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CONTENTS

BOOK REVIEWS

Plane waves and spherical means applied to partial differential equations. By Fritz John. Interscience Publishers, Inc., New York, 1955. viii + 172 pp. $4.50.

The present tract contains a collection of various results on partial differential equations, their unifying thread being the use of certain elementary identities for plane and spherical integrals of an arbitrary function. The author's principal aim is to show that many results on fairly general differential equations follow from those identities. The book begins with a concise introduction, which illuminates the paths to be followed. Chapter I deals with the decomposition of arbitrary functions into functions of the type of plane waves, i.e. into functions having parallel planes as level surfaces. The decomposition used here consists of expressing a function by spherical means of integrals over hyperplanes, and is due to J. Radon. This kind of decomposition is to be compared with that furnished by Fourier analysis (into plane waves of exponential type). The "Randon transform" has the advantage over the "Fourier transform" that the integrals appearing in it involve the given function itself, and not its Fourier transform. In Chapter II the Randon transform is applied to the solution of the Cauchy problem for the homogeneous hyperbolic equation with constant coefficients $L(u) = Q(\partial/\partial x_1, \cdots, \partial/\partial x_n, \partial/\partial t)\, u = 0$, where $Q(\eta_1, \cdots, \eta_n, \lambda)$ is a form of degree m in its arguments, with constant real coefficients; with the initial data $(\partial^k u/\partial t^k)_{t=0} = f_k(x_1, \cdots, x_n)$ for $k = 0, 1, \cdots, m - 1$. Chapter III gives a construction of a fundamental solution for a linear elliptic equation, and for a system, with analytic coefficients. The problem amounts to finding a solution of the symbolic equation $L(u) = \delta$, where δ is the Dirac function; and the method of solution employed is that of decomposing the δ function into plane wave functions, thus reducing the problem to that of solving $L(u) = f$, where f is a plane wave function. In Chapter IV are found formulas for an arbitrary function in terms of spherical integrals of the function. These formulas, which form the principal tool used in the later chapters, can be regarded as generalizations of the expressions of Chapter I, which give the arbitrary function in terms of its integrals over planes. They can also be viewed "as the analytic counterpart of the geometrical fact that spherical shells can be swept out by spheres in two different ways, just as the identities of Chapter I are connected with the fact that the exterior of a sphere can be swept out by planes". Chapter V is concerned with the mean value theorem of Asgeirsson for the ultra-hyperbolic equation, together with a somewhat more general identity, having to do with the ellipsoidal means of a function, due to A. S. Howard. Chapter V deals mostly with the problem of the determination of a function from a knowledge of its integrals over a sphere of fixed radius, which can be solved by means of the identities of Chapter IV. Chapter VII contains the major application of the identities on spherical means in Chapter IV, to the proof of the differentiability of solutions of linear or non-linear elliptic equations or systems, provided that the coefficients are sufficiently regular. Chapter VIII extends the results of the preceding chapter to non-linear elliptic equations, save that here the regularity not of the solutions themselves, but that of certain integral transforms of them is established. The lucidity of the exposition combines with the attractive format of the book to make it the delight of mathematicians interested in the subject.

J. B. DIAZ

Recent advances in science (Physics and Applied Mathematics). Edited by M. H. Shamos and G. M. Murphy. New York University Press, 1956. xi + 384 pp. $7.50.

The book is an outgrowth of the First Symposium on Recent Advances in Science held at New York University in the spring of 1954; it contains the following articles: Methods of Applied Mathematics (R. Courant); The Future of Operations Research (P. M. Morse); Atomic Structure (I.I. Rabi); Microwave Spectroscopy (C. H. Townes); Nuclear Structure and Transmutations (H. A. Bethe); Elementary Particles (V. F. Weisskopf); Electronuclear Machines (L. J. Haworth); Neutron Physics (N. F. Ramsey); Transistor Physics (W. Shockley); Ferromagnetism (R. M. Bozorth); Cryogenics: Very-Low Temperature Physics and Engineering (F. G. Brickwedde); Physics and the Engineer (E. U. Condon).

(*Continued on p. 32*)

BOOK REVIEWS

(*Continued from p. 20*)

Gas dynamics. By Klaus Oswatitsch. English version by Gustav Kuerti. Academic Press, Inc., New York, 1956. xv + 610 pp. $12.00.

The rapid development of supersonic aerodynamics during the past twenty years has created a need for authoritative text books. Since most people working in this field have been engaged very fully on highly specialized problems and since so many and varied contributions have been made to the subject it was inevitable that the first leading works of reference should have been prepared by teams of writers rather than by individuals. In the circumstances it was unavoidable that these works, excellent though they are, should suffer from a non-uniformity in style and, owing to difficulties in liaison, should have been partially outdated at the time of their publication.

Dr. Oswatitsch has therefore performed a great service in preparing the present volume single handed. He provides us with the opportunity of covering the complete range of Gas Dynamics in a style which is even, clear and easily followed throughout. The work is of considerable scope and covers almost every important aspect of the subject. While the treatment is thorough and rigorous the emphasis is on explaining the many interesting phenomena occurring in Gas Dynamics, without introducing complications due to Mathematical details or laboring particular practical applications. As the author himself states this is not an Engineering handbook, nor, it might be added, is it a Mathematical treatise. The aim is clearly to give an understanding of the physical character of the subject and this is certainly accomplished.

The book follows a logical order. Thermodynamic principles and one dimensional models of gas flows precede chapters on general equations and theorems. Three chapters follow on steady subsonic, steady supersonic and steady transonic flow respectively. This last chapter is of especial interest since it not only contains a much tidier account of recent work in the United States than has appeared elsewhere but also present the author's own approach to this subject with a clarity which the reviewer missed in earlier versions. After a fairly complete chapter on linearized theory the book concludes with a chapter on viscous effects and a chapter on experimental methods.

Dr. Kuerti is to be congratulated on his English translation of the book. Unlike some earlier English versions of German text books, which were often harder to follow than in the original, the present work reads as though it has been prepared in English. This is evidently the result of the translator taking the trouble to understand the whole contents and of his close collaboration with the author.

The book is very well set out; it is liberally illustrated with diagrams and remarkably clear photographs and the list of references is excellent both in its completeness and in being up to date. It would serve as a first rate text book in a graduate course but is also readily understood by the student working without supervision. It is strongly recommended to all those engaged in work on high speed aerodynamics.

M. Holt

Abacs or nomograms. An introduction to their theory and construction, illustrated by examples from engineering and physics. By A. Giet. Translated and revised by J. W. Head and H. D. Phippen. Iliffe & Sons, Ltd., London; Philosophical Library, New York, 1956. $10.00.

This book was originally published by Dunod in 1954 under the title "Abaques ou nomogrammes". The scope is indicated by the following chapter headings: Relations between two variables—Cartesian abacs—Alignment charts—Alignment charts not based on parallel coordinates—Relations between n variables. The mathematical treatment is elementary, avoiding even the use of determinants, but all important types of nomogram are thoroughly discussed and illustrated by numerous examples.

W. Prager

(*Continued on p. 46*)

BOOK REVIEWS

(*Continued from p. 32*)

Elements of partial differential equations. By Ian N. Sneddon. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1957. ix + 327 pp. $7.50.

In the author's own words: "the aim of this book is to present the elements of the theory of partial differential equations in a form suitable for the use of students and research workers whose main interest in the subject lies in finding solutions of particular equations rather than in the general theory." An attempt has been made to touch upon subjects from the entire field of partial differential equations. There are six chapters in all. Chapter 1 is introductory in character and deals with ordinary differential equations in more than two independent variables. Of special interest here is the treatment of Pfaffian differential forms, Carathéodory's theorem on integrability, and its application to thermodynamics. Partial differential equations of the first order occupy Chapter 2. Among the subjects covered are Cauchy's problem for first order equations, linear and nonlinear equations, Cauchy's method of characteristics, Lagrange's and Charpit's methods and applications, for example to stochastic processes and to birth and death processes in bacteria. Chapter 3 is concerned with a preliminary discussion of second (and higher) order equations, preparatory to the last three chapters, which consider in more detail the three main types of second order equations. One finds a treatment of linear partial differential equations with constant coefficients, canonical forms for second order equations, characteristics of equations in two and three independent variables, Riemann's method of integration, and integral transforms. Chapter 4 deals with Laplace's equation, the prototype of the elliptic second order linear partial differential equations. The Dirichlet, Neumann and mixed boundary value problems are dealt with, Green's functions, Kelvin's inversion theorem, and allied topics. The typical hyperbolic equation, the wave equation, constitutes the subject matter of Chapter 5. One, two, and three space dimensions are considered; the Riemann-Volterra solution of the one dimensional equation, and Helmholtz', Weber's and Kirchhoff's general solutions in two and three dimensions. There is a brief introduction to Marcel Riesz' method of solution, based on Riesz' generalization of the Riemann-Liouville integral. The concluding Chapter 6 has the heat equation, the typical parabolic equation, as its topic. There is an appendix on systems of surfaces, meant to provide a brief outline of some of the properties of systems of surfaces used in Chapter 2. The use of this book as a text is facilitated by the presence of a large number of problems, together with the solutions of all odd numbered ones.

J. B. Diaz

The plastic methods of structural analysis. By B. G. Neal. John Wiley & Sons, Inc., New York, 1956. xi + 353 pp. $7.50.

"The fundamental defect in the orthodox elastic design technique can now be exposed. It has been seen that a simply supported beam carrying a uniformly distributed load . . . has a load factor against collapse of 1.75, whereas if a similarly loaded but fixed-ended beam were designed elastically to the same working stress its load factor against collapse would be 2.34." With these words midway through the introduction, Professor Neal embarks on a careful and detailed study of the status of beams and frames. His own work and the work of former colleagues at Cambridge and at Brown play a central role in the text along with reference to the many other workers in this country and abroad. The book is intended as a step by step guide through most of the complexities of the actual calculation of collapse loads, deflection estimates, effects of shear and axial force on limit moment, minimum weight design, and problems of shakedown.

Mathematics is kept to the barest minimum even in the proofs of the theorems which are relegated to the Appendices. Except for providing the necessary warnings about the possibility of buckling, Bauschinger effects, non-homogeneity of material and similar difficulties, the author is not deflected in his purpose. He demonstrates the power and the relative simplicity of plastic methods of analysis for frames encountered by the practicing engineer who should read all of the text and check his solution to the problems with the answers in the back of the book.

D. C. Drucker

(*Continued on p. 72*)

BOOK REVIEWS

(*Continued from p. 46*)

Matrix calculus. By E. Bodewig. North-Holland Publishing Company, Amsterdam; Interscience Publishers, Inc., New York, 1956. xi + 334 pp. $7.50.

The author feels that the usual matrix notation does not exhibit "the true building blocks of a matrix, the rows and columns", and therefore causes "discrepancy between thought and calculation, and lack of elegance". To remedy this situation, the author introduces the following notations: $A_{i.}$ = i-th row, and $A_{.j}$ = j-th column of matrix A; e_i = column vector whose i-th element is unity while all others vanish; E_{ij} = matrix having the element at the intersection of row i and column j equal to unity and all other elements equal to zero. Thus, $A_{.i} = Ae_i$ and $A_{i.} = e_i'A$, where the prime indicates the transpose; $a_{ij} = e_i'Ae_j$. These notations enable the author to represent operations explicitly that otherwise would have to be described verbally. For instance, $A(I + cE_{ij})$ is the operation that adds the c-fold of the i-th column of A to the j-th column and leaves all other columns unchanged.

The book consists of four parts, the first of which provides an excellent survey of the general theory (85 pp.). The remaining parts are concerned with specific problems of computation, each part containing sections on direct and iterative methods. Part II (96 pp.) is devoted to Linear Equations, Part III (48 pp.) to Inversion of Matrices, and Part III (98 pp. to Eigenproblems).

While often highly concise, the exposition is always clear and easy to follow, though the English is occasionally awkward. Examples of a new approach to a familiar problem abound throughout the book. In this connection, Section III, C on Geodetic Matrices merits particular mention. The author's frequent remarks on the suitability of numerical methods for different types of computing equipment will be welcomed by many readers.

W. Prager

Nuclear reactor physics. By Raymond L. Murray. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1957. xi + 317 pp. $10.00.

This book is a very clearly written text on the fundamental features of reactors with emphasis on neutron physics and neutron flux distribution, on transient effects in reactors, and on the temperature effects on reactor operation. Included are chapters on reactor control, transport theory, and the slowing down of neutrons.

The author has included sufficient mathematical treatment to make the discussion useful and interesting but not so much that only the specialist would be interested. The book seems to have excellent balance between what a student of reactor fundamentals might need and what a well trained physicist or mathematician might want to know without becoming a specialist. The book seems excellent for those who simply want to know what problems enter into reactor design.

Rohn Truell

The International Dictionary of Physics and Electronics. D. Van Nostrand Co., Inc., Toronto, New York, London, 1956. xvii + 1004 pp. $20.00.

This book is an ambitious undertaking (even for fifteen editors) amounting to one thousand pages of laws, equations, definitions, principles and concepts of physics. The objective as stated by the editors has been to provide a book useful as a general reference in physics. This work will undoubtedly be useful to many people especially to those who seek information outside their field of specialization.

Rohn Truell

(*Continued on p. 94*)

BOOK REVIEWS

(*Continued from p. 72*)

The mathematics of diffusion. By J. Crank. Oxford University Press, New York, 1956. vi + 347 pp. $8.00.

This is the first book on the diffusion-heat conduction equation to appear since the publication of the well known work by Carslaw and Jaeger. Almost all the situations treated involve the transport of matter rather than of heat, and many show a number of features, such as concentration dependence of the diffusion coefficient, which are normally unimportant in the corresponding heat conduction problems. The diffusion processes are for the most part one dimensional, although quite a few cases with spherical and cylindrical symmetry are presented.

The first six chapters constitute a necessary review of fairly standard diffusion problems, but from this point on most of the material has never been presented in book form before; indeed, much of it is from the author's own publications. The chief topics discussed are moving boundary problems, diffusion with simultaneous chemical reaction, concentration-dependent diffusion coefficients, and numerical methods. In addition there is a chapter dealing with the different kinds of diffusion coefficients measured by various experimental procedures, plus one on the somewhat specialized problem of coupled diffusion and heat conduction in sheets of wool fiber.

One of the best features of the book is the inclusion of a very large number of plots of concentration-distance and sorption-time relationships; these greatly aid the reader's intuition, and many of them should be useful in actual computations. Also mentioned should be the appendix, which contains tables of a variety of useful quantities, such as integrals of the error function, as well as sample computation sheets for some of the numerical methods presented in the text.

If there is anything to be aid by way of criticism, it is not about the material which has been included, but rather about that which has been left out. Thus, coupling between the diffusion of different species in multicomponent systems is not mentioned, and no problems involving convection or external forces are treated. Perhaps the inclusion of these and other topics would have made the book excessively long. In any case, it is a valuable contribution as it stands, and no one engaged in or contemplating diffusion measurements should be unfamiliar with its contents.

STEPHEN PRAGER

Introduction to finite mathematics. By John G. Kemeny, J. Laurie Snell, and Gerald L. Thompson. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1957. xi + 372 pp. $5.00.

Here is a book which can safely be described as unique among college mathematics texts.

In recent years there has been an increasing demand for "terminal" courses in mathematics, running for one or at most two semesters and intended to provide appropriate material for students who may not intend to go further in mathematics. The usual introductory analytic geometry and calculus course is from some points of view unsatisfactory for this purpose, firstly, because one or two terms is hardly enough time to get started in the subject, and secondly, the material lacks motivation without a considerable amount of application to the physical sciences. "Introduction to Finite Mathematics" overcomes both of these difficulties, first, by taking for its mathematical subject matter not analysis but "finite mathematics," (actually certain branches of algebra), and second, by choosing motivating material from the social rather than the physical sciences, thus dealing with matters which are close to the students' every day experience.

In the authors' own words the book's purpose is "to introduce a student to some concepts in modern mathematics early in his college career. While primarily a mathematics course, it was to include applications to the biological and social sciences and thus provide a point of view, other than that given by physics, concerning the possible uses of mathematics. Our aim was to choose topics which were initially close to the student's experience, which are important in modern day mathematics, and which have interesting and important applications."

As to the somewhat mystifying title of the book, the authors explain "our purpose . . . was to develop several topics from a central point of view. In order to accomplish this on an elementary level, we restricted ourselves to consideration of *finite* problems, that is, problems which do not involve infinite sets, limiting processes, continuity, etc."

The program of the book in very brief outline, is the following: The first chapter "Compound Sentences," deals with the elementary propositional calculus, the central idea being that of truth tables. Also in this chapter the notion of the "logical possibilities" associated with a statement is introduced. This concept which appears to be a device invented by the authors (we return to this later) provides a link between the first and second chapter entitled "Sets and Subsets." Its main concern is the Boolean Algebra of sets and its relation to the propositional calculus of the previous chapter. The set notion leads naturally to Chapter III, "Partitions and Counting," which is concerned with elementary combinatorics, permutations, etc. The material of all the preceding chapters is brought together in Chapter IV, "Probability Theory," which, however, is quite a departure from the usual treatments of this subject in first year texts as indicated by the presence of sections entitled "Finite Stochastic Processes," "The Law of Large Numbers," "Markov Chains." This last section leads to Chapter V on "Vectors and Matrices" which, after introducing the standard terms and operations, applies them to the theory of Markov chains. These five chapters comprise the "basic core" of the book and represent material which could be conveniently covered in one college semester. The book concludes with two "optional" chapters, (so indicated by an asterisk). The first entitled "Linear Programming and the Theory of Games," contains an elementary treatment of some aspects of these two subjects. The final chapter "Applications to Behavioral Science Problems" treats five problems, "selected for their interest both to mathematicians and to behavioral scientists. One topic was chosen from each of five sciences; sociology, genetics, psychology, anthropology, and economics."

This book represents such a radical departure from traditional college texts that it would take considerable space to give it a really adequate review. Perhaps some notion of its unusual flavor can be conveyed by listing a few of the many (over 1000) varied and excellent problems which appear at the ends of each section.

In a starred (optional) section of Chapter I on "Statements Having Given Truth Tables" the student is lead in a series of exercises to the proof that every logical connective can be expressed in terms of "neither . . . nor." In a section on "Valid Arguments" the authors develop the theory of the syllogism so the student has a systematic procedure for checking the validity of arguments like "Father praises me only if I can be proud of myself. Either I do well in sports or I cannot be proud of myself. If I study hard then I cannot do well in sports. Therefore, if Father praises me then I do not study hard."

The final section (starred) of this chapter is on "Application to Switching Circuits". Problem: "A committee has five members. It takes a majority vote to carry a measure except that the chairman has a veto. Design a circuit for the committee, so that each member votes for a measure by pressing a button, and a light goes on if and only if the measure is carried."

In the chapter on sets the following problem is typical. "In a survey of 100 students the number studying various languages were found to be: Spanish, 28; German, 30; French, 42; German and French, 5; all three languages, 3. (a) How many students were studying no language? (b) How many students had French as their only language?" etc. In the chapter on partitions we find, "On a transcontinental airliner there are 9 boys, 5 American children, 9 men, 7 foreign boys, 14 Americans, 6 American males and 7 foreign females. What is the number of people on the plane?"

As a final example we quote one of the problems on Markov chains. "A professor tries not to be late for class too often. If he is late one day, he is 90% sure to be on time the next. If he is on time, then the next day there is a 30% chance of his being late. In the long run how often is he late?".

The foregoing examples show the way in which each new mathematical concept is illustrated by a non-trivial application. Even such seemingly mechanical matters as the rules for matrix multiplication are illustrated by realistic examples in which multiplying matrices is precisely the natural thing to do. Indeed, of the text's many virtues, perhaps the outstanding one is the remarkable balance which the authors have preserved between abstract theory and "common sense" applications. This is "applied mathematics" in the best sense.

A second notable feature of the book is its emphasis on pedagogical devices of a graphical nature. Simple problems in set theory are solved by the use of Venn diagrams. Another device which occurs in the first chapter and is used especially effectively in the probability chapter is that of "three diagrams" to represent either partitions, or "logical possibilities" at various stages of some process. This use of "picture writing" is thoroughly exploited to the great advantage of the student.

Having pointed out some of the books strong points, we now turn to what, in the reviewer's opinion, is a very serious flaw. The authors have chosen as their central notion the concept of a "statement." Almost all other notions in the book are derived from or closely related to this one. In the chapter on probability the fundamental notion is not the usual idea of the probability of an event but the "probability of a statement." It is therefore somewhat of a shock to find that this notion is used in a completely inconsistent manner. Specifically, the book opens with the following sentences, "A *statement* is a verbal or written assertion. In the English language such assertions are made by means of declarative sentences." But on page 21 we find, "Hence, also, the *statement* [the italics are the reviewer's] "Draw three white balls" is logically false." (Problem for the authors: what is a declarative sentence?). Next, on page 2 we find "The fundamental property of any statement is that it is either true or false (and that it cannot be both true and false)," but on page 19 we read "Normally, for a given statement there will be many cases in which it is true and many in which it is false." Which sentence is the student to believe? Worse, what is the instructor to tell him? This example does not represent a momentary lapse on the authors' part, but a deliberate ambiguity which is carried through the whole book.

There is no need to dwell on the morass of inconsistency caused by the authors' refusal to distinguish between sentences like "$x^2 = 4$" and "p implies q" which are *not* statements (they are "sentential functions") and are true in some cases, false in others, and honest to goodness statements like "Socrates is a man" and "$2 + 2 = 5$" which are either true or false,—no cases about it. The result of the ambiguity, however, is to put the instructor in an unhappy dilemma. In the authors' phraseology, he will be faced with the following "logical (or pedagocial) possibilities" (we refrain from drawing the tree diagram) he must either play along with the authors, hoping the students won't detect the deception, or he must rewrite a substantial portion of the exposition himself. It is unfortunate that this quite unnecessary mix-up had to occur in what is in many ways a superlative text. Perhaps these matters can be remedied in an eventual second edition.

The last two chapters of the book should really be considered as a separate unit since the level of difficulty, especially in the final chapter, is much higher than in the first five. The reviewer found the exposition here rather muddy in spots, especially the section on Este's learning model.

The examples treated were, however, uniformly interesting and, I suspect, entirely unfamiliar to nine out of ten teachers of college mathematics.

Finally, this book should be welcomed eagerly by college teachers in departments other than mathematics, especially behavioral scientists, as it provides precisely the sort of elementary mathematical training which will be useful to students whose main interest lies in this field.

DAVID GALE

Physics and mathematics. By R. A. Charpie, J. Horowitz, D. J. Hughes, and D. J. Littler. McGraw-Hill Book Co., Inc., New York, and Pergamon Press Ltd., London, 1956. x + 398 pp. $12.00.

This book is labeled Volume One of Series One (of eight series) arising mainly as the result of the review articles presented at the Geneva Conference. This volume contains articles on neutron physics (especially cross sections) mainly in connection with the physics of fission. There are eleven chapters with titles and authors as follows (1) Summary of Data on Cross Sections and Neutron Yields of U^{233}, U^{235}, and Pu^{239}—Harvey and Sanders; (2) Resonance Structure of U^{233}, U^{235}, and Pu^{239}—Egelstaff and Hughes; (3) Theoretical Analysis of Neutron Resonances in Fissile Materials—Bethe; (4) Techniques for Measuring Elastic and Non-Elastic Neutron Cross Sections—Cranberg, Day, Rosen, Taschek and Walt; (5) The Cross Section of the Fission Product Poison x_e^{135} as a Function of Energy—Bernstein and Smith; (6) Resonance Capture Integrals—Macklin and Pomerance; (7) Delayed Neutrons—Keepin; (8) Homogeneous Critical Assemblies—Callihan; (9) The Physics of Fast Reactors—Codd, Sheperd and Tait; (10) Heterogeneous Methods for Calculating Reactors—Feinberg; (11) Highly Enriched Intermediate and Thermal Assemblies—Hurwitz and Ehrlich.

ROHN TRUELL