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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x/2b)}{\cos (\pi a/2b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2b}}{\cos \frac{\pi a}{2b}}$$

BOOK REVIEWS

The workability of input-output analysis. By Michio Hatanaka. Fachverlag für Wirtschaftsheorie und Ökonometrie, Ludwigshafen am Rhein, 1960. xxiii + 310 pp. $5.00.

Dr. Hatanaka's book describes a series of tests undertaken to determine the usefulness of the input-output model. Both the validity of the assumptions underlying the model and the predictive accuracy (of the future outputs of each industry) of the model in contrast to alternative predictive tools are investigated. The tests are all based on the 1947 BLS input-output table.

The basic assumption of the input-output model is that the input coefficients (describing the interindustry flow of goods) remain constant over time. Since lack of data makes it impossible to observe comparable coefficients at different moments of time, Dr. Hatanaka has been forced to devise some rather clever tests which are based on implications of the constant coefficient hypothesis and utilize available data. The results of these tests are, briefly:

1) Random errors cannot explain the apparent changes in the coefficients.

2) At least in the longer run (7–10 years), there appears to be a definite trend in the movements of the coefficients.

3) About half of the industries tested seem to show "abnormal" changes in the values of the coefficients during World War II, thus suggesting the inadequacy of the model for war mobilization planning.

4) The occurrence of significant substitutions due to price changes (tested for only a few industries) could be neither substantiated nor rejected due to lack of data.

5) About ⅓ of the industries tested appear to have non-linear input functions, a serious matter for a model which hypothesizes linearity.

Predictive accuracy was determined by making predictions from the 1947 table for intervals up to 10 years and comparing these predictions with those produced by several alternative simple models. Four alternatives were used: 1) The output of an industry over time is a fixed fraction of the final demand in that industry. 2) The indirect demand for an industry is a fixed fraction of the total indirect demand for all industries. 3) The output of an industry is determined using a regression of past output on past GNP and time. 4) The output is determined using a regression of output on final demand for that industry and time. In the short run (1–3 years), the input-output model appears superior to alternative models 1, 3, and 4, but the results for these models for the longer run (7–10 years) are less conclusive. Alternative model 2 appears superior to the input-output model both in the short and the long run.

Two minor improvements might be suggested here. First, the alternative prediction models used appear too simple. A more challenging alternative to the input-output model for predicting outputs might be found among various econometric demand functions utilizing relationships suggested by economic theory. A true test of the input-output model demands that the best possible alternative be used for the comparison. Second, the observed differences between the prediction errors of the input-output model and the alternative models was accepted as evidence of the superiority of one or the other. Some of the conclusions were based on quite small differences in accuracy. Some test of the statistical significance of the observed differences would be helpful.

The amount of distortion introduced into the empirical estimates due to errors of observation has not been treated quantitatively since data in this area is completely lacking. Dr. Hatanaka has presented theoretical arguments concluding that such distortion is probably small in certain cases and may be large in others, but the actual amount of distortion due to this cause remains an open question.

The final conclusion from this work is that the present form of the input-output model does not appear to be completely satisfactory. Neither the underlying hypothesis nor the predictive superiority has been clearly sustained. Dr. Hatanaka is quite right in suggesting that many further studies need to be made to determine more precisely the nature of the limitations of the model and the types of modifications which will prove useful. This does not detract in any way, however, from the pathbreaking nature of his present work, both in his single-minded attitude towards determining the usefulness of the model rather than the elegance of the theoretical structure and in the depth and variety of analysis and testing performed under severe data limitations.

Mark B. Schupack

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BOOK REVIEWS

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Solutions numeriques des équations algébriques. By E. Durand. Volume 2. Masson et Cie., Paris, 1961. viii + 445 pp. $18.41.

The first volume, reviewed on p. 110 of volume 19 of this Quarterly, treated the solution of a single equation; the present volume is concerned with systems of equations. Most of the book is concerned with systems of linear equations, matrix inversion, and eigenvalues and eigenvectors of matrices. A first chapter (48 pp.) recalling the notations, definitions, and basic theorems of matrix algebra is followed by two chapters on direct and iterative solution of systems of linear equations (43 pp. each). There follows a chapter on matrix inversion (26 pp.). The fifth chapter (25 pp.) is the only one concerned with systems of non-linear equations. The remaining seven chapters deal with eigenvalues: determination of the coefficients of the characteristic polynomial (45 pp.), reduction to diagonal (27 pp.), triangular (47 pp.), and tri-diagonal and quasi-triangular forms (43 pp.), deflation (41 pp.), repeated multiplication by a vector (25 pp.), and various other methods (19 pp.).

The second volume retains the essentially practical approach of the first. The various methods are amply illustrated by worked-out numerical examples which reveal the power or weakness of the various methods.

W. Prager

The theory of graphs and its applications. By Claude Berge. Translated by Alison Doig. John Wiley & Sons, Inc., New York 1962. x + 247 pp. $6.50.

Though the original version appeared only in 1958, it has already become the standard reference in its field. The publication of an English translation is therefore most welcome. The translation is competently done, some minor deviations from the original having been made in agreement with the author. The only major change this reviewer could detect, is the suppression of Appendices III through V.

W. Prager

Operational calculus and generalized functions. By Arthur Erdelyi. Holt, Rinehart & Winston, New York, 1961. viii + 103 pp. $2.75.

This book is devoted to a rigorous presentation of the elegant theory developed by Mikusinski in which generalized functions are defined as convolution quotients. The last three of seven chapters give applications to the solutions of partial differential equations, and there is a detailed treatment of the one-dimensional diffusion equation.

The theory of convolution quotients is developed by considering the class C of all continuous functions of a nonnegative variable and defining, for $f,\ g \epsilon C,\ f^*g = \int_0^t f(u)g(t-u)\,du,\ t \geq 0$. An elementary proof of Titchmarsh's theorem is given, and it is then shown that, since there are no divisions of zero, it is possible to construct a field of ordered pairs of functions of C by following the familiar construction which yields rational numbers from integers. It turns out that under the natural embedding $f \epsilon C$ corresponds to $(h^*f,\ h)$ where h equals one for $t \geq 0$, and that the delta function corresponds to $(g,\ g),\ g \epsilon C$. It also follows that integration corresponds to multiplication by h and differentiation to division by h.

The book is intended for use by advanced undergraduate and beginning graduate students, especially those in engineering. Although the notion of generalized function introduced by Mikusinski is less general than that resulting from the theory of distributions, it is more amenable to elementary treatment and application; furthermore, it is sufficiently general to provide a satisfactory basis for operational calculus. It appears therefore to be an excellent choice from this point of view. There is a sufficient number of exercises and the book is written with great clarity. The author is to be congratulated in having succeeded in writing an excellent text-book on material which should be, and is not at present, studied at an early stage.

R. V. Chacon

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BOOK REVIEWS

(*Continued from p. 340*)

Finite-difference methods for partial differential equations. By G. E. Forsythe and W. R. Wasow. John Wiley & Sons, Inc., New York, London, 1960. x + 4444 pp. $11.50.

The aim of this book is to describe and explain the various difference methods in use for solving partial differential equations. No knowledge of the theory of partial differential equations is assumed, but the relevant theory is derived or at least described in the text. Thus the book can be read by first year graduate students or possibly by properly motivated people with even less experience; some experience with analytic manipulations is a prerequisite, and so is a rudimentary knowledge of linear algebra.

The authors have succeeded admirably in their aim. The book describes almost all the important difference methods in use at the time the book was written, so that readers previously unacquainted with the subject can master both the ideas and the details. The book makes lively reading thanks to the inclusion of brief discussions of theory and applications. This reviewer was particularly impressed by the masterful treatment of the rates of convergence of various iterative procedures for solving boundary value problems for elliptic equations.

The organization of the book is as follows: Chapter 1 is about hyperbolic equations in two variables, 2 about parabolic equations, 3 about elliptic equations and 4 about the initial value problem for hyperbolic and parabolic equations in more than one space variable. The sketchiest part is about hyperbolic equations, due largely to the shortage of effective methods for treating such problems; however, a description of the method of Friedrichs should have been included.

The book belongs (along with the treatise of Richtmyer) on the shelf of every numerical analyst, practicing or aspiring. The authors are to be thanked for their valuable service; let us hope that they will be around (in the field, that is) to write the revision when current and future rapid developments make this necessary.

Peter D. Lax

Summation of series. By L. B. W. Jolley. Second revised edition. Dover Publications, New York, 1961. xii + 251 pp. $2.00.

The first edition of this work was published in 1925. In the present paperbound edition numerous new series have been included, with a resulting increase of more than 50% in the number of series listed.

Introduction to elliptic functions. By F. Bowman. Dover Publications, New York, 1961. 115 pp. $1.25.

This new paperbound edition is identical with the first edition published in 1953, except for the correction of typographical errors and the addition of references to recent work.

Tables of indefinite integrals. By G. Petit Bois. Dover Publications, New York, 1961. xiv + 151 pp. $1.65.

This is a reprint of "Tafeln der unbestimmten Integrale" published in Germany in 1906. The Preface, the section on Notations, and headlines and foot-notes have been translated into English.

Nonlinear differential equations. By R. A. Struble. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1962. x + 267 pp. $7.50.

This is an excellent text for a one-semester course in nonlinear differential equations, either for beginning graduate students or for upper division students. Designed to furnish an introduction to modern developments, it fulfills its purpose admirably. In particular, it is well suited for those who wish to learn some of the mathematical prerequisites for the theory of control processes.

The text covers a number of the fundamental results concerning the existence and uniqueness, boundedness, stability and asymptotic behavior of periodic and nonperiodic solutions of linear and

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BOOK REVIEWS

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nonlinear ordinary differential equations. A large number of examples and exercises are given. This feature, the attractive and easily readable format, and the careful and lucid exposition, all combine to make this book a model of what an introduction to a significant contemporary field of research should be.

RICHARD BELLMAN

Chebyshev series for mathematical functions. By C. W. Clenshaw. (National Physical Laboratory, Mathematical Tables, Vol 5.) Her Majesty's Stationery Office, London, 1962, IV + 36 pp. $1.80.

Tables of high accuracy (up to 20 figures) of the coefficients in Chebyshev series expansions for a number of common mathematical functions are presented, and basic methods for evaluating and manipulating such series are discussed. The following functions are treated: trigonometric and inverse trigonometric functions, exponential, logarithmic, hyperbolic and inverse hyperbolic functions, gamma function, error function, exponential integral and Bessel functions.

Survey of numerical analysis. Edited by John Todd. McGraw-Hill Book Co., Inc., New York, 1962. xvi + 589 pp. $12.50.

The volume contains material prepared for an NSF-sponsored training program in numerical analysis for senior university staff. The contents are best indicated by the following chapter headings: Motivation for working in numerical analysis (by J. Todd)—Classical numerical analysis (by J. Todd)—The constructive theory of functions (by J. Todd)—Automatic computers (by M. Newman and J. Todd)—Use and limitation of computers (by H. Cohn)—Matrix computations (by M. Newman)—Numerical methods for finding solutions of nonlinear equations (by U. Hochstrasser)—Eigenvalues of finite matrices (by O. Taussky and M. Marcus)—Numerical methods in ordinary differential equations (by H. A. Antosiewicz and W. Gautschi)—Orthonormalizing codes in numerical analysis (by P. J. Davis)—The numerical solution of elliptic and parabolic partial differential equations (by D. M. Young, Jr.)—Numerical methods for integral equations (by H. F. Buckner)—Errors of numerical approximations for analytic functions (by P. J. Davis)—Numerical analysis and functional analysis (by H. A. Antosiewicz and W. C. Rheinboldt)—Discrete problems (by M. Hall, Jr.)—Number theory (by H. Cohn and O. Taussky)—Linear estimation and related topics (by M. Zelen). Although the jacket claims that the first third of the volume, which contains some exercises, could be used as a text for an introductory course, this is a guide for the teacher rather than a text for the student. The presentation is clear and frequently illustrated by numerical examples expressing the authors' conviction that "there should be no division between theoretical and practical numerical analysis, and that a lecture without numerical examples is a lecture wasted."

W. PRAGER

Tensor and vector analysis—with applications to differential geometry. By C. E. Springer. The Ronald Press Co., New York, 1962. ix + 242 pp. $8.50.

The orientation of this elementary textbook is toward Riemannian geometry rather than applications in engineering or physics. The approach is more analytic than geometric, and the usual order of presentation is reversed, in that the simpler aspects of general tensor analysis are covered before vector analysis is taken up as a subject unto itself. The final chapter, on geodesic and union curves, includes some of the author's own work on the subject. Although a knowledge of elementary calculus is the only prerequisite mentioned, the student probably should also understand the elements of vector algebra, since in the early part of the book certain results which are most easily proved vectorially are assumed to be known.

A. PIPKIN

Error-correcting codes. By W. Wesley Peterson. The M. I. T. Press, Cambridge, and John Wiley & Sons, Inc., New York, London, 1961. x + 285 pp. $7.75.

This book provides a readable and quite complete coverage of algebraic error correcting codes. Most of the material covered does not appear elsewhere in book form and there is a considerable amount of new material. The journal literature on algebraic coding theory has become so extensive recently that a book was necessary to give the theory perspective and order. Peterson's book not only fills this need but also improves greatly on the presentation in many of the journal articles.

A successful compromise has been made in the style between the mathematical rigor, formalism, and conciseness desired by mathematicians and the intuition and insight desired by engineers. As a result, the book will make algebraic coding theory accessible to a wide variety of people and also should broaden the concepts of those in the coding field. This book, in conjunction with the literature on the probabilistic schemes of coding and decoding, gives an essentially complete picture of coding theory as currently known.

The first chapter discusses the role of coding in communication and defines some basic concepts. Linear codes, which are group codes or parity check codes generalized to non binary alphabets, are discussed in the next four chapters. The first of these provides algebraic background, the second treats linear codes in general, the third contains some bounds on performance, and the fourth considers several specific types of linear codes.

Cyclic codes are emphasized in the rest of the book, starting in chapter 6 with an excellent introduction to polynomial rings and Galois fields. Chapter 7, on linear switching circuits, complements chapter 6 by providing ways to implement Galois field and polynomial operations along with giving a mathematical framework for treating shift register circuits. Chapter 8 applies the results of these chapters to the theory and implementation of cyclic codes.

Bose-Chaudhuri codes, which are the most important of the known algebraic codes, are elegantly treated in Chapter 9. A simple derivation of their error correcting ability is given and two decoding techniques are derived. The remaining chapters treat burst error correction, other approaches to decoding, recurrent codes, and the checking of arithmetic operations.

Throughout the book, the applicability of the various codes to actual communication systems was considered, although perhaps more emphasis here would have been helpful.

The appendices include a table of irreducible polynomials over the field of two elements. They are arranged in order as minimum polynomials of the elements of a Galois field, and this makes it possible to find generator polynomials for Bose-Chaudhuri codes almost by inspection.

The book is highly recommended to engineers and mathematicians interested in coding, information theory, communication, and computers.

R. G. Gallager

An introduction to applied anisotropic elasticity. By R. F. S. Hearmon. Oxford University Press, New York, 1961. viii + 136 pp. $4.93.

This is a short (136 pages) reference book for engineers and physicists. The first part of the volume is concerned with the stress-strain relations (assuming small strains) for various anisotropic materials, including the thirty two crystal classes, and such laminated plates as plywood. Some tables of numerical values of the elastic coefficients are given for specific materials.

The remainder of the book is devoted to applications such as the bending and torsion of bars, plane stress, thermal stresses, waves in an anisotropic solid, and bending and buckling of anisotropic plates. The author generally confines himself to the simpler problems in these topics but gives frequent references to the literature for more extensive treatment. The selected references and the bibliography are up to date and cover the Russian as well as the western literature. The book well serves its purpose as an introduction to the subject.

J. Lyell Sanders, Jr.

Numerical methods of curve fitting. By P. G. Guest. Cambridge University Press, New York, 1961. xii + 422 pp. $15.00.

The author has brought together a vast amount of material on fitting observed data by polynomials and other functions. There are three parts, labelled (i) Single Variables; (ii) Regression Theory and the Straight Line; (iii) Polynomials and other Curves, of which the first presents the theoretical statistical background without pretence at rigour. In the second part, much attention is paid to problems where either one variable or both variables are subject to error, and in the third numerical methods proper are treated. Various classical methods for solving the normal equations are explained in detail, with numerical examples, and the usefulness of orthogonal polynomials, in this context is rightly stressed. Additional material includes, for instance, the treatment of missing data and harmonic analysis.

The book could, perhaps, have been improved along three lines. Firstly, a less formal development of the underlying probability and statistics material would make the work more palatable to mathematicians. Secondly, a more energetic use of matrix notation (although employed to some extent) would have made the exposition more translucent. Thirdly, and most seriously, the book is almost completely oriented towards desk-computations. There is some mention of automatic computers, such as in the exposition of Forsythe's method of generating orthogonal polynomials, but in general no attempt is made to distinguish methods appropriate for nonautomatic from those appropriate for automatic computation.

Nevertheless, we have here an invaluable reference work on statistical and numerical techniques for scientists (the author is a physicist) who wish to analyse data, with many numerical examples, practical hints, reference tables and a bibliography of over 200 items.

W. Freiberger

The simplex method of linear programming. By F. A. Ficken. Holt, Rinehart and Winston, New York, 1961. vi + 58 pp. $1.50.

This is a neat little textbook on Linear Programming whose brevity makes it particularly suitable as a rapid introduction for persons with some mathematical background while its elementary character would recommend it also to the non-mathematician. The linear programming problem is introduced in intuitive terms, the concepts of convexity and duality are developed, and Dantzig's theorem is demonstrated that the solution may always be found among the extreme points of the constraint set. The simplex method and tableaus are presented after Charnes so that the treatment extends also to the so-called degenerate case. This is standard material, but it is presented clearly and economically.

M. Beckmann

Mathematical programming. By S. Vajda. Addison-Wesley Publishing Co., Inc., Reading, Mass., and London, 1961. ix + 310 pp. $8.50.

Dr. Vajda's previous texts on game theory and linear programming have been well received abroad and in this country. The present texts ranges over a somewhat broader subject but employs with advantage the same method of exposition with emphasis on numerical examples. Of particular interest are an early chapter on Graph and Combinatorial Theory which discusses various aspects of networks (shortest paths, min cut- max flow) and of the transportation and assignment problems. The simplex method and its variants are presented in detail with examples of cycling resulting from degeneracy. Special algorithms are given for the transportation and assignment problem including Kuhn's Hungarian method. The standard economic interpretation of the dual variables as efficiency prices and marginal value products are presented, but an explicit statement of Koopmans' efficiency price theorem would have added to clarity: Necessary and sufficient for a solution is the existence of prices such that active activities have zero profits and all other activities non-positive profits. From the many known applications of Linear Programming some well chosen practical examples are presented. The last chapters, justifying the title of the book, give brief outlines of discrete (or integer), stochastic, non-linear and dynamic programming. The treatment of non-linear programming includes piece-wise linear approximations, algorithms for quadratic programming, and the method of Lagrangian Multipliers in the special form of the Kuhn-Tucker theorem. The remaining topics are handled too briefly to do more than whet the reader's appetite. Throughout the book there is a large number of exercises, to which the answers are given at the end. This book is a welcome addition to the growing literature on programming. It is particularly useful for the student who seeks a quick orientation in this field which at the present time is still very much in flux.

M. BECKMANN

Classical electrodynamics. By J. D. Jackson. John Wiley & Sons, Inc., New York, London, 1962. xvii + 641 pp. $13.00.

In addition to writing an excellent book on conventional classical electromagnetic theory, the author has carried out extensive treatments of some less conventional areas not usually dealt with in books on electromagnetic theory. The author has considered many of the problems in modern physics which require the application of electromagnetic theory. Topics such as magnetohydrodynamics, plasma physics, collisions between charged particles and scattering, bremsstrahling, method of vertical quanta, and radiative beta processes are treated in this work. The book seems to be suitable either as a text or for advanced study.

The chapter headings are as follows: 1. Introduction to Electrostatics—2. Boundary-Value Problems in Electrostatics I—3. Boundary-Value Problems in Electrostatics II—4. Multipoles Electrostatics of Microscopic Media, Dielectrics—5. Magnetostatics—6. Time Varying Fields, Maxwells's Equations, Conservation Laws—7. Plane Elecromagnetic Waves—8. Wave Guides and Resonant Cavities—9. Simple Radiating Systems and Diffraction—10. Magnetohydrodynamics and Plasma Physics—11. Special Theory of Relativity—12. Relativistic-Particle Kinematics and Dynamics—13. Collisions Between Charged Particles, Energy Loss and Scattering—14. Radiation by Moving Charges—15. Bremsstrahling, Method of Virtual Quanta, Radiative Beta Processes—16. Multipole Fields—17. Radiation Damping, Self Fields of a Particle, Scattering and Absorption of Radiation by a Bound System—Appendix, Units and Dimensions.

R. TRUELL

An introduction to Fourier analysis. By R. D. Sturat. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1962. 126 pp. $3.00.

According to the preface, this book was written with the needs of students of science and engineering in mind. The first two chapters consist of a fairly standard discussion of Fourier series, the next two chapters consist of a discussion of the Fourier integral. The fifth chapter is mostly concerned with applications of the Fourier integral in solving elementary network problems. Two topics are included here which are not often found in books of similar scope. These are the bandwidth-risetime relationship and the method of paired echoes. The final chapter discusses applications of the Fourier intergral in optics, especially with regard to a diffraction grating, and then includes a treatment of various forms of modulation. The method of paired echoes is used in the discussion of the latter topic. In general the exposition is concise and clear, if not especially imaginative. The author does not feel compelled to maintain a high level of mathematical rigor (*e. g.* he interchanges the order of integration without apologies). It seemed surprising to the reviewer that the convolution integral was not included nor that the commonly used frequency modulation was not mentioned in the discussion of phase modulation. A scientist or engineer desiring an elementary discussion or a brief review of Fourier analysis would probably find this book to be quite valuable.

B. HAZELTINE

Numerical analysis (with emphasis on the application of numerical techniques to problems of infinitesimal calculus in single variable). By Zdenek Kopal. John Wiley & Sons, Inc., New York, 1961. xvi + 594 pp. $12.00.

The first edition of this well-known text appeared in 1955 and was reviewed on p. 445 of volume 14 of this Quarterly. Chapters I through VIII of the present edition are substantially identical with the corresponding chapters of the first edition. A ninth chapter on Operational Methods in Numerical Analysis has been added. The Appendix on Trigonometric Interpolation has been expanded, and an Appendix on Rational Approximations to the Integration Operators in Terms of Central and Diagonal Differences has been added.

W. PRAGER

Electromagnetic waves. Edited by Rudolph E. Langer. University of Wisconsin Press, Madison, 1962. xii + 396 pp. $6.00.

This volume is another of the hard-cover proceedings of symposia on electromagnetic theory, this one having been held at the Mathematics Research Center at the University of Wisconsin in April, 1961. This reviewer is doubtful about the propriety of issuing such proceedings in the guise of a book, for while the papers are sometimes confined to a reasonably narrow range of topics, they never form the logical and comprehensive treatment of the subject one expects when he buys a book. In fact such volumes often resemble a random selection of papers on the favorite topics of the perennially active contributors to the field.

The present work, while no exception to the above criticism, has perhaps more unity than most, if only because so many of the papers are concerned with electromagnetic diffraction and because it begins with an admirable and lucidly written review by Morris Kline of the relation between electromagnetic theory and geometrical optics. It contains papers by sixteen of the more prolific contributors to the recent development of electromagnetic theory, ranging in viewpoint from the purely mathematical consideration of C. H. Wilcox to the more specific and practical problems of Siegel, King, and Goubau. Eleven of the papers deal with diffraction or scattering, and the remainder are concerned with antennas or propagation in non-homogeneous media. On the basis of such a distribution one might be justified in complaining that the title of the book is misleadingly general.

Presumably the motivation for the multifarious activity and the frequency of conferences in this field in recent years is the extreme practical importance of high frequency electromagnetic waves for communication and detection over large distances. The problems whose solutions are required unfortunately lie in the nebulous region between classical electromagnetic theory, where the problems are too difficult, and geometrical optics, which is not an adequate approximation. Hence the interest in the mathematical connection between these two regimes is justifiable. However, it seems that the ultimate hope for practical results from all this effort lies in new approximate theories, such as Keller's geometrical theory of diffraction, on which he has a short contribution in this book, rather than in the multitude of specific approximate solutions or in purely formalistic work, of which this book also contains examples.

The book is reproduced by photo-offset and has a fair number of typographical errors (even the title of Siegel's paper came out as nonsense); it also has an index, which is of very limited utility. However, since the articles have not been published in the journals, the book should be available in libraries to workers in the field of electromagnetic theory.

E. T. KORNHAUSER