BOOK REVIEWS

An introduction to mathematical economics. By D. W. Bushaw and R. W. Clower. Richard D. Irwin, Inc., Homewood, Ill., 1957. xii + 345 pp. $7.00.

As stated in the preface, the purpose of the book is "to provide an introductory account of mathematical methods of economic analysis which is accessible to persons with a limited training in mathematics." Part I ("The Economics"), contains a very readable account of the theory of price determination; its scope is indicated by the following chapter headings: Introduction—Macroeconomic Statics—Macroeconomic Dynamics I: Isolated Markets—Macroeconomic Dynamics II: Multiple Markets—Microeconomics I: The Theory of Consumer Behavior—Microeconomics II: The Theory of Business Behavior—Concluding Generalizations. Part II ("The Mathematics") contains material on functions, graphs, and equations; the elements of calculus; quadratic forms and determinants; difference equations and differential equations; and maxima and minima. While the book aims primarily at introducing economists to the mathematics that is useful in their field, the first part would serve equally well to introduce applied mathematicians to the field of mathematical economics.

W. Prager

The analysis of structures. By Nicholas John Hoff. John Wiley & Sons, Inc., New York and Chapman & Hall, Ltd., London, 1956. x + 493 pp. $9.50.

Theory of structures is presented in this book mainly from the point of view of the principle of virtual displacements and the theorems of minimum potential and minimum complementary energy. Linear and non-linear elastic materials are considered and there is an introduction to plastic limit analysis and to inelastic buckling. Various topics treated on conventional lines are included, e.g. moment-distribution and slope-deflection methods for continuous beams and frames. Mathematical topics are developed from scratch; nothing beyond elementary calculus is assumed, and sometimes not even that. More knowledge of structures is occasionally presumed; e.g. results are quoted without proof from the theories of thin plates, of St. Venant torsion, and of non-uniform torsion.

The book is in four parts, the first giving an exposition of the principle of virtual work and its direct application to structural problems. The second part presents the theorem of minimum potential energy and shows how this can be used to derive differential equations and as the basis for methods of successive approximation (Rayleigh-Ritz and Galerkin). Part Three deals with buckling problems, using the theorem of minimum potential energy to derive differential equations of adjacent equilibrium states, and also illustrating the Rayleigh-Timoshenko procedure. Part Four presents the theorem of minimum complementary energy and includes an introduction to plastic limit analysis. Throughout the book the structural problems chosen for illustration are taken from both the aeronautical and civil engineering fields.

The book should be of enormous interest and usefulness to those working in advanced structural analysis. The main defect of the book, in the reviewer's opinion, is the author's tendency to add discussions of subsidiary topics to such an extent that the main physical and mathematical ideas do not stand out clearly and strongly. As one example, the author's treatment of plastic limit analysis methods is based on rather full discussions of several examples of limit load calculations, using collapse mechnisms and virtual work. But, although the ideas are all exemplified, statements of the two fundamental theorems (furnishing upper and lower bounds, respectively) are not explicitly given. The reviewer believes that the whole book could have been made more readable and more useful as an introductory text by sharper separation of basic concepts from illustrative and explanatory material. Still, these are matters of style and pedagogy; what is most impressive about the book is the obvious mastery of the author of both the practical and theoretical aspects of the subject, and his delight in it.

P. S. Symonds

(Continued on p. 154)

BOOK REVIEWS

(*Continued from p. 146*)

Models of man—social and rational; mathematical essays on rational human behavior in a social setting. By Herbert A. Simon. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xiv + 287 pp. $5.00.

The book consists of a collection of sixteen articles, previously published in various statistics and social science journals, together with introductory material which has been added to clarify the articles' mutual relations and unity. They are grouped under four main headings: I. Causation and Influence Relations; II. Social Processes; III. Motivation: Inducements and Contributions; IV. Rationality and Administrative Decision Making. The collection represents a concentrated effort to bring to bear on various social science theories the powerful symbolism of mathematics, especially mathematics of a non-statistical nature, e.g. algebra and calculus. In view of the considerable range of both subject matter studied and methods employed, the book should be very interesting to anyone concerned with the investigation of social science topics by means of mathematics.

G. MORGAN

Light scattering by small particles. By H. C. Van De Hulst. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1957. xiii + 470 pp. $12.00.

This book is a treatise on the scattering of light in which relatively few phenomena are dealt with. The assumptions made throughout are that the frequency is unaltered by the scattering process, that the scatterers are independent and without any cooperative effect among them, that there is no multiple scattering present. This book is concerned mainly with the scattering theory for a single particle and it gives first of all general theorems for particles with arbitrary size and shape. It is shown that the scattering by any finite particle can be expressed by four amplitude functions which are complex functions of the incidence and scattering directions, and knowing these functions permits calculation of all quantities involved.

The book contains the amplitude functions and cross sections for various particles, and finally it deals with specific applications in a number of branches of physics including various problems in chemistry, meteorology, and astronomy.

ROHN TRUELL

Physical properties of crystals. By J. F. Nye. Oxford University Press, New York, 1957. xv + 322 pp. $8.00.

The purpose of this book (as stated by the author), is to formulate the physical properties of crystals systematically in tensor form. The book is an excellent detailed survey of macro crystal physics; there is no attempt to deal with the quantum theory of solids.

The chapter headings are as follows: (1) The Groundwork of Crystal Physics, (2) Transformations and Second Rank Tensors, (3) Para and Diamagnetic Susceptibility, (4) Electric Polarization, (5) The Stress Tensor, (6) The Strain Tensor and Thermal Expansion, (7) Piezoelectricity. Third Rank Tensors, (8) Elasticity. Fourth Rank Tensors, (9) The Matrix Method, (10) Thermodynamics of Equilibrium Properties of Crystals, (11) Thermal and Electrical Conductivity, (12) Thermoelectricity, (13) Natural and Artificial Double Refraction. Second Order Effects, (14) Optical Activity, plus seven appendices on various topics related to the text.

This textbook is recommended to both physicists and mathematicians interested in crystal physics problems.

ROHN TRUELL

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BOOK REVIEWS

(*Continued from p. 154*)

Cours de Mathématiques. By J. Bass, with a preface by G. Darmois. Masson, Paris, 1956. xi + 916 pp. $24.55.

The comprehensive text is based on the author's courses at the Ecole Nationale Supérieure de l'Aéronautique and the Ecole Nationale Supérieure des Mines in Paris. These courses are designed to acquaint students of engineering with the mathematical tools of the modern engineer.

As a detailed survey of the contents of the volume would far exceed the space available for this review, the following condensation of the table of contents will have to serve as an indication of scope: Linear Algebra (vector spaces, matrices, Hermitian space, tensors); Integrals (sets and integrals, definite integrals, reduction of integrals, numerical quadrature, improper integrals); Functions Defined by Series or Integrals (Fourier series, orthogonal functions, Fourier integral); Curves, Line Integrals (differential geometry of curves, line integrals, application of line integrals to mechanical quadrature); Surfaces, Multiple Integrals (double integrals, change of variables, elements of differential geometry of surfaces, multiple integrals, surface integrals, integral formulas of vector analysis, improper multiple integrals, gamma and beta functions, elements of operational calculus); Analytic Functions (derivative of a function of a complex variable, elementary analytic functions, integration, series, theorem of residues); Ordinary Differential Equations (general properties, linear differential systems, linear differential equations of the second order, Bessel functions, numerical integration); Partial Differential Equations (linear first-order equations, non-linear first-order equations, linear second-order equations, examples (vibrating strings and membranes, sound waves, heat conduction), Laplace equation and Newtonian potential); Appendix (calculus of variations, nomography).

The exposition is clear and easy to follow; results are always stated with useful generality and rigor even if, for the sake of brevity, they have not been derived in full generality.

W. Prager

Science and information theory. By Leon Brillouin. Academic Press, New York, 1956. xvii + 320 pp. $6.80.

This book is a series of essays on topics in the theory of information, in the physics of thermodynamics and fluctuations, and on relations, established or surmised, between these domains of discourse. The essays themselves are based on lectures given by the author to engineers of the International Business Machines Corporation, and upon other lectures and papers by the author.

About one-third of the book, Chapters 1 through 8, is devoted to aspects of the theory of information. The coverage is restricted to the introductory material needed for the author's later discussion, and to some special topics. A comprehensive introduction to the theory is not attempted.

The remainder of the book is devoted to the author's main thesis—that the entropy of physics and Shannon's measure of information are one and the same. His treatment of this thesis is that of the physicist; it is in the same spirit as several briefer studies of the subject by others, studies to which the author makes generally adequate references.

In more detail: The book's first four chapters (50 pages) present in the author's words some of the elements of Shannon's first paper on his theory of communication. ("A Mathematical Theory of Communication" Bell System Technical Journal, v.27, pp. 379–423, pp. 623–656.)

Chapters 5, 6, and 7 (28 pp.) treat the minimal redundancy codes of Shannon and Fano, and the error correcting codes of R. W. Hamming. The author exhibits some interesting alternatives of his own to these latter. Shannon's formula for the capacity of the binary channel with symmetric, independent, and memoryless noise is introduced, but the general concept of channel capacity is not defined.

Chapter 8 (36 pp.) discusses some topics in Fourier analysis which are useful for studying the statistical aspects of electrical communication: the uncertainty relation between time and frequency, the so-called "sampling theorem" for a function whose Fourier transform vanishes except over an interval, (electrical) filters, the autocorrelation of a function (introduced as a time average, not an ensemble average), and the Wiener-Khintchine representation thereof.

Chapters 9, 10 and 11 (38 pp.) present a summary of phenomenological thermodynamics, and discussions, both phenomenological and in the idiom of Gibbs, of Brownian motion and of thermal fluctuations in simple mechanical and electrical systems.

With Chapter 12 (11 pp.), "The Negentropy Principle of Information," begins the author's discussion of his main theme: the principle to which he has given the name just quoted. Chapter 12 enunciates the principle, with illustrations, as a generalization of the Second Law of thermodynamics. Chapter 13 (24 pp.) applies the principle to the special problem raised by Maxwell's Demon, who, by defying the Second Law for almost ninety years, has outraged three generations of physicists. The author exercises this demon by invoking his negentropy principle, in this way going somewhat beyond the more common observation that the demon could not function, at least anthropomorphously, if the local radiation field were in thermal equilibrium with the ensemble he was upsetting. The arguments in these two chapters are a mixture of phenomenological thermodynamics and quantum kinetic theory. The reviewer interprets these chapters as the author's proof that his principle is valid, a proof obtained by showing that the principle is necessary in order to preserve the Second Law, in particular against daemonic attacks.

Chapters 14, 15, and 16 (62 pp.) apply the negentropy principle to problems of measurement in many physical contexts. Again, the methods are a mixture of phenomenology and appeals to the principle of equipartition in kinetic theory.

Chapter 17 (16 pp.) applies the negentropy principle specifically to the physical problem of communicating electrically in the presence of thermal fluctuations. The argument appeals to equipartion; it results in the formula of Shannon for the capacity of a channel with additive Gaussian noise. A definition of channel capacity adequate for this particular case is enunciated.

Chapter 18 (8 pp.) treats problems, troublesome in the light of the author's principle when it is interpreted as a conservation principle, which arise because printed information can be duplicated and somehow multiplied by dissemination. These problems go far beyond the scope of any mathematically formulated theory of information, but seem to yield to the author's phenomenological methods.

Chapter 19 (18 pp.) treats some topics in computing, and Chapter 20 (14 pp.) several unsolved problems, some of which also go far beyond any presently formulated mathematical theory.

Just as there are, broadly, three views of thermodynamics, the phenomenological, that of kinetic theory, and that of Gibbs—so there can be three views of the author's negentropy principle of information. It is difficult for a mathematician to catch the author in a fully satisfactory statement of any of these views. The reviewer will attempt some clarifications.

First, a word about signs. Shannon regards $-\Sigma\, p_i \log p_i$, quite naturally, as a measure of "uncertainty." When this "uncertainty" is eliminated or reduced, "information" results. Both quantities are treated as being intrinsically positive, just as a solvent person who is not an accountant naturally regards both his bank balance, and the pocket money he has just withdrawn from that balance, as intrinsically positive. Brillouin follows this usage with good consistency, and does not display the confusion of some of his successors on the matter. To avoid here any such confusion, interpret "information" as "information obtained as the result of a physical change." When this quantity is positive it represents a withdrawal from a balance of "uncertainty." Therefore "negative information" is, from the point of view of an accountant, "uncertainty."

Phenomenologically, the negentropy principle of information is a postulate, a conservation postulate, in which "negative information" (i.e. "uncertainty" in the reviewer's words above) is introduced as a balance to entropy. In a reversible transformation of a closed system the sum of "negative information" plus entropy then remains constant. At this point, "negative information" appears, like potential energy and the neutrino, as a concept physically necessary to preserve a conservation law. In this way the conservation postulate can be regarded as defining "physical information" (reviewer's term introduced for convenience).

Brillouin chooses to extend this formulation, as is usually done with the restricted form of the Second Law, to an inequality applicable to irreversible processes: he asserts that, in a closed system, the change in entropy-minus-information (entropy plus negative information) is always non-negative. This is his enunciation of the negentropy principle as a generalization of Carnot's principle (the Second Law).

Since physical information is defined above by the conservation law only as a quantity whose negative balances entropy in a reversible change, the author must find some other, independently defined, form of "information" to insert in his generalization of the Second Law. For this, phenomenological concepts do not suffice, and he turns essentially to kinetic theory. He chooses for his information the quantity $\kappa \ln(P_0/P_1)$ where (page 152) P_0 is the "... number of possible cases" existing *a priori*, and P_1 the number of cases which are possible after the physical change has taken place. Both *a priori* and *a posteriori*, the cases possible are postulated to be all equally likely. The numerical factor κ is, quite naturally, Boltzmann's constant (dimensions: energy per temperature).

After this essentially kinetic statement of the negentropy principle, three arguments, or kinds of argument, intertwine. The first is the phenomenological argument of Szilard that the conservation postulated above, (i.e., that physical information is a balance to entropy) is necessary to preserve the Second Law. The second is an argument from kinetic theory defending $\kappa \ln(P_0/P_1)$ as that which balances entropy in this conservation: by this then the author identifies what above was called physical information with concepts from kinetic theory. Finally, there is much use of the language of the theory of communication to associate $\kappa \ln(P_0/P_1)$ with concepts found in that theory.

A complete formulation of the author's negentropy principle in terms appropriate to kinetic theory would be mathematically quite interesting. The subject is too complex for discussion in a book review, but it is evident that the author has missed some of the interesting features. He has, for example, failed to observe that in kinetic theory entropy appears most naturally not as a function of state variables nor as a functional of the ensemble, but as a random variable—in fact, that random variable whose expectation is Shannon's average conditional entropy of the complexion given the macroscopic state. Once one recognizes this, Shannon's theorem on the asymptotic approximate constancy of that variable applies to large systems and can be used to justify the equiprobability that has been assumed by Brillouin, and by all physicists since Boltzmann.

The author omits any discussion of the negentropy principle in terms appropriate to Gibbs' view of thermodynamics. In such terms, the negentropy principle is simply this: in a canonical or grand canonical ensemble of (e.g., quantum) physical systems Shannon's functional $-\Sigma p_i \log p_i$, in which p_i is the probability that the i^{th} state be occupied, is, apart at most from a linear and monotone change of scale, equal to the thermodynamic entropy of that ensemble. In this form, the "principle" is tautologous, and it is false when applied to any but the canonical or grand canonical ensembles. From the Gibbs point of view, then, the interesting physical fact is not the negentropy principle but the universal usefulness of these canonical ensembles.

This book suffers from hasty editing; many of its chapters are clearly derived directly from lecture notes, they overlap in places and often use with brief, or no, explanation ideas introduced more carefully later. More seriously, the author has not defined his intended audience: a wide range of topics is covered, each in some detail and often with some kind of sophistication, but almost no topic is treated adequately for a beginner therein.

The author is generous with his references. He does, however, overlook a particularly precise and satisfying treatment of Szilard's argument that negative physical information (as the term is used in this review) balances entropy. This is the paper "The Well Informed Heat Engine" by Richard C. Raymond (American Journal of Physics, v.19, pp.109–112, 1951; compare its title with Section 6 of Chapter 13).

As it appears in this book the substance "information" has several forms, some of them quite anthropocentric. In the typical argument, more than one form is displayed. There results an impression of elusiveness and the reader is often challenged to be sure that the author has indeed limited all applications of his principle to strictly closed systems. In contrast, the less ambitious treatment by Raymond, just mentioned, is explicit and precise on this point.

To a mathematician, it is clear that the author is a physicist. He exhibits no interest in logical completeness—for example, Shannon's full definition of channel capacity, and his fundamental theorem concerning it, are never mentioned. The author depends throughout upon phenomenological arguments in terms of "gedanken-experimente," and upon the most naive formulations of the mathematical concept of information. Little effort is made to distinguish between random variables and quantities which define their distributions. Sample spaces, when they are mentioned at all, are specified only in the loosest terms; this latter despite the author's own admission that "information" depends explicitly upon the sample space i.e., upon the ensemble of *a priori* possibilities.

This reviewer feels that the author has a valid message and that a negentropy principle of information could be made mathematically precise. He regrets that, in failing to make the principle precise, Brillouin's book has added the considerable weight of its author's support to an already flourishing mystical school of information theory.

B. McMillan

Quantum chemistry—an introduction. By Walter Kauzmann. Academic Press, New York, 1957. xii + 744 pp. $12.00.

The outstanding feature of this new introductory text on chemical quantum mechanics is its excellent mathematical section, which is much more extensive than in most books of this sort. After presenting a certain amount of necessary background material, the author goes into a detailed treatment of vibrating systems, starting with the vibrating string, and continuing on to cover membranes of various shapes, the uniformly flooded planet, and three dimensional continua. In the course of this discussion all the basic mathematics of quantum mechanics, including perturbation theory and the variational method, is developed in terms of physical situations relatively familiar to the beginning student.

The basic principles of quantum mechanics are presented next in a purely postulational manner which, while of course unassailable from a logical standpoint, is unlikely to appeal to most readers encountering this material for the first time. Following this there are two large sections on atomic and molecular quantum mechanics, well done in a more or less conventional fashion. A regretable omission here is the absence of any presentation of group theory, although symmetry considerations are extensively used.

The final part of the book is devoted to non-stationary situations, in particular the interaction of atoms and molecules with electromagnetic radiation, discussed in considerable detail both from classical and quantum viewpoints. Here there is a good but somewhat lengthy treatment of optical rotation, given apparently at the expense of equally important topics such as the Raman effect, which is not mentioned at all.

The exercises scattered throughout the text deserve especial mention. They are entirely original, workable without being trivial, and well chosen to clarify the accompanying material.

On the whole, in spite of the few criticisms offered in this review, this is probably the best beginning text on quantum chemistry currently available.

STEPHEN PRAGER

Viscous flow theory II—turbulent flow. By Shih-I Pai. D. Van Nostrand Co., Inc., New Jersey, 1957. xi + 277 pp. $6.75.

Turbulence represents a field of fluid mechanics which is both very important and very difficult. The literature on the theory of turbulent flows ranges therefore from subtle mathematical studies to the crudest approximations. Experimental work on turbulence covers a similar range. To penetrate from the outside into the field of turbulence research through reading of original papers is difficult and the non-specialist or student trying to do this must find the literature very confusing indeed.

A text which covers the elements of turbulence research and the applicable results so far obtained is thus very much needed. According to the table of contents of Dr. Pai's book it seems to fill at least part of this need. After closer study however I came to the conclusion that this is not the case and that the book in its present form will add to the confusion rather than help to eliminate it.

The fundamental shortcoming in the book is the lack of a personal point of view of the author and of a definite scientific standard. Work with high mathematical standards and little physical insight, work with much physical insight and less rigor, and crude ad hoc computations to check in one way or other experimental data are compiled indiscriminately. Thus in the chapters on the statistical theory of turbulence the author insists on using Lebesgue instead of Riemann integrals, and worries about an ergodic theory. In computing shear flow problems on the other hand, computations which amount to little more than interpolation formulae for a specific set of experiments are presented as "theories" (e.g. Laufer's measurements on pipe and channel flow).

Reading the book I recognized chapter by chapter the impact of different personalities and schools of turbulence research which the author has neither assimilated into his own way of thinking or even style of writing, nor put into the proper context. The use of experimental results is equally unsatisfactory because experiments are used only to "verify" computations, and sometimes single experimental curves are taken out of their original context, thus defeating the purpose of the original work. Measurements on the turbulent boundary layer at high Mach numbers are cited in one sentence: "Most of the experimental values for the skin friction coefficient agree within about 10% with the theoretical values of van Driest, Wilson and Rubesin et al." Not long ago there existed over twenty different theories for the turbulent boundary layer at high Mach number and the choice of the one "most likely to succeed" is entirely based on a few sets of first rate experiments like Coles' and Chapman and Kester's. The author does not seem to realize that good theory like a good experiment can stand on its own feet and that agreement between the two does not improve a bad theory or a poor set of experiments.

A number of misleading and meaningless statements which appear throughout the text can be traced to the same basic cause, i.e. the failure to assimilate opinions of others within a unifying point of view; e.g. (p. 20): "The law of heat loss (of a hot wire at high speeds) is non-linear while that for low speed flow, King's law, is linear." Linear in what quantity?—(p. 109): "The fact is that neither the turbulent Prandtl number nor the laminar Prandtl number are unity but rather about 0.75 for air." *Is* the turbulent Prandtl number 0.75?—(p. 165): "The joint-probability distribution of fluctuating velocities at two points and the probability distributions of the derivatives of the velocity components are not normal, due to the fact that these quantities satisfy the Navier-Stokes equation; hence they are not entirely random in nature." Cannot statistics be applied to any mechanical syst m? This list could be extended. Every one of these statements can be interpreted correctly by somebody familiar with the subject matter. For an outsider loose sentences like these must be puzzling and for a beginner utterly confusing. Here a good deal of editing would have helped.

As far as the selection of subject matter is concerned the overall choice of the chapters is in my opinion quite good. However everywhere one finds detailed manipulation of equations rather than critical and constructive explanations of why certain assumptions make sense and others not. Thus four different approaches to turbulent boundary layer theory at high Mach number are given in great and cumbersome detail; on the other hand one of the most fruitful concepts of recent years, Clauser's "equilibrium layer," is barely mentioned. I also miss Donaldson's simple and intuitive approach to Mach number effects on turbulent skin friction. In general the simple and general laws which would be most helpful to the reader are not explained: e.g. the law of the wall, the linear stress-distance relation in pipe and channel, the similarity solution for free turbulence. All of these *are* there but they are smothered under lengthy manipulations of equations. Here too, I believe, one notices again that the book is assembled rather than written.

Summarizing, I regret that I cannot recommend the book as it stands since it will tend to give the reader a distorted and often misleading impression of what turbulence research is and what has been achieved. To read up on turbulence the student still has to be referred to Goldstein's "Modern Developments" and to some survey papers and original work until he is ready to read Batchelor's and Townsend's recent monographs.

H. W. LIEPMANN

Applied probability. Proceedings of Symposia in Applied Mathematics. Vol. VII. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1957. v + 104 pp. $5.00.

This volume contains nine papers presented at the seventh symposium in Applied Mathematics of the American Mathematical Society. The intention of the symposium was to explore three principal subjects, the theory of diffusion, the theory of turbulence and probability in classical and modern physics. No attempt is made to review these very extensive subjects although a few papers review special topics and others deal with problems of more recent origin than the above titles might suggest. The papers describe work of the high quality that would be expected from the following very impressive list of authors: Paul Lévy, J. L. Doob, William Feller, Eberhard Hopf, Guido Münch, G. K. Batchelor, Mark Kac, S. M. Ulam, and B. O. Koopman.

G. F. NEWELL

Relaxation methods in theoretical physics, Vol. II. By R. V. Southwell. Oxford University Press, New York, 1956. 274 pp. $8.80.

This book presents the latest developments in the solution of boundary value problems in partial differential equations by the numerical, hand computation scheme originated in the 1930's by the author. The book is a second volume on this subject, the first volume of which appeared in 1946. These two volumes, together with the book "Relaxation Methods In Engineering Science" which appeared in 1940, present essentially all that has been accomplished in development of the method up to 1955.

The method of solution basically is to convert the differential equation to a finite difference net and to assume a numerical value as the solution at each net point. These incorrect values are then changed at one point at a time in such a way that the successive values approach the correct ones.

In the present volume, as in the past ones, the methods of carrying out the solution in detail are discussed by specific examples. In large part the illustrations are taken from elasticity; stresses and deflections of plates and solids of revolution, vibrations of plates and membranes, and elastic stability. A discussion of electromagnetic oscillations forms an obvious application of some of the same methods. As an example the lower modes of oscillation of a Klystron tube are computed.

Chapters XIII and XIV (Vol. II contains chapters VII to XV) deal with particularly difficult non-linear problems; large deflections of plates, the flow of viscous fluids and plastic strain.

The final chapter makes brief mention of a number of recent, only partially developed ideas. Some three dimensional problems are solved and methods of applying relaxation techniques to parabolic and hyperbolic type equations by converting them to elliptic equations of higher order are noted.

The illustrative problems and many of the ingenious tricks of solution are taken with acknowledgement from a number of workers. Many of these are former students of the author and have collaborated with him in making the present as well as the former volumes possible.

For many problem solvers who have become accustomed to think in terms of the large scale computation machine, the relaxation method may appear like a good idea that arrived a century too late. From the nature of the problems solved, it is clear that considerable time must be spent with a desk computer to find them. However, there are many people who for years will not have a readily available large scale computing machine. Furthermore, some of the problems solved have a complexity calling for judgment during solution which is not yet available in a computing machine. At present, of course, computing machines sometimes use an identical finite difference net with an iterative method of solution. If a future machine is ever built which can scan a field of numbers as much faster than it can compute as is true of the human computor, the relaxation method will emerge as a most significant development and the present volume will be one of the classics.

H. W. Emmons

Digital computer programming. By D. D. McCracken. John Wiley & Sons, Inc., New York, 1957. vi + 253 pp. $7.75.

A first book on any subject is always welcome, even if it did not fulfill its purpose as admirably as does the book under review.

It is elementary, to be sure. A book is still to be written (or, at least, published) on modern high speed computing methods applied to advanced problems in numerical analysis. But there has not even been available before a systematic introduction to the art of coding for an automatic stored program computer. The computer on which we learn to solve problems is TYDAC, a mythical computer whose approximate counterpart in real life the experts will easily be able to guess: it is a decimal machine with 2000 words high speed storage, index registers and four tapes. The discussion includes number systems, fixed and floating decimal point methods, address computation with and without index registers, loops, subroutines, flow charts, input-output methods, and all too short an anatomical discursion into interpreters and compilers. The explanations are uniformly lucid, the presentation is systematic and purposeful, the layout excellent.

As a textbook for a first programming course and as a compendium to numerical analysis seminars the book is heartily recommended.

W. F. Freiberger

Automatic digital computers. By M. V. Wilkes. John Wiley & Sons, Inc., New York, 1956. x + 305 pp. $7.00.

The contents of this volume provide a general introduction to the principles underlying the design and use of digital computers. After a historical introduction the author, who writes with authority of many years pioneering effort in the field, presents the principles of logical design and of program construction. The latter are illustrated by means of the order-code of the Cambridge EDSAC. Storage, electronic switching and computing circuits, design and operation of digital computer, both relay and electronic, are discussed in considerable detail.

The book is by no means easy to read, but the expert will find in it a wealth of information otherwise hard to come by. The solution of particular problems is not discussed—the application of computers to the solution of advanced problems in numerical analysis is, by self-imposed limitation, outside the scope of the work which is thus of relatively little interest to mathematicians. Its value lies in giving an up-to-date picture, at the time of writing, of the state of computer design on both sides of the Atlantic.

W. F. FREIBERGER