

where  $a$  and  $b$  are any positive real constants satisfying

$$2b + abp - a^2q^2 + 2a^2b^2 > 0, \quad ap - 1 + b^2a^4 > 0. \quad (16)$$

Lessen's criterion [3] corresponds to  $p = 0$ ,  $q = 1$  of (16).

We may choose some values for  $a$  and  $b$  to get a criterion for jet flow. If we put

$$a = K/\alpha p, \quad b = Kq^2/\alpha p^2, \quad (17)$$

where  $K$  is an arbitrary constant such that  $K \geq 1$  and  $K^2q^4/p^4 \geq 1$ , Eq. (16) is satisfied for all  $\alpha$  and (15) becomes

$$(Rq)^2 < 8 \left[ \alpha^4 + \frac{p^2}{Kq^2} \alpha^3 - \frac{p^4}{K^2q^4} \alpha^2 + \left( 1 - \frac{p^2}{K^2q^2} \right) \frac{p^4}{Kq^4} \alpha + \frac{p^6}{K^2q^6} \right]. \quad (18)$$

Equation (18) seems to be the criterion which is suitable for jet type flow. As  $\alpha$  tends to be zero, the flow will be stable for a finite Reynolds number.

#### REFERENCES

- (1) J. L. Synge, *Hydrodynamical stability*, Semi-centennial Pub. Am. Math. Soc. Vol. 2, New York 1938, pp. 227-269.
- (2) C. C. Lin, *On the stability of two-dimensional parallel flow: Part I—General theory*, Q. Appl. Math. **3**, 117-142 (1945).
- (3) M. Lessen, *Note on a sufficient condition for the stability of general, plane parallel flows*, Q. Appl. Math. **10**, 184-186 (1952).

A note on my paper

### A FORMULA FOR AN INTEGRAL OCCURRING IN THE THEORY OF LINEAR SERVOMECHANISMS AND CONTROL-SYSTEMS

Quarterly of Applied Mathematics, X, 205-213 (1952)

By HANS BÜCKNER

Replace  $D_n^{-1}$  by  $D_{n-1}^{-1}$  in formula (36'), replace  $a_2^2$  by  $a_1^3$  in formula (44), replace  $2Y$  by  $Y$  in formula (45). Formula (46) is correct and corresponds to corrected formula (44).

The author wishes to thank Mr. Kenneth Geohegan and Mr. Edwinn Kinnen for their check of formulae (36') and (44).

### BOOK REVIEWS

*50-100 Binomial tables*. By Harry G. Romig. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1953. xxvii + 172 pp. \$4.00.

The value of the individual terms and the sum of the first  $x$  terms of  $[P + (1 - P)]^n$  are given for  $n$  in the range 50 to 100 in steps of 5 and  $p$  in the range 0.01 to 0.50 in steps of 0.01 to six decimal places although the last place is doubtful. The introduction, written in a clear and concise manner, describes the use of the tables, its application to quality control and its relation to the ratio of the incomplete  $\beta$ -function to the complete  $\beta$ -function. The tabulated data are easy to read but the nature of the entries prevents a more uniform tabulation.

S. L. LEVY

*Stress waves in solids.* By H. Kolsky. At the Clarendon Press, Oxford, 1953. x + 211 pp. \$5.00.

This book, the thirteenth in the Monographs on the Physics and Chemistry of Materials series, is a welcome addition for those interested in the propagation of stress waves in solids. The first part of the book treats the classical theory of stress waves in elastic materials and unites many topics which have only appeared in the literature. It is unfortunate that the author did not insert the results for the reflection and refraction of a stress wave at a plane boundary between two anisotropic materials.

The second portion of the book treats "imperfectly elastic" materials and contains a well written account of the present state of affairs of internal friction in solids. Chapter VI, Experimental Investigation of Dynamic Elastic Properties, presents the various techniques used in the measurement of stress waves in solids and discusses the results of the several types of measurements. The last two chapters of the book treat plastic and shock waves in materials and the experimental results in this field—a large portion contributed by the author.

It is the reviewer's belief that the author has accomplished the aim stated in the introduction that the book should be easily read by anyone trained in mathematical physics.

S. L. LEVY

*Principles of numerical analysis.* By Alston S. Householder. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1953. x + 274 pp. \$6.00.

This is a mathematical text which is concerned with the theoretical rather than the practical aspects of certain computational methods. It describes and examines the principles underlying these methods in an effort to provide the reader with a capital of knowledge which he can use to evaluate and develop new techniques. The treatment throughout has high speed digital computers in mind, but much of the material is relevant when hand methods are considered. The subjects treated fall conveniently into two categories. The first concerns the solution of equations and systems of equations; the chapter headings being *Matrices and Linear Equations*, *Nonlinear Equations and Systems*, and *The Proper Values and Vectors of a Matrix*. The second deals with the approximate representation of functions and the chapters here are entitled *Interpolation*, *More General Methods of Approximation*, and *Numerical Integration and Differentiation*. A short introductory chapter discusses the nature and sources of errors, and the final chapter is a brief description of the Monte Carlo method of estimating accuracy. Each topic is developed from an intermediate level and every chapter concludes with an up to date, (1952), Bibliographic section. Differential and integral equations are not treated. In the opinion of this reviewer, the value of the book would be greatly improved if a few worked examples were inserted in the text.

J. FOULKES

*Discontinuous automatic control.* By Irmgard Flügge-Lotz. Princeton University Press, Princeton, New Jersey, 1953. viii + 168 pp. \$5.00.

In this book the author presents, in graphical form, an analysis of discontinuously operating controllers (on-off controls). By using oblique cartesian coordinates in phase space, the trajectories of the motion are logarithmic spirals and a graphical analysis of the "motion" is easily plotted yielding design criteria. Part I treats ideal systems whereas in Part II the author investigates the effects of imperfections in the control mechanism. In Part III the analysis for three degrees of freedom with discontinuous position control is treated.

The author has a clear concise style and has included many diagrams. The reviewer noticed one small error in connection with the transition points which lead to end points. He recommends this book to engineers working with on-off controls as well as those interested in solving non-linear differential equations by means of piecewise continuous solutions.

SHELDON LEVY

*Statistical theory in research.* By R. L. Anderson and T. A. Bancroft. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1952. xiv + 399 pp. \$7.00.

The book is divided into two parts: Part I. Basic Statistical Theory—150 pages. Part II. Analysis of Experimental Models by Least Squares—227 pages. Tables include the normal, chi-square, "t" and "F" distributions.

Chapter headings in Part I are in order: probability, univariate parent population distributions, properties of univariate distributions, bivariate and multivariate distributions and their properties, derived sampling distributions and orthogonal linear functions, derived sampling distributions: normal parent population, introductions to point estimation and criteria of "goodness," principles of point estimation: maximum likelihood, interval estimation, tests of hypotheses, special uses of chi-square.

Two purposes of the book as stated by the authors are to provide (1) a reference book on statistical theory in connection with research and (2) a textbook in statistical theory.

The reviewer prefers to endorse the reference book objective for Part I. The beginning student of statistics who has a good mathematical background may be a bit confused by the statistical terminology of the examples while the student who is weak in mathematics will encounter trouble in the brevity of that part of the discussion. On the other hand Part I would certainly be a good reference for a course primarily aimed at the material in Part II. Part II provides an excellent reference or text for problems of regression and analysis of variance.

Chapter headings in Part II are in order: regression analysis general regression model with  $r$  fixed variates, computational methods and methods of analysis for a general regression model, curvilinear regression: orthogonal polynomials, least squares for experimental design models, the analysis of designs in complete blocks, the analysis of incomplete block designs, factorial experiments, the analysis of covariance, variance components: all random components except the mean, analysis of data with both random and fixed effects (mixed model), the recovery of interblock information in incomplete-block designs, other topics concerning components of variance: summary of needed research.

D. R. WHITNEY

*Complex variable theory and transform calculus with technical applications.* By N. W. McLachlan. Cambridge University Press, New York, 1953. xi + 388 pp. \$10.00.

This book, like its first edition, contains those contributions of complex variable theory which are most useful in treating boundary value problems by transform techniques. The use of these techniques is then illustrated by the treatment of a large variety of instructive physical problems, both mechanical and electrical. This edition has an improved presentation and contains applications of greater technological interest than those of the first edition.

G. F. CARRIER

*Elasticity, plasticity and structure of matter.* By R. Houwink. Harren Press, Washington, D.C., 1953. xviii + 368 pp. \$7.50.

This new edition does not differ greatly from the 1937 original. The only changes are in the chapter on rubber, where the earlier discussion of rubber-like elasticity has been replaced by a presentation of the modern statistical theory, and in the chapter on proteins, where some new figures illustrating the folding of linear to globular proteins have been introduced. In the latter chapter it is surprising to find mentioned neither the helical structures proposed for proteins by Pauling and others, nor the role played by the hydrogen bond in the stabilization of the folded structures.

STEPHEN PRAGER