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CONTENTS

BOOK REVIEWS

Wave motion and vibration theory. Edited by A. E. Heins. Proceedings of Symposia in Applied Mathematics. Volume 5. McGraw-Hill Book Co., Inc., New York, 1954. v + 169 pp. $7.00.

The volume contains the papers presented at the Fifth Symposium in Applied Mathematics of the American Mathematical Society held at the Carnegie Institute of Technology in June 1952. The four sessions of the Symposium were devoted to Stability of Fluid Motion, Hydrodynamic Waves, Diffraction and Scattering Problems, and Vibration Theory. The contributors to the volume are: C. C. Lin, S. Chandrasekhar, P. R. Garabedian, W. Bleakney, N. W. McLachlan, H. Levine, W. Magnus, A. E. Heins and H. Feshbach, G. F. Carrier and W. H. Munk, J. J. Stoker, E. W. Montroll and J. M. Greenberg, E. H. Lee, A. Weinstein, S. Lefschetz, and R. J. Duffin and A. Schild.

W. PRAGER

Relaxation methods. By D. N. de G. Allen. McGraw-Hill Book Company, Inc., New York, 1954. ix + 257 pp. $7.50.

The aim of the present volume is clearly and accurately stated in the author's preface: " . . . to explain as clearly as possible how to relax; its contents were entirely chosen with this end in view . . . not intended to be a reference book . . . no attempt to discuss the more advanced and difficult applications [such as] non-linear equations. Nor . . . to discuss the physical facts and assumptions. . . Another volume could be written dealing with such topics as convergence, finite difference and truncation errors, parallel procedures used in relaxation and other computational methods, and so on . . . . I have deliberately kept off these subjects. . ."

When an author defines his aim, the duty of the reviewer is to discuss the extent to which he has achieved it; the prospective reader or purchaser can make his own decision as to whether his purposes will be satisfied by the author's aim. Viewed in this manner, the task of reviewing the present volume is a pleasant one. Mr. Allen writes with great clarity. The book is well arranged, progressing from simple to difficult problems. The chapters are reasonably self contained so that a reader already familiar with the beginning subject matter may turn at once to the later chapters. Although the author does not discuss in detail any physical problems as such, he does not hesitate to draw upon physical analogies to explain portions of the relaxational technique. An important example of this is in connection with Poisson's equation with curved boundaries where the relaxational net is viewed as a physical entity, being an approximation to a continuous membrane.

The detail of the book appears to be ideal for the serious student. Beginning examples are worked out in full, and the new parts of each new concept are fully illustrated. Once the problem has been reduced to the point where further detail would be repetitious the remainder is left as an exercise. Frequently the final result is quoted so the reader can check his result. Finally, each chapter concludes with a few problems. As the author points out in his preface, "The examples at the end of each chapter have been designed to give the student . . . practice; if they seem to be uninspiring drill problems, well, so they are. No relaxer can learn too early to take the rough with the smooth. . ."

The first four chapters of the book are devoted to linear algebraic equations, frameworks, and ordinary differential equations. Next come six chapters on partial differential equations including Laplaces's, Poisson's, quasi-plane-potential, biharmonic, and simultaneous. Also discussed in these chapters are curved boundaries, normal-gradient boundary conditions, and triangular nets. Next come two chapters on eigenvalue problems and two on internal and free boundaries. The penultimate chapter gives an ingenious technique by which the relaxation method can be used to solve non-elliptic differential equations. Finally, the last chapter applies the relaxation method to an example of Laplace's equation in three dimensions. The book then closes with four appendices, two of which summarize useful formulae and two of which give mathematical details omitted in the text.

Despite the blurb on the jacket, this is the second book on relaxation, the first being "An Introduc-

tion to Relaxation Methods" by F. S. Shaw (Dover, 1953). Both books are written by former students of Sir Richard Southwell, and both are approximately the same length and cover approximately the same ground. Shaw's book is entirely mathematical, while as noted Allen's makes some use of physical analogies. Shaw treats complex examples in considerably more detail, but does not contain any problems for the reader. On the other hand, Shaw has nothing corresponding to Allen's last two chapers. If a conclusion is necessary, this reviewer would probably prefer to teach a course using Allen's book as text, but would like a copy of each in his own bookcase.

P. G. HODGE, JR.

Four-place tables of transcendental functions. By W. Flügge. McGraw-Hill Book Co., Inc., New York, and Pergamon Press Ltd., London, 1954. 136 pp. $5.00.

According to a statement in the preface, these tables were prepared for "those who are old-fashioned enough to do their daily chores with the slide rule, but modern enough to use mathematics beyond sines and cosines whenever that will help solve a problem." The useful volume contains tables, of slightly more than slide rule accuracy, of those transcendental functions that are most likely to occur in problems of physics and engineering (trigonometric and hyperbolic functions; exponential function and logarithm; Bessel functions; Thomson functions; elliptic integrals; error function; Fresnel integrals; sine, cosine, and exponential integral; gamma function). The volume also contains a collection of formulas concerning these functions.

W. PRAGER

Stochastic processes. By J. L. Doob. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1953. vii + 654 pp. $10.00.

This book contains the first complete and detailed treatment of stochastic processes and is essential for anyone concerned with the mathematical basis of the subject.

The first two chapters deal with the necessary probability background and definition of the various classes of stochastic processes—Gaussian, Markov, stationary, martingales, etc. There follow detailed sections on processes with mutually independent, uncorrelated and orthogonal random variables, discrete and continuous parameter Markov processes, martingales, processes with independent and orthogonal increments and discrete and continuous parameter stationary processes. The final chapter treats the theory of linear least squares prediction.

There is a supplement at the end of the book in which the author outlines those aspects of measure theory most needed for the subject matter of the book. However, a fairly thorough knowledge of measure theory may nevertheless be considered to be an essential prerequisite for the profitable use of this work.

LEONARD C. MAXIMON

Theory of matrices. By Sam Perlis. Addison-Wesley Press, Inc., Cambridge 42, Mass., 1952. xiv + 237 pp. $5.50.

The book gives a concise and clear treatment of the theory of matrices with emphasis on the basic ideas rather than particular applications. It can serve well as a text for a year course, especially if supplemented by applications of the material presented in the book. The approach of the text emphasizes strongly the algebra of matrices, with chapters on vector spaces, equivalence, rank and inverses, congruences and Hermitian congruences. Other topics considered include determinants, polynomials over a field and matrices with polynomial elements, similarity, characteristic roots and vectors, and linear transformations. Exercises are given at the end of each chapter.

The book provides a good background for anyone who is to specialize in either pure mathematics or physics and applied mathematics. Although some prior acquaintance with the basic notions of modern algebra would help the student using this text, the concepts are not presented in a form so abstract that the student without such background will feel lost.

LEONARD C. MAXIMON