Equation (3.4) indicates that for each j the Re $(\alpha_j^2) < 1$ when $| k/k_c | \geq 1$, but this inequality does not hold when $| k/k_c | < 1$. Thus we are in agreement with the principal result obtained in [2] but note that the model of that paper has broader validity than was claimed.

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Comments on the paper

WAVES PRODUCED BY A PULSATING SOURCE TRAVELLING BENEATH A FREE SURFACE*

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There appears to be an error in the analysis performed in the subject paper by Dr. Tan [1], which has led to erroneous conclusions in regard to the character of the resulting wave pattern and the asymptotic conditions at infinity that are used to arrive at a unique solution. Other treatments of this same problem have been carried out by Becker [2], Kaplan [3], and Wu [4], and they have arrived at different conclusions from those of Tan. Their results show two harmonic wave trains on the downstream side of the source when $\tau > \frac{1}{4}$, corresponding to the roots K_3 and K_4, in agreement with the results of Tan. However, when $0 < \tau < \frac{1}{4}$, Tan indicated that four harmonic wave trains were on the downstream side, with no disturbance found to propagate to infinity upstream. This led to his conclusion that the imposition of an asymptotic upstream condition of "vanishing disturbance at infinity" is sufficient to render his solution unique. The results of Becker, Kaplan and Wu were all in agreement, and showed that when $0 < \tau < \frac{1}{4}$ there were three harmonic wave trains on the downstream side, and one harmonic wave train on the upstream side. It can be shown that the wave in the upstream side corresponds to the root K_2 and that the error in Tan's analysis was not recognizing the fact that, in the notation of [1],

$$\lim_{\beta \to 0} ImK_2 = 0- \qquad (0 < \tau < \tfrac{1}{4})$$

not $0+$, as he indicated.

If this error were corrected, and the resulting analysis carried out in the same manner as done in the paper, the results would have been in agreement with that of the other authors. The imposition of the requirement of no disturbance upstream would not be valid, but the use of the technique of Rayleigh's artifice in order to secure uniqueness would not be affected, in spite of the artificial nature of this factor. A more realistic

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approach, which has more physical appeal, is to formulate the problem as an unsteady initial value problem with the motion assumed to start from a state of rest, and the steady state oscillatory solution is obtained when the time $t \to \infty$ and "transients" have disappeared. The behavior of the waves at infinity, i.e. the radiation conditions, are determined directly from that type of analysis by only imposing boundedness conditions at infinity, without any *a priori* assumptions about the character of the waves. This technique has been utilized by Kaplan and Wu in their treatments of this problem.

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