QUARTERLY

OF

APPLIED MATHEMATICS

Volume XI April 1953–January 1954 Numbers 1–4

QUARTERLY

OF

APPLIED MATHEMATICS

This periodical is published quarterly by Brown University, Providence 12, R. I. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following industrial companies are gratefully acknowledged:

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The subscription price for the Quarterly is $6.00 per volume (April-January), single copies $2.00. Subscriptions and orders for single copies may be addressed to: Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., or to Box 2-W, Richmond, Va.

Entered as second class matter March 14, 1944, at the post office at Providence, Rhode Island, under the act of March 3, 1879. Additional entry at Richmond, Virginia.

Reprinted 1974 in U. S. A.

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VOLUME XI 1953

CONTENTS

4. Pulsed point and line sources in an infinite medium.

For the point source, suppose that heat is supplied at the origin at the rate of q units per unit time in the time intervals

$$nT < t < nT + T_1 , \qquad n = 0, 1, 2, \cdots \tag{24}$$

no heat being supplied at other times. Then, when steady conditions have been attained, the temperature at distance r from the origin at a time bT after the beginning of a heating period is, for $0 < b < a$,

$$\frac{qa}{4\pi kr} + v_P , \tag{25}$$

where the periodic part v_P is

$$v_P = \frac{q}{4\pi kr}\left\{\operatorname{erfc}\frac{C}{2b^{1/2}} - a + \frac{2}{\pi}\int_0^\infty \frac{e^{-b\xi^2}[e^{-(1-a)\xi^2} - e^{-\xi^2}]\sin C\xi}{\xi(1 - e^{-\xi^2})}\,d\xi\right\}, \tag{26}$$

and

$$a = T_1/T, \qquad C = r(\alpha T)^{-1/2}. \tag{27}$$

This is the limiting case of the problem of §3.

For the line source, suppose that heat is supplied along an infinite line at the rate q units per unit length per unit time during the time intervals (24), no heat being supplied at other times. Then, for large values of the time the temperature at distance r from the line is, for $0 < b < a$,

$$-\frac{qa}{4\pi k} Ei\left(-\frac{r^2}{4\alpha t}\right) + v_P , \tag{28}$$

where the periodic part v_P is

$$v_P = -\frac{q(1 - a)}{4\pi k} Ei\left(-\frac{C^2}{4b}\right) - \frac{q}{2\pi k}\int_0^\infty \frac{e^{-b\xi^2}J_0(C\xi)[(1 - a)e^{-\xi^2} - e^{-(1-a)\xi^2} + a]\,d\xi}{\xi(1 - e^{-\xi^2})}, \tag{29}$$

and a and C are defined in (27), Ei is the exponential integral, and J_0 the Bessel function of the first kind of order zero.

BOOK REVIEWS

Tensor analysis for physicists. By J. A. Schouten. The Clarendon Press, Oxford, 1951. $x + 275$ pp. 50s.

The first five chapters of this extremely well written book are devoted to an exposition of tensor analysis and Riemannian geometry. The author uses his well known expository skill to give a clear geometric interpretation to the various quantities discussed. The last five chapters are devoted to applications of this material to various physical problems. The two groups of chapters are separated by a summary of the salient points of the theory described in the first part of the book. This section should prove to be very useful.

Chapters I and II deal with the algebra of affinors (tensors) and contain a detailed discussion of p-vectors and p-vector densities (anti-symmetric tensors and tensor densities with p indices). The identification of various affinors obtained by restricting the group of the space is made extremely clear.

This discussion is used in Chapter IV to make clear the meaning of Stokes' Theorem which is the name he assigns to the theorem relating integrals over a $q + 1$ dimensional subspace of a space and integrals over the q dimensional boundary of the space.

Chapter V is devoted to a discussion of manifold which have an affine connection and contains in particular a discussion of those parts of Riemannian geometry needed in subsequent applications.

Chapter VI is devoted to a discussion of dimensions of physical objects. The relation between dimensions of physical quantities and the choice of the underlying group is discussed.

The applications of tensor calculus that the author discusses are: Elasticity and peizo-electricity (Chapter VII), Classical dynamics (Chapter VIII), Relativity (Chapter IX). Chapter X on Dirac's matrix calculus gives a brief exposition of the calculus introduced into quantum mechanics by Dirac. Unfortunately the spinor calculus is not discussed.

The space allotted by the author to these applications is relatively small. He has "endeavoured to avoid an incoherent enumeration of interesting facts— and has tried to make each chapter a short but systematic introduction to some branch of theoretical physics." In the main he has succeeded admirably. However, the author devotes more than twice the space allotted to a discussion of graviation to a phenomenological discussion of hydrodynamics. The reviewer feels that the book would have been more useful if this allocation of space were reversed.

In spite of this minor lapse, the book is to be highly recommended for use in a course on tensor analysis and as a reference book.

A. H. Taub

The rise of the new physics. By A. D'Abro. Dover Publications, Inc., New York, 1951. ix + 426 pp. (Volume I), 553 pp. (Volume II). $8.00 the set.

This is a corrected and enlarged edition (two volumes) of The Decline of Mechanism in Physics (1939); it is essentially unchanged except for the addition of 36 portraits of leading physicists and mathematicians. The book is an account of the development of classical physics and approximately the first decade of quantum mechanics. The account has a strong philosophical flavor especially in the first volme. The book is certainly worthwhile for the student of physics and perhaps even more so for the mathematician interested in physics.

Rohn Truell

Mémoire sur l'intégration graphique des équations aux dérivées partielles. Par J. Massau. Edition du Centenaire par les soins du Comité National de Mécanique, Palais des Académies, Bruxelles, Belgique, 1952. $x + 544$ pp. 200 Belg. frs.

As a matter of general editorial policy, space in these columns is considered too valuable to be available for the review of reprint editions. In the present case, however, there is ample justification for deviating from this policy.

In reprinting this treatise, the Belgian National Committee on Mechanics has rendered a significant service, not only to the memory of an outstanding Belgian scientist, but also to all those who are interested in the application of the method of characteristics to the approximate integration of partial differential equations of hyperbolic type. The volume contains a sequence of papers which originally appeared in the "Annales des Ingenieurs sortis des Ecoles de Gand" around the turn of the century, and which do not seem to be sufficiently well known among scientists working on problems of this kind.

Pages 1 to 58 are devoted to the general theory of characteristics and pages 59 to 120 to the one-dimensional treatment of problems of unsteady flow in canals and rivers. By far the greatest portion of the volume (pages 121 to 391) is concerned with the study of limiting states of equilibrium in a cohesionless soil. A 152-page note on the wave equation concludes the volume.

There is no doubt that acquaintance with this material would have saved much time and effort to the scientists who, during the last twenty years, applied the method of characteristics to numerous problems in hydro-and aerodynamics, detonics, plasticity, soil mechanics, and other branches of mechanics of continua. To mention only a few examples, in the section on soil mechanics the author

establishes the basic geometric properties of the net of failure lines (p. 259), recognizes the possibility of limiting lines (p. 163 ff.), and gives an exhaustive discussion of stress discontinuities (p. 267 ff.). In the theory of plasticity, the corresponding results have been developed *ab initio* whereas they could have been taken almost without modification from Massau's work. The other sections of the book contain similar cases where Massau has anticipated results usually considered to be of much more recent origin.

W. PRAGER

Matter and motion. By J. Clerk Maxwell (Reprinted with notes and appendices by Sir Joseph Larmor). Dover Publications, Inc., New York, 1951. xii + 163 pp. $2.50 clothbound ($1.25 paperbound).

The first edition of this famous little book appeared in 1876 under the imprint of the Society for the Promotion of Christian Knowledge. An American edition, published by D. Van Nostrand Co., was brought out in 1875. The original edition was reprinted with notes and appendices by Sir Joseph Larmor in 1920 and it is this reprinting which has been reproduced in the edition under review.

The book was originally intended by the author to provide in terms of the most elementary mathematics an exposition of the fundamentals of mechanics sufficient to serve as a foundation for the general study of physical science. As such it is still eminently readable for the elementary student. The notes added in the later edition by Larmor help to place the contents in proper perspective with respect to twentieth century revisions of classical physics, notably the theory of relativity.

R. B. LINDSAY

Dialogues concerning two new sciences. By Galileo Galilei. Translated from the Italian and Latin into English by Henry Crew and Alfonso de Salvio. Dover Publications, Inc., New York, 1951. xxi + 300 pp. $1.50 (paperbound).

The publishers are to be congratulated for making this celebrated book once more readily available in the fine translation of Crew and de Salvio, which appeared originally under the imprint of the Macmillan Company in 1914 and was reprinted without change in 1939 by Northwestern University.

The original edition of the Dialogues was published in Leiden in 1638, four years before the author's death, at a time when his difficulties with the ecclesiastical authorities made publication in Italy impossible. It is unnecessary to stress here the importance of the book in the history of physical science. In it the man, acknowledged by all to be the founder of modern physics, sets forth in simple terms the basic concepts of mechanics essentially in the physical form in which they have been used ever since. Every student of elementary physics should read the words of Galileo on the motion of falling bodies and on the mechanical properties of solids and fluids. The present inexpensive edition brings them within the reach of all who can read English.

R. B. LINDSAY

Vektor-und Dyadenrechnung für Physiker und Techniker. By Erwin Lohr. Walter de Gruyter & Co., Berlin W. 35, 1950. xv + 488 pp. DM 24.00.

This is the second edition of a book on vector and dyadic calculus which has apparently enjoyed some popularity in Europe since its publication in 1939. The notation is essentially that of Gibbs, and, as the title indicates, the book is of interest principally to physicists. No extensive alterations of or additions to the first edition are made. The changes consist largely in the addition of a final chapter of miscellaneous applications of vector and dyadic calculus.

The book consists of three parts: I) The arithmetic and algebra of extensive quantities, II) The calculus of extensive quantities, and III) Physical applications. These topics, especially the first two, are covered extensively, but the author's enthusiasm for his subject leads him to apply vector methods to some topics where little is to be gained and dyadics to some where the use of tensors is more satisfactory.

The book will be useful to applied mathematicians and, in particular, to physicists.

WILLIAM PELL

Differential equations. By Robert C. Yates. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1952. vii + 215 pp. $3.75.

This is a text book suitable for an introductory course for engineering students. Equations treated include the various standard types of first order, some special forms of first order, (mostly non-linear), and linear second order with constant coefficients. There is a brief discussion of solutions by numerical methods and by power series, the latter illustrated by the Legendre and Bessel equations.

Partial differential equations are introduced by means of problems in vibrations and heat flow with solutions by separation of variables.

Finally, there is a brief treatment of Fourier series.

The book contains several review chapters inserted at appropriate stages and an abundance of problems with answers.

The illustrative applications of the various equations range over a great variety of physical and geometrical problems, emphasis being placed on first analyzing the problem, then formulating it in terms of a differential equation and finally solving it.

The treatment is non-rigorous throughout and the writing is clear.

GEORGE W. MORGAN

The scientific papers of James Clerk Maxwell. Edited by W. D. Niven. Dover Publications, Inc., New York. xxix + 801 pp. (two volumes bound as one). $10.00.

This collection of papers is bound as a single volume and contains the papers formerly published in 1890 in two volumes. This edition contains the published papers, lectures, and addresses of Maxwell and probably includes all of his work outside of his texts on "Theory of Heat", "Electricity and Magnetism", and "Matter and Motion".

While it may be superfluous to list the contents or to review something published in 1890, it is, I believe, not out of place to recommend as very interesting reading almost all of these papers by the first Cavendish Professor of Experimental Physics.

ROHN TRUELL

Proceedings of the Second Canadian Mathematical Congress, Vancouver, 1949. University of Toronto Press, Toronto, Canada, 1951. xxxi + 255 pp. $6.00.

This volume contains the papers presented at the Second Canadian Mathematical Congress, held at Vancouver, B. C., from September 5–9, 1949. The eighteen papers presented cover a wide variety of topics. Historical and survey papers are presented as well as technical work on pure and applied mathematics.

The papers included are: A. Zygmund, *Polish Mathematics between the two wars;* P. A. M. Dirac, *The relation of Classical to Quantum Mechanics;* A. W. Conway, *Hamilton, his life, work, and influence;* H. J. Bhabha, *Recent scientific developments in India;* G. Szegö, *Principal frequency, torsional rigidity and electrostatic capacity;* L. Schwartz, *Les Mathématiques en France pendant et après la guerre;* W. H. Watson, *The role of the National Research Council in the education of Canadian mathematicians and physicists;* B. Jessen, *Mean motions and almost periodic functions;* R. L. Jeffery, *Non-absolutely convergent integrals;* G. F. D. Duff, *A development in the theory of the F-Equation;* Josephine Mitchell, *An example of a complete orthonormal system and the kernel function in the geometry of matrices;* P. Mandl and J. R. Pounder, *Wind tunnel interference on rolling moment of a rotating wing;* R. De Vogelaere, *Une nouvelle famille d'orbites périodiques dans le problem de Störmer: les ovales;* B. Davison, S. A. Kushneriuk, and W. P. Seidel, *Influence of a small black cylinder upon the neutron density in an infinite non-capturing medium;* W. P. Thompson, *Thermal convection in a magnetic field;* E. Leimanis, *The application of infinitesimal transformations to the integration of differential equations of exterior ballistics by quadratures;* A. E. Scheidegger, *On gravitational radiation;* M. A. Melvin, *Symmetry and affinity of electromagnetic fields, charges, and poles.*

W. H. PELL

Advanced engineering mathematics. By C. R. Wylie, Jr. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1951. xiii + 640 pp. $7.50.

This text book for third or fourth year engineering students treats material of proven value to the analytical engineer. The main mathematical topics are: linear differential equations with constant coefficients, Fourier series, the Laplace transformation, separable partial differential equations, Bessel functions, complex variables, conformal mapping, vector analysis, and numerical analysis. In the development of these topics the relation to physical problems is constantly brought out. Thus, analytic functions are related to fluid mechanics. A valuable feature is a chapter on electrical and mechanical vibrations which explains electrical circuits and how to set up electromechanical analogies. There are an extremely large number of diagrams and worked out examples; this should serve to make the book easy to read.

R. J. DUFFIN

Tensor analysis. By I. S. Sokolnikoff. John Wiley & Sons, Inc., New York and Chapman & Hall, Ltd., London, 1951. ix + 333 pp. $6.00.

This text should furnish a good introduction to tensor analysis for students on the senior-graduate level. The exposition is clear and is illustrated by examples. The subject matter is arranged so that the formal processes of tensor analysis are developed and then applications are made to geometry, analytical dynamics, relativity, and the mechanics of continuous media. In particular, the author's development of the theory of non-linear elasticity should prove of interest. There are a few minor errors and misprints.

The text covers a considerable amount of material which may be subdivided into: (1) matrix theory; (2) the theory of tensor analysis; (3) applications of tensor analysis. In the first part, the author discusses: linear vector spaces, linear transformations in terms of matrices, characteristic values and the reduction of two quadratic forms. The tensor theory is developed by introducing the group of coordinate transformations and studying the induced group of transformations for the covariant and contravariant components of tensors. This is followed by a study of tensor algebra, the metric tensor, the Christoffel symbols, covariant differentiation of tensors, the e-systems and generalized Kronecker deltas, and the Riemann-Christoffel tensor. In the applications of tensor analysis to geometry, the above results are interpreted in terms of curvilinear coordinate systems and base vectors in Euclidean three space. Parallelism of vectors, in three space, the Frenet formulas for curves, and an introduction to surface theory (imbedded in three space and intrinsic) are discussed. In the sections on analytical dynamics and relativity, the author considers the Lagrange equations of motion of a system of particles, the principle of least action, Hamilton's equations, some elements of potential theory, the Lorentz-Einstein transformation law of restricted relativity, and the Schwarzchild line element. In the final chapter the mechanics of continuous media are studied by following the general approach of F. D. Murnaghan, Finite Deformations of an Elastic Solid, Am. Journal of Math. vol. 59, 1937. This leads to a unified and interesting development of the strain tensor in non-linear elasticity.

N. COBURN

Conformal mapping. By Zeev Nehari. First Edition. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1952. viii + 396 pp. $7.50.

Professor Nehari's clearly written and thorough monograph on conformal mapping can be read with profit by all mathematicians who have an interest in this important subject. His very detailed study of special conformal transformations, and their analytical expressions, fills a real gap in the literature. Especially to be commended are the elegant discussions of circular polygons and hypergeometric functions, distortion theorems, the symmetry principle, and of elliptic functions and the Picard theorem. The introductory discussion of multiply connected domains is also useful.

The style of the book is modern, and all discussions very rigorous and complete. Because of this

fact, and the presence of ample exercises, it should serve as an excellent text for a course on conformal mapping. Because of its specialized emphasis, it would seem more useful as a reference than as a basic text for a course on complex variable theory.

Physicists and engineers who are interested in transformations of special domains of the classical type, should also find the book an excellent reference. (Other references for this purpose are A. Betz, "Konforme Abbildung" and H. Kober's "Dictionary of Conformal Transformations," published by the British Admiralty Computing Service.) Those who desire a sound basic understanding of the theory of conformal transformations, can hardly do better than study it carefully.

On the other hand, there is little discussion of numerical methods applicable to general domains; reductions to tabulated real functions and problems of determining numerical parameters are not considered; and the applications to non-Euclidean geometry and to problems of electromagnetism and fluid mechanics are hardly mentioned. For these reasons, and because of its general tone (viz., the proof on p. 278 of the elementary formula for $\tan(z+\zeta)$, and the discussion of $z = \frac{1}{2}(\zeta + \zeta^{-1})$ as a special case of $z = (a\zeta^2 + b\zeta + c)/(d\zeta^2 + e\zeta + f)$), the book is less suitable as an "applied" mathematics text.

However, "applied" mathematicians already familiar with practical aspects of the subject, and prospective mathematical analysts, will find the volume a nearly ideal reference, both for the general theory of conformal transformations, and for the treatment of special cases.

GARRETT BIRKHOFF

Foundations of high-speed aerodynamics. Facsimiles of nineteen fundamental studies as they were originally reported in the scientific journals. With a bibliography compiled by George F. Carrier, Professor of Engineering, Brown University. Dover Publications, Inc., New York, 1951. 286 pp. $3.50.

In addition to an extended bibliography, the book contains photo-offset reproductions of nineteen basic papers on the dynamics of compressible fluids. Obviously, space limitations have excluded some important contributions to the subject; otherwise, the collection illustrates well the rapid development of this field. The bibliography is arranged according to the following headings: Hodograph Method. The Rayleigh-Janzen Method. The Prandtl-Glauert Method. Supersonic Flow. Shock Waves. Boundary Layer. The Oscillating Airfoil and Other Unsteady Flow Phenomena. General.

W. PRAGER

The theory of electromagnetic waves. (A Symposium). Interscience Publishers, Inc., New York, 1951. vii + 389 pp. $6.50.

This book is a series of papers presented at a symposium at Washington Square College of New York University in June 1950. The symposium was sponsored by the Air Force Cambridge Research Laboratories and New York University. This book contains eighteen complete papers and three abstracts of papers as follows: On the Theory of Electromagnetic Wave Diffraction by an Aperture in an Infinite Plane Conducting Screen (37pp.), by H. Levine and J. Schwinger. On Systems of Linear Equations in the Theory of Guided Waves (18 pp.), by W. Magnus and F. Oberhettinger. Wiener-Hopf Techniques and Mixed Boundary Value Problems (16 pp.), by S. N. Karp. Asymptotic Solutions of a Differential Equation in the Theory of Microwave Propagation (12 pp.), by R. E. Langer. Criteria for Discrete Spectra (11 pp.), by K. O. Friedrichs. Extension of Weyl's Intregral for Harmonic Spherical Waves to Arbitrary Wave Shapes (10 pp.), by H. Poritsky. Kirchhoff's Formula, Its Vector Analogue, and Other Field Equivalence Theorems (17 pp.), by S. A. Schelkunoff. On the Diffraction Theory of Gaussian Optics (14 pp.), by H. Bremmer. Diffraction and Reflection of Pulses by Wedges and Corners (20 pp.), by J. B. Keller and A. Blank. Vector Wave Functions (10 pp.), by R. D. Spence and C. P. Wells. The W. K. B. Approximation as the First Term of a Geometric-Optical Series (21 pp.), by H. Bremmer. Remarks Concerning Wave Propagation in Stratified Media (12 pp.), by S. A. Schelkunoff. The Theory of Magneto Ionic Triple Splitting (32 pp.), by O. E. H. Rydbeck. An Asymptotic Solution of Maxwell's Equations (38 pp.), by Morris Kline. Field Representations in Spherically Stratified Regions (53 pp.), by N. Marcuvitz. Propagation in a Non-homogeneous Atmosphere (34 pp.), by B.

Friedman. Reflection of Electromagnetic Waves from Slightly Rough Surfaces (28 pp.), by S. O. Rice. The Theory of Scattering of Radio Waves in the Troposphere and Ionosphere (abstract), by H. G. Booker. Properties of Guided Waves on Inhomogeneous Cylindrical Structures (abstract), by R. B. Adler. Evaluation of Integrals Associated with Wave Motion in Dispersive Media and the Formation of Transients (abstract), by M. Cerrillo. Electromagnetic Research in the U. S. Air Force Research Program, by N. C. Gerson.

This book is obviously a discussion of mathematical methods applied to special problems in propagation, diffraction, dispersion, etc., of electromagnetic waves. The book will undoubtedly be of value to the student of advanced electromagnetic theory and to the research worker.

ROHN TRUELL

Advanced calculus. By Wilfred Kaplan. Addison-Wesley Press, Inc., Cambridge, Mass., 1952. xiii + 679 pp. $8.50.

This text on advanced calculus is an excellent addition to the list of books on this subject, because of its laudable emphasis on vector methods in all phases of the calculus in which they can be applied.

The material contained covers all the usual subjects found in a course of advanced calculus, but the emphasis seems definitely slanted toward the advanced engineer or the applied mathematician. The topics discussed include differential and integral calculus of functions of several variables, infinite series, Fourier series and orthogonal functions, functions of a complex variable, and ordinary and partial differential equations. In addition, there is an excellent treatment of vectors and their properties, vector differential calculus, and vector integral calculus, both in the plane and in space. The consistent use of vectors is well illustrated in the discussion of orthogonal functions, where the functions in question are thought of as elements in a vector space. Thus, the engineering student, to whom the concept of a general set of orthogonal functions and their properties may appear as an intellectual abstraction, will be able to relate these functions to what is to him a more or less familiar concept, vectors.

There is a large number of problems of which a goodly portion is of the so-called applied type, i.e., relating to the fields of mathematical physics, hydromechanics, elasticity, potential theory, etc. The proofs of some of the theorems are given as problems. The rest of the problems, though not trivial, are of the standard drill type. Answers are given for all problems which are not self-contained.

Some of the longer and more difficult results are given without proof, but whenever this is done, references are given where the proofs may be found. As the author states, "A competent teacher can easily fill in these gaps, if so desired, and thereby present a complete course in real analysis." Beyond this, the work is completely rigorous, clearly presented, and concisely written.

At the end of each chapter there is a long list of suggested supplementary reading.

HARRY J. WEISS

The principle of relativity. By H. A. Lorentz, A. Einstein, H. Minkowski and H. Weyl. Notes by A. Sommerfeld. Translated by W. Perrett and G. B. Jeffrey. Dover Publications, Inc., New York, 1951. viii + 216 pp. $3.50 (clothbound) and $1.50 (paperbound).

This collection of the classical papers by the above-named authors has been made available to the American market at a very attractive price and with the added convenience of being in English throughout. This edition will make it possible for graduate students so inclined to read the key papers that established relativity, and about as easily as they could get the same material out of a text book on the subject. One can only regret that this collection has not been supplemented by the most important papers that have appeared since, but such additions would undoubtedly have driven the price up to the point where purchase by students would no longer be feasible.

It is interesting to note that the so-called Lorentz transformation is not due to Lorentz at all (I owe this remark to Dr. H. Zatzkis of the University of Connecticut) cf. page 14. Lorentz correctly describes the deformation of moving scales, but the transformation law for the time is quite different from the one postulated by Einstein (page 48).

Einstein's fundamental paper on the general theory of relativity occupies some fifty pages in this

small format. In a modern U.S. physics journal it would probably take about twenty pages. In that amount of space, Einstein manages to give a comprehensive introduction to the general theory of relativity to the extent he had then been able to develop the theory, including an almost complete presentation of the foundations of Riemannian geometry. Very commonly the authors of today's theoretical papers excuse their lack of comprehensibility on the grounds that the editor will not let them use enough space. This paper by Einstein, covering the complete foundations of a new theory, presenting all of its physical and mathematical aspects, is "experimental" proof that within the length of a paper perfectly acceptable to today's editors it is possible to write lucidly, provided the author is willing to go to the trouble of organizing his material with the reader in mind. It has been this reviewer's privilege to see Professor Einstein at work on several papers intended for publication and to watch the infinite pains with which he went over the drafts, sentence by sentence, until they were satisfactory to him. The results are well worth the effort.

The volume concludes with a paper by H. Weyl, which now has mostly historical significance. In this paper Weyl presented his generalization of Riemannian geometry, in which the length of a displacement vector is not an invariant quantity. Occasionally, one reads in the literature references to Weyl's geometry which convey the impression that Weyl's geometry reduces the *length* of a vector to a non-integrable status in the same way that Riemannian geometry makes the direction of a parallel displaced vector non-integrable. This impression is not quite correct, insofar as Weyl's geometry permits the construction of vectors (those of weight-$\frac{1}{2}$) whose norm is invariant. It is true, though, that the elementary displacement vector dx has no invariant norm. Unfortunately, in spite of its intrinsic conceptual beauty, Weyl's geometry has not led to a viable physical theory. This may have been the fault of those trying to apply it, including Weyl himself, and it is conceivable that another attempt may succeed.

Altogether, this collection of original articles may be warmly recommended to those who find the German original articles a bit too difficult and who, nevertheless, appreciate the value of studying the source material. Undoubtedly, many readers will find it worthwhile to expand their reading beyond the papers available here. There are numerous references (such as to the original papers by Michelson and Morley, which were, of course, published in English), but no index.

PETER G. BERGMANN