BOOK REVIEWS

Théorie des Circuits de Telecommunication. By Vitold Belevitch. Librairie Universitaire Louvain, 1957. viii 384 pp. $9.00.

This is a well conceived exposition of electric circuit theory for the use of engineers having the usual mathematical preparation. The contents of the book are indicated by the titles of the fourteen chapters, which can be translated freely as follows: I, Linear systems; II, Analysis of Kirchoff networks; III, Energetics of passive networks; IV, Reflection and transmission; V, Scattering matrix; VI, Image parameters; VII, Analytic theory of passive networks; VIII, Image theory of low-pass filters; IX, Bandpass filters; X, Filters with pre-determined attenuations; XI, Inductances and transformers; XII, Amplifiers; XIII, Transient phenomena; XIV, Complements on the synthesis of passive networks.

Thus the book is devoted chiefly to the *steady state* theory of circuits consisting of discrete elements. It appears that the author's knowledge of this subject is very extensive, and that he has made many significant contributions to it; and, so far as the reviewer can judge, he expounds the subject with great competence and thoroughness. On the other hand, the parts of the book (notably Chapter XIII) which deal with other branches of circuit theory seem to be too sketchy to serve as more than rudimentary introductions.

As has been indicated above, the mathematical preparation expected of the reader is quite modest. Not only does the mathematics used not go appreciably beyond that usually taught in engineering curricula, but also in many places intuitive physical arguments are used, instead of rigorous mathematical demonstrations, in the derivation of formulae. On the logical level which the author has chosen, the exposition is generally satisfactory. However, there are occasional slips which may give unwary readers trouble. Thus the author states the Hurwitz stability condition (i.e. the condition that the real parts of the roots of an algebraic equation be negative) in a special simplified form which applies only to the case in which the leading coefficient in the equation is positive, but he does not mention this restrictive assumption concerning the coefficients. Again, in the discussion of Fourier integrals, the author states that the absolute integrability of $f(t)$ over the interval $(-\infty, \infty)$ is a *necessary* condition for the representability of $f(t)$ by a Fourier integral.

There is an extensive list of errata. However, this is concerned mainly with trivial typographical errors, and does not seem to touch on any of the more serious faults.

On the whole, the book affords a comprehensive and lucid exposition of a large and important part of curcuit theory, and it should be welcome to engineers who are concerned with the practical design and use of circuits. An excellent feature of the book is a fifteen-page critical bibliography. This will afford adequate guidance to readers who wish to study the subject more deeply and extensively.

L. A. MacColl

Wahrscheinlichkeitstheorie. By Hans Richter. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1956. xi 435 pp. $16.65.

The author states in the preface that his objective is to supply a remedy for the lack of any modern German text on probability theory. He seems to have accomplished not only this modest goal but also to have made a valuable contribution to the international literature.

The reader is assumed to have some knowledge of analysis and linear algebra plus some appreciation of pure mathematics. Starting at this relatively low level, the author devotes about half the book to a rigorous development of the experimental and mathematical foundations including the necessary background from the theory of measure and integration. The last half of the book includes discussions of random variables (expectations, etc.), properties of common types of distributions (Poisson, Gaussian, etc.) and finally some of the well-known limit theorems (laws of large numbers, zero-one laws, central limit theorems, etc.).

Although the book covers only the above limited range of topics, it treats these very thoroughly. There is little if any discussion of applications or more advanced topics such as Markov chains or stochastic processes.

G. F. Newell

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Neutron transport theory. By B. Davison. With the collaboration of J. B. Sykes. Oxford University Press, New York, 1957. xx 450 pp. $12.00.

This book has for its central theme the study of the Boltzmann equation for neutron transport, the fundamental equation that describes the migration of neutrons through material media. It is evident that this study is of principal importance in the design of nuclear reactors. It is also of direct interest in astrophysics in the consideration of problems of radiative transfer which are very similar to those of neutron transport theory.

In this monograph the author has set for himself the task of giving a thorough review of all the important mathematical methods used in neutron transport theory. The subject matter is developed from first principles, and so a previous familiarity with the theory is not strictly necessary. Nevertheless, this book is not meant to serve as a first introduction to the subject. It is an advanced work which assumes a fair degree of mathematical maturity on the part of the reader; it requires comparatively little, however, in the way of a knowledge of nuclear physics and quantum mechanics.

The material covered in this work is conveniently divided into four parts. The first part is devoted to a precise formulation of the laws of neutron transport, the derivation of the basic integral and integro-differential equations, and the relation of stationary and time-dependent problems. The second part, by far the largest in the book, is concerned with the methods of solution of the transport equation in the constant cross-section approximation, i.e., for the case of one-group theory. Apart from a consideration of the few situations which can be treated exactly, this section develops in considerable detail the various approximation methods that have been proposed to date; the spherical harmonics method, in particular, is discussed at great length. The remainder of the book is concerned with energy-dependent problems for the cases of spectrum-regenerating media (Part III) and slowing-down media (Part IV).

This book fills an important gap in the literature, tying together, as it does, a considerable amount of material which had previously been available only in the form of technical reports and periodical articles. It is a serious mathematical work which is very well organized and very well written. It should prove extremely satisfying to theoretical physicists and applied mathematicians who are interested in an up-to-date account of the problems of neutron transport which goes far beyond the simplified discussion of neutron diffusion theory as it is usually presented in textbooks on nuclear reactors.

While one may question the nature of certain limitations on the scope of the subject which were imposed by the author (thus, the discussion is limited throughout to homogeneous media, with little or no reference being made to experimental methods or results), there is no doubt but that these have made possible a very well-knit and unified presentation; in any case, the book is already of considerable length. However, it would have been useful, especially in a comprehensive treatise such as this, if the author had documented this work much more extensively. The book is highly recommended.

DAVID FELDMAN

Elementary theory of angular momentum. By M. E. Rose. John Wiley & Sons, Inc., New York, 1957. 248 pp. $10.00.

The quantum theory of angular momentum occupies a central position in present-day atomic and nuclear physics. Evidently, in the description of complicated systems, it is important to be able to separate out those aspects which can be traced directly to the existence in nature of certain fundamental symmetries from those which depend upon the detailed characteristics of the systems themselves (the specific shape of the nuclear force, for example). Notwithstanding the recent experimental verification that the so-called weak interactions are not invariant under spatial reflections, it still appears that rotational invariance is an absolute symmetry principle; hence, angular momentum is always conserved.

The book under review is an outgrowth of a series of lectures given recently by the author at the Oak Ridge National Laboratory. As is perhaps evident from the title, it is a textbook and not a treatise.

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It is addressed to physicists, experimental as well as theoretical, who are familiar with the elements of quantum mechanics and who are approaching the subject of angular momentum for the first time.

Accordingly, it is not surprising that the author, in his presentation, avoids group-theoretical and abstract algebraic methods. Instead, he relies upon an inductive approach to his subject which leans rather heavily on the assumption that the reader, from previous experience, already has an intuitive idea as to what is meant by "angular momentum" and "spin." There is an inconsistency here in this assumption which may not be out of place in a spoken lecture but which some, like the reviewer, may find annoying to see in a more formal treatment of the subject.

The book is divided into two parts of approximately equal length, entitled General Theory and Applications. Under General Theory, one finds a terse review of some basic principles of quantum mechanics (which the reviewer feels could well have been omitted), a discussion of the angular-momentum operators (these are defined in terms of the transformation properties of wave functions under rotations), together with chapters dealing with the addition of two angular momenta and the Clebsch-Gordan coefficients, the transformation properties of the angular-momentum wave functions under rotations, irreducible tensors and the Wigner-Eckart theorem, and, finally, the addition of three angular momenta and the Racah coefficients. Applications include chapters on the expansion of the electromagnetic field into multipole fields, the multipole moments of static charge distributions, spin one-half particles, oriented nuclei and angular correlations, angular distributions in nuclear reactions, and wave functions for systems of identical particles.

The discussion of the general theory is sufficiently detailed and up to date so as to give the reader a useful first approach to the theory of angular momentum. The applications are sufficiently varied so as to illustrate the considerable scope of the subject; it is inevitable, however, that the author cannot go into as much detail here as in the discussion of the general theory. It is for this reason that the second half of the book does not hang together nearly so well as the first part; it can, however, serve as a very useful point of departure to the periodical literature.

Since it seems fairly certain that this book will be widely used as a first introduction to the subject, one would have hoped that certain inconsistencies and ambiguities, which may trouble the beginner, had been avoided. For example, on p. 16, there is the remark that, in quantum mechanics, "no more than one component operator (of the angular momentum) can be a constant of the motion." On p. 81, we have the statement that "a scalar, as the term is used here, may be a tensor of any rank." Finally, the author consistently uses the term "projection quantum number" in place of the old-fashioned "magnetic quantum number" except for the several instances where he forgets himself and reverts to the more conventional nomenclature.

DAVID FELDMAN

Applied group-theoretic and matrix methods. By Bryan Higman. Oxford University Press, New York (American Branch), 1955. xii + 454 pp. $9.60.

This book is an outgrowth of lectures given at the University College of the Gold Coast, the object being to present the theory of matrices and group representations in a form palatable to chemists and physicists. The style of the book, which is quite unique, reveals its origin in a rather informal lecture series and the author probably succeeds in conveying the intuitive picture that goes along with the formal theory. The author, however, shows excessive disregard for mathematical rigor to the point of making rather vague statements of theorems. Most of the book is devoted to the applications in almost all the obvious branches of physics and chemistry. At places these are discussed in monotonous detail and one is left with the impression that the author, in amplifying his lecture notes, did not know where to stop.

G. F. NEWELL

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BOOK REVIEWS

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The hypercircle in mathematical physics. By J. L. Synge. Cambridge University Press, Cambridge, 1957. xii + 424 pp. $13.50.

In 1947, the author and W. Prager collaborated on a paper entitled, *Approximations in elasticity based on the concept of function space*, in which the "method of the hypercircle" was developed to furnish approximate solutions to boundary value problems of elasticity theory. At the time of this research, the reviewer was privileged to read the correspondence between the authors. It is especially gratifying, therefore, to read the present book which represents a summation of over a decade of work and thought during which the ideas developed and new applications materialized.

The hypercircle method is one of functional approximation in Hilbert space. Given a boundary value problem, a function space is defined with an appropriate metric. The solution of the problem is characterized as the intersection of two orthogonal linear subspaces. This is typified in the Dirichlet problem where one of the subspaces consists of all vector fields obtained as gradients of functions satisfying the boundary conditions, the other subspace consists of all divergence free vector fields. The metric is the Dirichlet integral. By choosing finite sets of particular vectors one constructs a finite dimensional subspace in each of the given subspaces. Minimizing the metric distance between the subspaces determines a pair of vectors, termed "vertices." The solution of the problem lies on a hypercircle having the segment joining the vertices in the space as diameter. This gives upper and lower bounds for the norm of the solution. Knowing the center and radius of this hypercircle, one takes the function associated with the center as an approximation to the unknown solution with error measured in the sense of the metric by the radius. Increasing the dimension of the subspaces reduces the error and improves the approximation.

In certain problems, such as determining the torsional rigidity of a prismatic bar the norm of the solution function is itself of interest. This can be accurately determined with precise error estimates. The function associated with the center of the hypercircle can be evaluated to yield pointwise approximation in the physical domain to the solution, e.g. in the torsion problem to yield approximate stress and warping values. Although the pointwise accuracy of the approximation at any stage is unknown, convergence in the sense of the metric implies pointwise convergence. However, the method can be extended to determine point bounds on the solution and on its derivatives. This is done by bounding the projection of the solution vector on a specially chosen vector which plays the role of a Green's function, although no singularity appears.

Turning to the practical computational problem the author has devised a systematic way of constructing subspaces of increasing dimension so as to contain functions which approximate the solution and its first derivatives to any desired accuracy. These functions are termed "pyramid functions." For a boundary value problem in a domain in two dimensions, such a pyramid function corresponds to a polyhedron constructed over a triangularization of the plane domain with the vertices of the polyhedron located above the vertices in the domain.

The book is divided into three parts. In Part I, elementary properties of linear function space are explored which do not depend on metric.

Part II is devoted to the consequences of the introduction of a positive definite metric and occupies the bulk of the book. In this part the hypercircle method is developed, together with the computational method of the pyramid functions. The results are applied to various boundary value problems of mathematical physics beginning with the Dirichlet and associated problems in potential theory. Particularly detailed calculations are carried out for the problem of determining the torsional rigidity and stress and warping of regular hexagonal and hollow square sections. Excellent bounds are obtained. As examples of mixed boundary value problems the author carries out calculations for the flow of a viscous fluid through a semi-circular channel and the deflection of an elastically supported triangular membrane under uniform pressure. Other boundary value problems for which the machinery of the hypercircle method is set up are those of equilibrium of a three dimensional elastic body and problems associated with the biharmonic equation in hydrodynamics and elasticity.

Part III considers the implications of an indefinite metric in the function space. The hypercircle becomes a "pseudohypercircle" and due to the presence of non-vanishing null vectors in the space, reducing the radius of the hypercircle to zero will not insure convergence of the approximating functions to the solution. However, in those cases where the metric is the difference of two positive-definite mettrics, as in Minkowski space-time, projections of the hypercircle on the "space-like" and "time-like" subspaces yield certain bounds. Minkowski type metrics arise in problems of vibration. The method of the hypercircle is discussed for the cases of forced elastic and electromagnetic vibrations.

The book is written in the characteristically lucid and interesting style of the author. It is intended for the student as well as the specialist and for the engineer or physicist as well as the mathematician. The pace is leisurely and considerable effort and ingenuity are combined to make the geometry of function space as familiar to the novice as ordinary Euclidian 3-space. There are a good number of interesting problems to be worked out by the reader.

The power of the hypercircle method as seen by the author lies in its wedding of geometry to analysis enabling one to visualize and use intuition to suggest both valid theorems and their proofs. In his introduction, the author refers to other treatments of the problem of bounding solutions of boundary value problems which proceed without diagrams or geometrical ideas. These he recommends to readers who "prefer to take their analysis neat." This reviewer, however, heartily recommends to all readers the stimulating mixture which Professor Synge has concocted for us.

H. J. GREENBERG

Nonparametric methods in statistics. By D. A. S. Fraser. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. x + 299 pp. $8.50.

The title of this book is somewhat misleading; the reader who wants a comprehensive collection of nonparametric statistical methods will not find it here. The author's aim seems rather to have been to give a theoretical treatment of those nonparametric techniques which admit some theoretical justification, and in the state of knowledge extant when the book was written this meant excluding mention of many important problems and many useful and commonly employed techniques.

In the first two chapters (almost a half) of the book, the author gives a general introduction to statistical inference which for the most part follows lines of development similar to those employed by E. L. Lehmann in his mimeographed Berkeley notes. Many of the examples here are the standard parametric ones. Thus, the reader will find an introduction to statistical decision theory and such topics as sufficiency, unbiasedness, and invariance in estimation and testing hypotheses. However, the reader who believes the prefatory remark that the calculus and Hoel are the only prerequisites, may find the rapid measure-theoretic excursion of Chapter 1 overwhelming; and the professional mathematician may be a bit disturbed that the measure-theoretic care is dispensed with at the start of the next chapter and that most of the hard and interesting proofs are omitted while those given are usually trivial. The omissions can perhaps be justified on the grounds that these generalities are not the main content of the book.

The last five chapters deal with various nonparametric problems and properties of certain procedures, especially tolerance regions, unbiased estimators, and rank tests. Some of the emphasized topics are treated rather completely, e.g., limit theorems concerning procedures of the last two types just mentioned, as well as the common conditional and run tests. As has been mentioned, many important topics are unfortunately omitted entirely; among these may be mentioned the use of the sample quantiles and sample distribution function in estimation and testing hypotheses. The χ^2 techniques receive only a brief historical mention.

Problems at the end of chapters supplement and extend the text. Within the framework of the latter, they should be instructive to students.

J. KIEFER

An introduction to matrix tensor methods in theoretical and applied mathematics. By Sidney F. Borg. J. W. Edwards, Publisher, Inc., Ann Arbor, Michigan, 1956. 202 pp. $4.75.

The reviewer could find no excuse for the publication of this book, which contains many examples of erroneous thinking in both mathematics and physics. To give only one example, on page 61 the following statement can be found. "In general, the requirements that a function $u(x,y)$ be continuous at a point (x_0, y_0) are that all partial derivatives be finite and continuous at (x_0, y_0) and that the remainder term of the Taylor Series approach zero as the number of terms increases."

R. T. SHIELD.

Economic models: an exposition. By E. F. Beach. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xi + 227 pp. $7.50.

For the non-genius, a knowledge of mathematical techniques is a sine-qua-non for work in economic theory; and it is highly desirable for the economic theorist to learn some pure mathematics. To be sure, a genius can get by on his high school plane geometry; but I venture the guess that the complexity of problems is growing at a faster rate than the mental capacity of human beings. As a result, it will soon be necessary for even a genius to learn mathematical techniques. Professor Beach realizes that "many students become interested in economic theory only after they have left the study of mathematics at an early stage some years previously. It is rather difficult for them to return to the study of mathematics when they are rather fully occupied with the requirements of advanced degrees." When the student finishes reading this book, Professor Beach believes, he should know whether he wants to delve deeper into the literature of mathematical economics and statistics.

An applied mathematician will not learn economics or mathematical economics from this book. If he is innocent of statistics, he could read part two with profit. The author provides a lucid and interesting introduction to sampling theory, simple and multivariate regression theory and the book concludes with a very useful chapter on the Cowles Commission's Simultaneous Equations Approach. Koopmans' famous article on Identification Problems in Economic Model Construction is discussed in this chapter, and it can be read with profit by many students. A bibliography is provided for those whose appetites for statistics have been whetted.

The cardinal deficiency of this book is that it fails to concentrate on a few economic problems and show how mathematical techniques enable the economists to derive a more profound understanding of the economic processes at work. For example, Beach tells the student that Hicks and Modigliani enabled economists to understand Keynes' contribution more fully, when they translated his model into mathematical terms. Beach then writes down their equations, but neglects to show the student how the mathematical model is more revealing than the literary model. Hicks translated Keynes into mathematics and then described his mathematical results in terms of two dimensional geometry; then, Hicks translated the geometry back into literary economics. Thereby, the mathematical reader, the semiliterate mathematical reader and the literary economist could benefit from Hicks' mathematics. Beach, on the other hand, merely writes down Hicks' equations and leaves it there. In the beginning of chapter three Beach interweaves algebra and geometry (familiar to economists) in a lucid and interesting manner. The student is lead back and forth between familiar geometry and unfamiliar algebra; and is thereby taught some algebra. Unfortunately, this method of exposition is not continued.

As the book progresses the exposition deteriorates in quality. The student is given an introduction to the derivative and is taught that $Dx^n = nx^{n-1}$. Then twenty pages later the author begins to solve differential equations, using the term integration and techniques of integration; but he never taught the student anything about an integral or integral calculus. A student who was familiar with integral calculus would not need to study this book; and one unfamiliar with integrals would be mystified by his chapter on continuous dynamic models.

Parts of this book are excellent introductions to the use of mathematical techniques in economics. Moreover, it is lucid and interesting. I have no doubt that Beach's major purpose will be satisfied: the student will learn whether or not he wants to learn mathematical economics.

J. L. Stein

Mathematical analysis—a modern approach to advanced calculus. By Tom M. Apostol. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1957. xii + 553 pp. $8.50.

The author properly describes his book by the statement in the preface, ". . . most of the topics which usually fall under the heading 'Advanced Calculus' are treated in this book. The author's aim has been to provide a development of the subject matter which is honest, vigorous, up-to-date, and, at the same time not too pedantic." The book is designed primarily as a text for (pure) mathematics students but is at a level of difficulty somewhat below the usual advanced analysis. The book is written in a clear leisurely style with ample foreshadowing and motivation of the developments even though there is practically no discussion of applications.

G. Newell