SOME INTEGRATED PROPERTIES OF SOLUTIONS OF THE WAVE EQUATION WITH NON-PLANAR BOUNDARIES*

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Abstract. Integrated properties of solutions of the wave equation with non-planar boundaries are found and applied to three dimensional supersonic flow problems and two dimensional diffraction problems.

For the problem of supersonic flow outside a cylindrical surface with generators parallel to the flow direction, a theorem is proved concerning the integrated properties of the linearized pressure distribution and the prescribed normal velocity on the surface. The theorem is a generalization of the integral relationships obtained previously and is useful in the evaluation of total lift and drag of wing-body combinations when the linear dimensions of the cross section of the body are not small as compared to the chord length.

For the diffraction of a pulse or a weak shock over a rectangular notch, a pressure integral theorem is obtained. Its usefulness is demonstrated in reducing the labor of obtaining the mean pressure distribution along any depth inside the notch at different instants for various width-height ratios of the notch.

Introduction. Theorems concerning certain integrated properties of the linearized pressure field due to planar source distributions in a supersonic stream have been presented by Lagerstrom and Van Dyke [1] and by Bleviss [2]. Extension of the theorems and their applications to a class of three dimensional problems involving biplanes or cruciform wing arrangements were presented by Ferri [3] and by Ferri and Clarke [4].

Ferri, Clarke and Ting [5] obtained a theorem regarding the pressure integral along the line of intersection of a forward Mach plane with a specified non-planar surface which represents a prismatic body of rectangular cross section mounted on a planar wing with supersonic edges. The pressure integral is related to the integral of the source distribution which in turn is related to the integral of the prescribed normal velocity on the top (or bottom) surface of the body and that on the wing surface. A similar relationship can be established if the normal velocity is prescribed on the side walls of the body. The wing surface together with the surface of the body above (or below) the wing represents a cylindrical surface in the form of a "single" step with generator parallel to the direction of the undisturbed supersonic flow.

By a further extension of the theorem in [5], it can be shown that a similar integral relationship is valid if the cylindrical surface is of the form of a "stairway" with a finite number of steps. As the number of steps becomes infinite and the size of each step infinitesimal, then a sequence of "stairways" is obtained whose limit can approximate the shape of any given cylindrical surface. Hence, the generalized integral relationship to be presented in this paper is conjectured. However, the argument which leads to the conjecture cannot be accepted as a proof of the validity of the relationship unless it can be shown that the limit of the sequence of the corresponding disturbance pressures or potentials exists and equals the corresponding value for the given cylindrical surface.

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Such a convergence proof is by no means simple. An attempt to verify the relationship directly by the same procedure used in the preceding investigations has not been successful owing to the difficulty of finding the proper source distribution which corresponds to the prescribed boundary condition on the cylindrical surface. The fact that the velocity potential is a solution of the wave equation is expressed implicitly in the preceding investigations through the relationship between the integral of the pressure distribution and that of the source distribution. In the present paper the integral relationship between the pressure distribution and the prescribed normal velocity is verified by observing the fact that the velocity potential obeys the wave equation. Thus, the necessity of finding the proper source distribution is avoided.

Generalization of integral relationship. As shown in Fig. 1, a cylindrical surface, $y = F(z)$, is placed in a supersonic stream with its generator parallel to the x-axis,

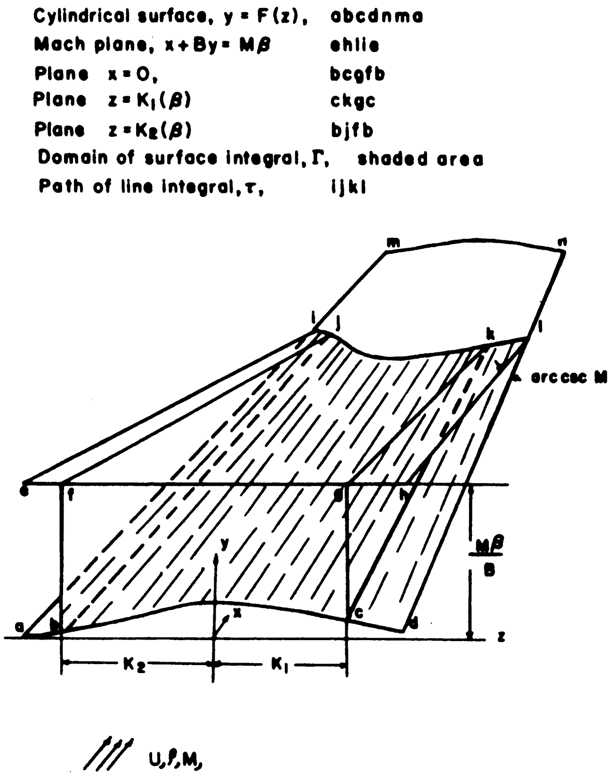

FIG. 1. The generalized integral relationship.

which is the direction of the undisturbed supersonic flow with velocity U and Mach number M. $q_n[x, y = F(z), z]$ represents the prescribed small normal velocity on the cylindrical surface with

$$q_n/U \ll 1 \tag{1}$$

and

$$q_n[x, F(z), z] = 0 \quad \text{for} \quad x < 0. \tag{2}$$

It will be assumed that $q_n[x, F(z), z]$ is piece-wise continuous and that for any given Mach plane, $x + By = M\beta$, at a finite distance β from the origin, there exist two finite values $K_1(\beta)$ and $K_2(\beta)$ with $K_1 > K_2$, such that on the Mach plane, the disturbance potential φ, due to q_n, is confined inside the region $K_1 > z > K_2$, i.e.,

$$\varphi(x \le M\beta - By, y, z) = 0$$

for

$$\infty > z \ge K_1(\beta) \quad \text{and} \quad K_2(\beta) \ge z > -\infty. \tag{3}$$

The disturbance pressure, $p(x, y, z)$, is related to the prescribed normal velocity on the cylindrical surface by the following integral relationship.

Theorem I. The integral of the y-component of the disturbance pressure force acting on the portion of the cylindrical surface, $y = F(z)$, which lies ahead of any given Mach plane parallel to the z-axis, $x + By = M\beta$ is equal to $\rho U/B$ times the integral of the prescribed normal velocity on the cylindrical surface, over the same domain of integration, i.e.

$$\iint_\Gamma p(x, y, z)\mathbf{n}\cdot\mathbf{j}\, dS = \frac{\rho U}{B} \iint_\Gamma q_n(x, y, z)\, dS, \tag{4}$$

where $\mathbf{n}$ is the unit vector normal to the cylindrical surface, $\mathbf{j}$ is the unit vector parallel to y-axis, and Γ represents the domain of integration on the cylindrical surface for

$$0 \le x \le M\beta - By.$$

Proof. The disturbance potential $\varphi(x, y, z)$ obeys the wave equation

$$B^2 \frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2} - \frac{\partial^2\varphi}{\partial z^2} = 0. \tag{5}$$

If a vector $\mathbf{Q}(x, y, z)$ is defined as

$$\mathbf{Q} = (B\varphi_x + \varphi_y)(B\mathbf{i} - \mathbf{j}) - \varphi_z\mathbf{k} \tag{6}$$

then Eq. (5) yields

$$\operatorname{div} \mathbf{Q} = 0.$$

If the volume which is confined by the cylindrical surface $y = F(z)$, the Mach plane $x + By = M\beta$, the planes $z = K_1(\beta)$, $z = K_2(\beta)$ and $x = 0$, is designated by V and its surface by S, then Gauss' theorem [6] states

$$-\iint_S \mathbf{n}\cdot\mathbf{Q}\, dS = \iiint_V \operatorname{div} \mathbf{Q}\, dV = 0, \tag{7}$$

where n denotes the unit vector normal to the surface S and points inward. Equation (7) is valid if $\mathbf{Q}$ is continuous inside V, i.e., if φ and its first derivatives are continuous. This is certainly true when $q_n[x, F(z), z]$ is continuous. It will be shown later that Eq. (7) is valid even if q_n is piecewise continuous.

Since φ and its first derivatives vanish on planes $x = 0$, $z = K_1$ and $z = K_2$, $\mathbf{Q}$ vanishes thereon. Since the unit normal vector $\mathbf{n}_\beta$ to the Mach plane $x + By = M\beta$ is $\mathbf{n}_\beta = (\mathbf{i} + B\mathbf{j})/M$, it is clear that on the Mach plane $-\mathbf{Q}\cdot\mathbf{n} = \mathbf{Q}\cdot\mathbf{n}_\beta = 0$. Consequently on the surface S, $\mathbf{n}\cdot\mathbf{Q}$ vanishes except the portion on the cylindrical surface, and Eq. (7) becomes

$$\iint_{\Gamma} \mathbf{n}\cdot\mathbf{Q}\, dS = 0. \tag{8}$$

The domain of integration on the cylindrical surface has been extended beyond the region of $K_2 < z < K_1$ because φ vanishes for $z \geq K_1$ or $z \geq K_2$.

For the cylindrical surface $y = F(z)$, $\mathbf{n}\cdot\mathbf{i} = 0$, and Eq. (7) becomes

$$\iint_{\Gamma} [(B\varphi_x + \varphi_y)(\mathbf{n}\cdot\mathbf{j}) + \varphi_z\mathbf{n}\cdot\mathbf{k}]\, dS = 0. \tag{9}$$

With disturbance pressure $p = -\rho U\varphi_x$ and the disturbance velocity,

$$\mathbf{q} = \varphi_x\mathbf{i} + \varphi_y\mathbf{j} + \varphi_z\mathbf{k},$$

Eq. (9) leads to

$$\begin{aligned}\iint_{\Gamma} \mathbf{j}\cdot\mathbf{n}p\, dS &= -\rho U \iint_{\Gamma} \varphi_x\mathbf{n}\cdot\mathbf{j}\, dS \\ &= \frac{\rho U}{B}\iint_{\Gamma} [\varphi_y\mathbf{n}\cdot\mathbf{j} + \varphi_z\mathbf{n}\cdot\mathbf{k}]\, dS \\ &= \frac{\rho U}{B}\iint_{\Gamma} \mathbf{q}\cdot\mathbf{n}\, dS = \frac{\rho U}{B}\iint_{\Gamma} q_n\, dS.\end{aligned} \tag{10}$$

The remainder of the proof of the theorem is to show that Eq. (7) is valid even when q_n is only piecewise continuous.

If $q_n[x, F(z), z]$ has a jump discontinuity across a curve on the cylindrical surface Γ, and the first derivatives of the disturbance potential are discontinuous across a surface S_c inside the volume V, the discontinuity surface S_c will be a characteristic surface—a Mach cone or an envelope of Mach cones.

A unit vector, $\mathbf{n}_c$, normal to the discontinuity surface, S_c , is also normal to a Mach cone and can be written in general as

$$\mathbf{n}_c = (\mathbf{i} - B\cos\lambda\mathbf{j} - B\sin\lambda\mathbf{k})/M \tag{11}$$

where λ is a parameter. Due to the discontinuity surface, a term

$$\iint_{S_c} \delta(\mathbf{Q}\cdot\mathbf{n}_c)\, dS$$

should be added to the right side of Eq. (7). Here $\delta(\quad)$ express the jump of the quantity inside the parenthesis across the surface S_c .

From Eqs. (6) and (11), it follows:

$$\begin{aligned}\mathbf{Q}\cdot\mathbf{n}_c &= [(B\varphi_x + \varphi_y)(1 + \cos\lambda) + \varphi_z \sin\lambda]B/M \\ &= \operatorname{grad}\varphi\cdot\boldsymbol{\tau}_c \\ &= |\,\boldsymbol{\tau}_c\,|\,\varphi_\tau\,,\end{aligned}$$

where

$$\tau_c = [(B\mathbf{i} + \mathbf{j})(1 + \cos\lambda) + \sin\lambda\mathbf{k}]B/M$$

and φ_τ is the derivative of φ in the direction of τ_c .

With $\tau_c \cdot n_c = 0$, or τ_c tangential to the surface S_c , φ_τ represents a derivative of φ along the surface S_c . Since φ is continuous across the surface S_c , φ_τ is also continuous, i.e., $\delta(\varphi_\tau) = 0$. Hence, the additional term to Eq. (7) vanishes,

$$\iint_{S_c} \delta(\mathbf{Q}\cdot\mathbf{n}_c)\, dS = \iint_{S_c} |\, \tau_c \,|\, \delta(\varphi_\tau)\, dS = 0$$

and Eqs. (7) to (10) are valid when q_n is piecewise continuous.* Thus concludes the proof of the theorem.

If Γ_1 and Γ_2 are the areas on the cylindrical surface $y = F(z)$ ahead of the Mach planes $x + By = \beta_1 M$ and $x + By = \beta_2 M$, respectively, the theorem or Eq. (4) yields

$$\iint_{\Gamma_1 - \Gamma_2} p\mathbf{n}\cdot\mathbf{j}\, dS = \frac{\rho U}{B} \iint_{\Gamma_1 - \Gamma_2} q_n\, dS. \tag{12}$$

When the two Mach planes are very close to each other, i.e., $M\beta_1 - M\beta_2 = dx$, the following result is obtained.

Corollary. The integral of the disturbance pressure along the curve of intersection, T, of the cylindrical surface $y = F(z)$ with the Mach plane $x + By = M\beta$ is related to the integral of the prescribed normal velocity $q_n[x, F(z), z]$ as follows

$$\int_T p \sin\sigma\mathbf{n}\cdot\mathbf{j}\, dT = \frac{\rho U}{B} \int_T q_n \sin\sigma\, dT, \tag{13}$$

where σ represents the angle between the x-axis and the tangent to the curve T.

Equation (13) is obtained readily from Eq. (12) when dS is replaced by $dx\, dT \sin\sigma$ and the area $\Gamma_1 - \Gamma_2$ becomes a strip along the curve T with thickness equal to $dx \sin\sigma$.

On the basis of linearized theory, the prescribed normal velocity q_n on the cylindrical surface $y = F(z)$ can be created by deforming the cylindrical surface slightly into a surface $y = F(z) = \epsilon E(x, y, z)$, if

$$q_n[x, F(z), z] = \epsilon U E_x[x, F(z), z](1 + F_z^2)^{-1/2} \tag{14}$$

where $\epsilon \ll 1$.

With this equation the integrals of disturbance pressure on an almost cylindrical surface in a supersonic stream are obtained from Eqs. (4) and (13).

Application to wing-body interference. In studying wing-body interference, the body is usually represented by an almost cylindrical surface with generator parallel to the direction of the supersonic stream (x-axis). The wing, which is planar with supersonic edges, coincides with the x-z plane while the x-y plane is the plane of symmetry. The surface of the body above (or below) the wing together with the upper (or lower) surface of the wing form the cylindrical surface $y = F(z)$ (Fig. 2).

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*This is a sufficient condition but not a necessary condition for the validity of the theorem.

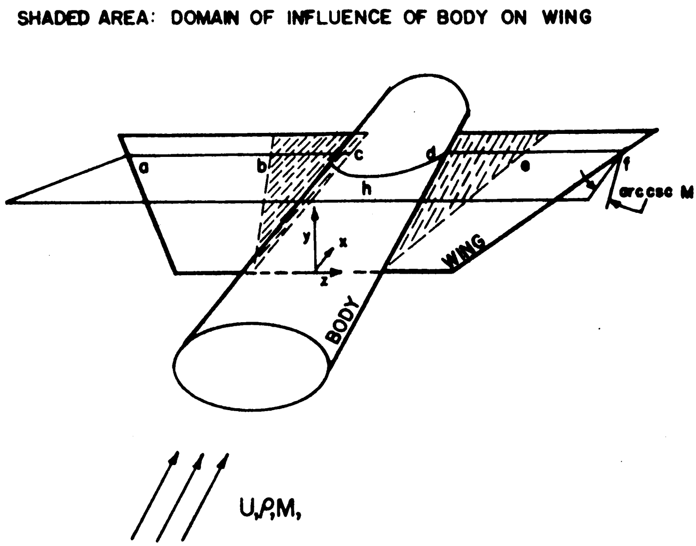

Fig. 2. Wing-body interference.

If T_B and T_W represent the line of intersection of the Mach plane, $x + By = \beta M$, with the body and the wing respectively, the corollary, Eq. (13) becomes:

$$\int_{T_B} p \sin \sigma \mathbf{n}\cdot\mathbf{j}\, dT + \int_{T_W} p\, dz = \frac{\rho U}{B} \int_{T_B} q_n \sin \sigma\, dT + U \int_{T_W} \vartheta\, dz, \tag{15}$$

where $\vartheta(x, z)$ is the inclination of the wing surface with respect to x-axis.

This relationship can serve as a check to the numerical results of the pressure distribution obtained by analytic methods [7, 8, 9, 10].

If the wing has a straight unswept trailing edge, the total lift acting on the region Γ of the upper (or lower) surface of the wing-body combination ahead of the Mach plane passing through the trailing edge is expressed by the term on the left side of Eq. (4). This lifting force which consists of the total lift acting on the wing and the lift acting on the part of the body is equal to the integral of prescribed normal velocity represented by the right side of the same equation. Therefore, if the pressure distribution on the body is obtained by analytical methods [7, 8, 9, 10], the total lift and the center of pressure can be obtained by virtue of the theorem and the corollary, i.e., Eq. (15). The labor of calculating the pressure distribution on the wing is saved.

In the special case where the inclination of the wing surface inside the region of influence of the body is constant in the spanwise direction, the total drag can be evaluated with the help of the theorems without calculating the pressure distribution on the wing due to the interference of the body [5, 11].

For wings with subsonic edges at an angle of attack, the theorem, in general, will not be helpful in the evaluation of lift or drag. However, for the special case of a rectangular wing, the theorem can be applied to obtain expressions for the lift, the drag, and the center of pressure of the wing without calculating the detailed pressure distribution [11].

Integral relationship in diffraction problems. Since the linearized two-dimensional unsteady flow obeys the wave equation,

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} - \frac{1}{C^2}\frac{\partial^2 \varphi}{\partial t^2} = 0, \tag{16}$$

it is expected that an integral relationship similar to that for the linearized supersonic three-dimensional flow can be found. In order to demonstrate its usefulness, an integral relationship is established for a special diffraction problem—the diffraction of a weak shock or pulse over a rectangular notch.

As shown in Fig. 3, a plane weak shock ($x = Ct$), which travels parallel to the ground

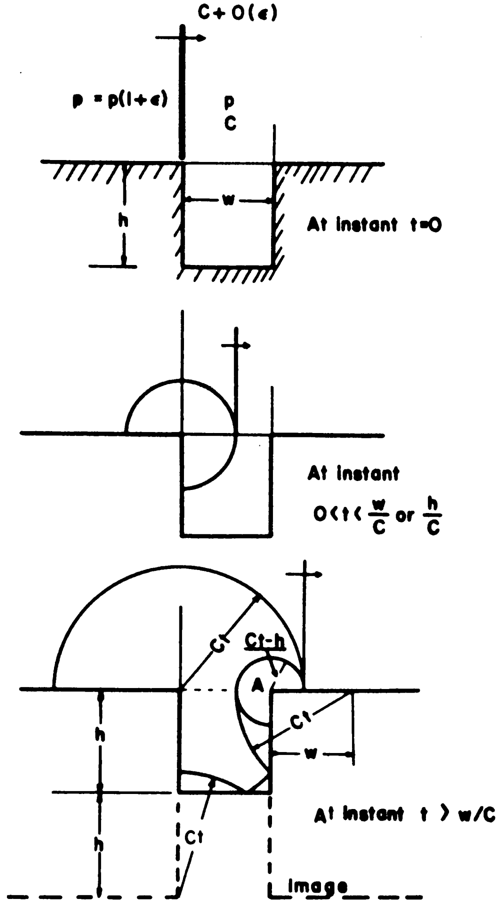

FIG. 3. Diffraction of a weak shock over a rectangular notch.

($y = 0$) is passing over a two-dimensional rectangular notch of width w and depth h. The pressure, density and speed of sound of the air at rest in front of the shock are P, ρ and C respectively. Deviations from the states at rest are defined as disturbance quantities. For example, if the notch is absent, the disturbance pressure and potential are $\epsilon PH(Ct - x)$ and $\epsilon C(1/\gamma).\ (x - Ct)H(Ct - x)$ respectively, where ϵ is the strength of the shock and $H(Ct - x)$ is the unit function, i.e., $H = 1$ for $Ct > x$ and $H = 0$ for $Ct < x$. The disturbance due to the notch will be confined inside the forward cone $Ct = (x^2 + y^2)^{1/2}$. Across the cone, the disturbance potential and its first derivatives are continuous.

Experimental investigations of this problem in shock tubes were reported by Smith [13] and Coulter [14]. Theoretical solution to this problem can be obtained by the method outlined in [9] and [12]. Nevertheless, an integral relationship will be established and save much labor in getting numerical results.

Theorem II. Twice the value of the line integral of disturbance pressure across the notch at a depth $d(0 > y = -d > -h)$ at an instant t_0 equals the difference of the integral of disturbance pressure on the ground from the corresponding integral if the notch is absent at the "retarded" instant $t = t_0 - d/C$ plus the same at $t = t_0 - (2h - d)/C$, i.e.,

$$2\int_0^w p(x, -d, t_0)\, dx = \left[\epsilon P \int_{-Ct}^{Ct} H(Ct - x)\, dx - \int_{-Ct}^{0} p\, dx - \int_{w}^{Ct} p\, dx\right]_{\substack{y=0\\ t=t_0-d/C}} + \left[\epsilon P \int_{-Ct}^{Ct} H(Ct - x)\, dx - \int_{-Ct}^{0} p\, dx - \int_{w}^{Ct} p\, dx\right]_{\substack{y=0\\ t=t_0-(2h-d)/C}}. \tag{17}$$

*Proof.** With characteristic coordinates $\alpha = Ct - y$ and $\beta = Ct + y$, the wave equation becomes

$$4\varphi_{\alpha\beta} = \varphi_{xx}\,. \tag{18}$$

The α- and β-axes together with the x-axis form an orthogonal system. The plane $\beta = Ct_0 - d = \text{const.}\ \beta_0$ passes through the depth $y = -d$ at the instant $t = t_0$. On

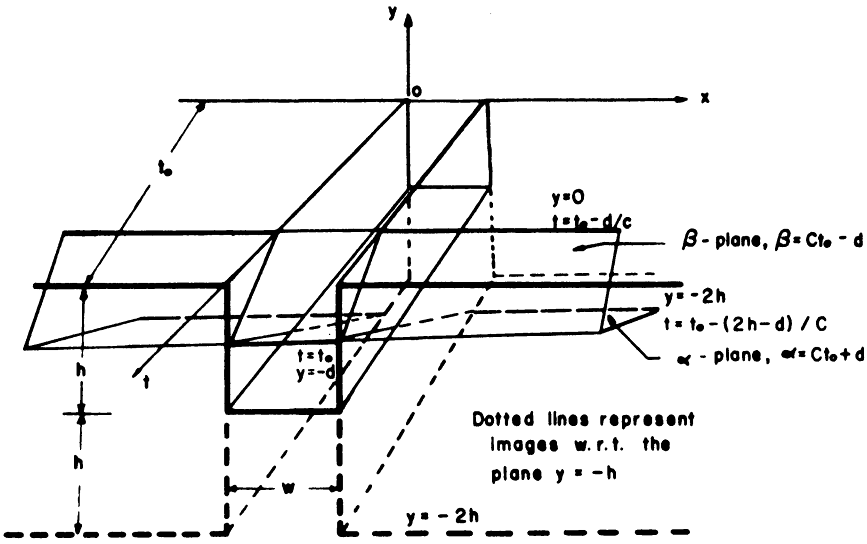

FIG. 4. Integral relationship for diffraction problems.

this plane, the cone is represented by a parabola $x^2 = \alpha\beta_0$ and the shock front is represented by a straight line $2x + \alpha = \beta_0$ which is tangential to the parabola at point $x = \beta_0$ (Fig. 4).

If S designates the area confined by the parabola, the ground $\alpha = \beta_0$, the side walls

*For the convenience of those who are interested in diffraction problems, the proof is given here without relying on the previous theorem.

of the notch $x = 0$ and $x = w$, and the line $\alpha = \beta_0 + 2d$ at the depth $y = -d$, then the divergence theorem in the plane $\beta = \beta_0$ states (p. 88, Ref. [6]):

$$\int_\Lambda (4\varphi_\beta\, dx + \varphi_x\, d_\alpha) = \int_\Lambda [4\varphi_\beta \cos(\lambda, x) - \varphi_x \cos(\lambda, \alpha)]\, d\lambda$$
$$= \int_S (4\varphi_{\alpha\beta} - \varphi_{xx})\, dS = 0, \tag{19}$$

where Λ designates the contour of the area S. If Λ_1, denotes the part of the contour on the ground, Λ_2 on the side walls, Λ_3 along the depth $y = -d$, and Λ_4 and Λ_5 denote the portions of the parabola above and below the ground level respectively (Fig. 5) then

$$\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 + \Lambda_5 .$$

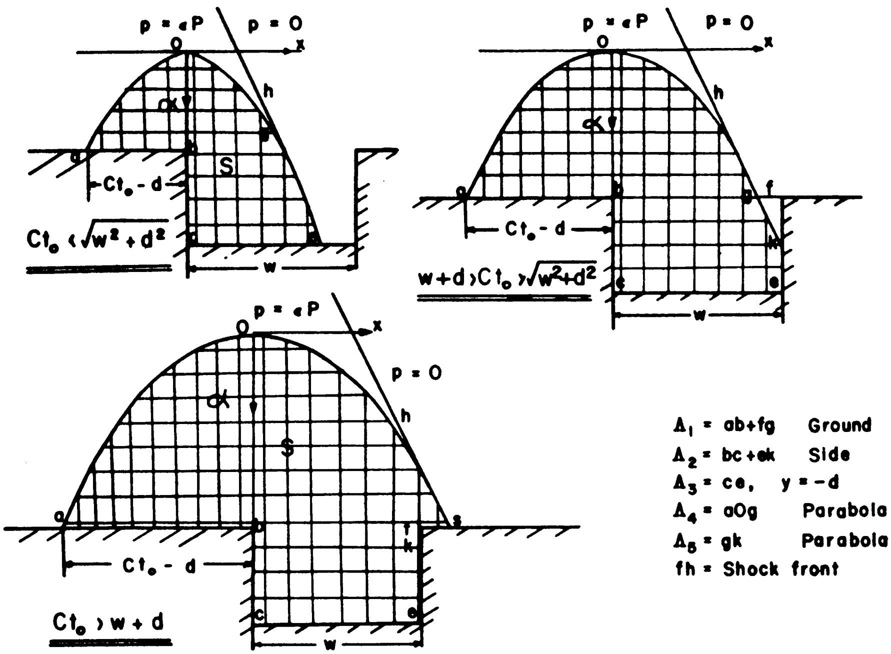

FIG. 5. β-planes, $Ct + y = Ct_0 - d$.

On the side walls, $\varphi_x = 0$ and $\cos(\lambda, x) = 0$, therefore, the line integral of Eq. (19) along Λ_2 vanishes. Across the parabola (Mach cone), φ and its first derivatives are continuous, therefore, along Λ_5, φ_β, φ_x and the corresponding line integral vanish.* Along Λ_4, $\varphi_x = \epsilon C/\gamma$ and $\varphi_\beta = -\epsilon C/(2\gamma)$ and the corresponding line integral becomes:

$$\int_{\Lambda_4} [4\varphi_\beta\, dx + \varphi_x\, dx] = 4\epsilon C(Ct_0 - d)/\gamma.$$

*When $Ct > d + w$, Λ_5 disappears from Λ, however, it does not affect the result, because the line integral along Λ_5 vanishes anyway.

Along Λ_1, $\varphi_y = 0$ and $\varphi_\beta = (\varphi_y + \varphi_t/C)/2 = -p/(2\rho C)$, and Eq. (19) becomes

$$\begin{aligned}\int_{-(Ct_0-d)}^{0} p(x, 0, t_0 - d/C)\,dx &+ \int_{w}^{Ct_0-h} p(x, 0, t_0 - d/C)\,dx \\ &- \int_{-(Ct_0-d)}^{Ct_0-d} \epsilon PH(Ct_0 - d)\,dx \\ &= -\int_0^w p(x, -d, t_0)\,dx + \rho C \int_0^w \varphi_y(x, -d, t_0)\,dx,\end{aligned} \tag{20}$$

the terms on the right side corresponding to the line integral along Λ_3. For $t_0 < (d^2 + w^2)^{1/2}/C$, Λ_3 has been extended to the full width of the notch, because the integral in the additional segment vanishes.

By the method of images, the bottom of the notch plane $y = -h$, can be removed and considered as a plane of symmetry, i.e.,

$$\varphi(x, y, z) = \varphi[x, -(2h + y), z]. \tag{21}$$

Instead of the plane $\beta = Ct_0 - d$, the other characteristic plane $\alpha = Ct_0 + d$ is considered and the result corresponding to Eq. (20) is obtained,

$$\begin{aligned}\left[\int_{-Ct}^{0} p(x, -2h, t)\,dx + \int_{w}^{Ct} p(x, -2h, t) - \int_{-Ct}^{Ct} \epsilon PH(Ct - x)\,dx\right]_{t=t_0-(2h-d)/C} \\ = -\int_0^w p(x, -d, t_0)\,dx - \rho C \int_0^w \varphi_y(x, -d, t_0)\,dx.\end{aligned} \tag{22}$$

With the condition of symmetry Eq. (21), the sum of Eq. (20) and Eq. (22) is equivalent to Eq. (17). Hence, the theorem is proved.

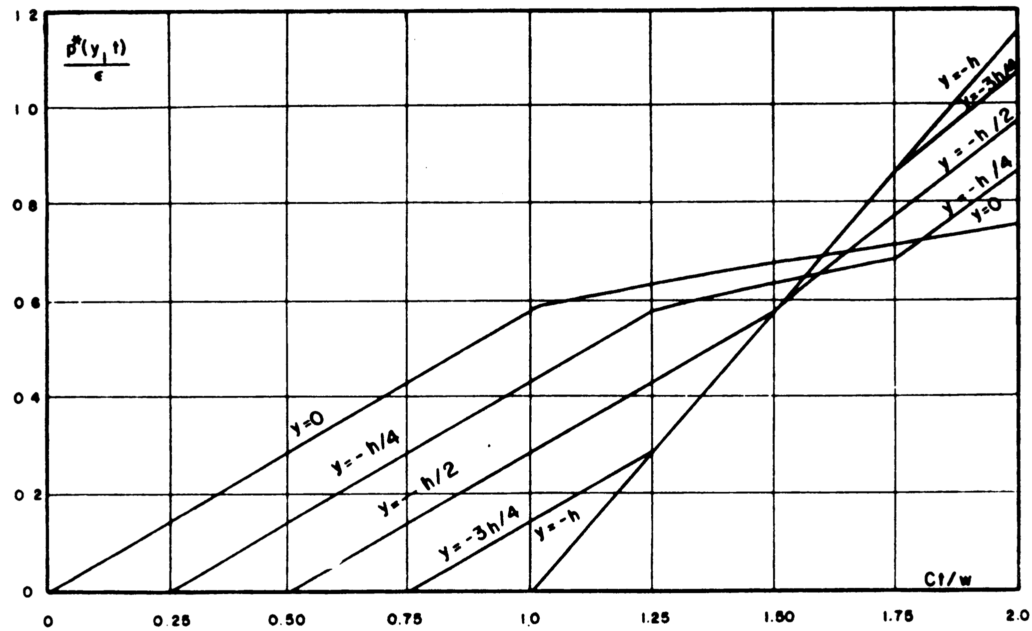

FIG. 6. Mean disturbance pressure across the width of the notch for $h/w = 1$.

The mean value of disturbance pressure across the width of the notch at depth, $y = -d$, is defined as

$$p^*(y = -d, t) = \frac{1}{w} \int_0^w p(x, -d, t)\, dx$$

and Theorem II, Eq. (17), becomes

$$p^*(y = -d, t_0) = \frac{1}{2w} \left[\epsilon P \int_{-Ct}^{Ct} H(Ct - x)\, dx - \int_{-Ct}^{0} p\, dx - \int_{w}^{Ct} p\, dx \right]_{\substack{y=0 \\ t=t_0 - d/C}}$$
$$+ \frac{1}{2w} \left[\epsilon P \int_{-Ct}^{Ct} H(Ct - x)\, dx - \int_{-Ct}^{0} p\, dx - \int_{w}^{Ct} p\, dx \right]_{\substack{y=0 \\ t=t_0 - (2h-d)/C}}. \tag{23}$$

With this relationship, the mean value of disturbance pressure along any depth inside the notch can be obtained from the pressure integral on the ground. The latter is obtained by the method presented in [9] and [12]. Detailed steps are given in [11]. The variations of mean pressure across the width of the notch at $y/h = 0, -\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}$ and -1 with respect to time are shown in Figs. 6 and 7 for notches with width-height ratios equal to 1 and $\frac{1}{2}$ respectively.

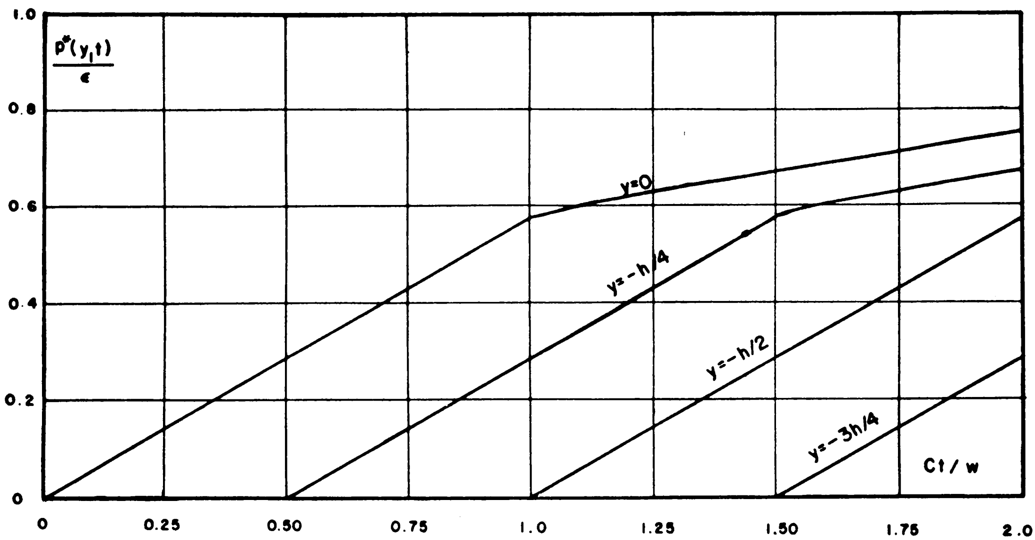

FIG. 7. Mean disturbance pressure across the width of the notch for $h/w = 2$.

Concluding remarks. For the problem of supersonic flow outside a cylindrical surface with generators parallel to the flow direction, a theorem is proved concerning the integrated properties of the linearized pressure distribution and the prescribed normal velocity on the cylindrical surface. This theorem can be extended readily to the case where there is more than one cylindrical surface present in the flow field and will be useful in the evaluation of total lift and drag of wing-body combinations and additional cylindrical bodies representing engines and (or) pylons.

For the diffraction of a pulse or a weak shock over a rectangular notch, a pressure integral theorem is obtained. By the same procedure, the theorem can be generalized for a notch of any shape, although the analytical solution of the diffraction problem is not available. By virtue of the theorem, the time history of the mean pressure along any depth inside the notch can be obtained from the measured pressure distribution on the boundaries.

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