress should be addressed to one of the undersigned members of the Organizing Committee.

Professor B. J. Lazan, *Chairman*
Professor R. Plunkett, *Secretary*
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Minneapolis 14, Minnesota