weaker than the one obtained by Synge but it has wider application. Such two-dimensional, parallel flows as boundary layers, jets, and wakes can be quickly considered preliminary to more detailed investigation.

Consider Eq. (4) as applied to the laminar boundary layer between parallel streams (Ref. 3).

Let $\eta = K/\alpha^2$, $\xi^2 = 0$. Equation (6) then becomes

$$(Rq)^2 > 8\left(1 + \frac{1}{K} - \frac{1}{K^2} - \frac{1}{K^3}\right)\alpha^4$$

If the right hand side is maximized with respect to K, the corresponding value of K is $K = 3$. The value of q for the problem under consideration is

$$q = 0.2000.$$

Therefore, for certain stability

$$R < 15.8\alpha^2.$$

It is interesting to note that, although no lower branch of the stability curve is obtained by the above method, the actual calculations to a second approximation for the free boundary layer also do not yield a lower branch of the neutral curve.

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GENERALIZATION OF A PROBLEM OF RAYLEIGH*

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Emmons has recently published a note [1] giving dissipative temperature distributions for Rayleigh's classic problem of the incompressible viscous flow set up by an infinite plate suddenly moved at constant velocity in its own plane [2]. This suggests sufficient interest to warrant exposition of a particular solution of the problem for a compressible fluid. This was worked out several years ago as a simple pedagogical demonstration of some characteristics of the compressible boundary layer, much as the Rayleigh case is often used to introduce students to the incompressible boundary layer.

We shall restrict the problem by taking low Mach number, $v/u \ll 1$, insulated wall, Prandtl number unity, perfect gas, $\mu \sim \lambda \sim T^{3/4}$ and c_p = constant. The plate lies on the plane $y = 0$ and moves in the x-direction; u is velocity along x, v is along y; c_p, μ, λ, ρ, T are specific heat at constant pressure, viscosity, thermal conductivity, density and absolute temperature, respectively.

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The x momentum equation with $v/u \ll 1$ is

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right). \tag{1}$$

For low Mach number, the energy equation is

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{2}$$

and the state equation is

$$\frac{\rho}{\rho_1} = \frac{T_1}{T}, \tag{3}$$

where the subscript 1 refers to $y = \infty$. Also,

$$\frac{u}{\mu_1} = \frac{\lambda}{\lambda_1} = \left(\frac{T}{T_1}\right)^{3/4}. \tag{4}$$

Assumption of unity Prandtl number permits satisfying (1) and (2) simultaneously with $c_pT = f(u)$ only [3]. Substitution of this into (2) shows that (2) is identical with (1) only if

$$\frac{d^2f}{du^2} = -1$$

whence,

$$c_pT = A + Bu - \frac{u^2}{2}. \tag{5}$$

The boundary conditions for u and T are $u = 0$ and $T = T_1$ at $y = \infty$ or $t = 0$; $u = U$ and $T = T_0$ at $y = 0$ or $t = \infty$. T_0 is stagnation temperature for a fluid flowing with velocity U and temperature T_1, since the plate is insulated.

Then (5) can be transformed to

$$T = T_0 - (T_0 - T_1)\left(1 - \frac{u}{U}\right)^2. \tag{6}$$

Since there is no heat transfer between wall and fluid at $y = \infty$, the one-dimensional energy equation is applicable:

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2, \tag{7}$$

where the effect of pressure ($\sim \partial p/\partial t$) has been neglected with the restriction to low Mach number, $M_1 = U/a_1$ and a_1 is the velocity of sound at $y = \infty$; γ is the ratio of specific heats.

With (7) and the new velocity variable $\omega = (1 - u/U)$, (6) becomes,

$$\frac{T}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2(1 - \omega^2) \tag{8}$$

which gives $T(y, t)$ after $u(y, t)$ is known.

With (4) and (8), the x momentum equation becomes

$$\frac{\partial \omega}{\partial t} = \nu_1[1 + N(1 - \omega^2)]^{7/4} \frac{\partial^2 \omega}{\partial y^2} - \frac{3}{2} \nu_1 N[1 + N(1 - \omega^2)]^{3/4} \omega \left(\frac{\partial \omega}{\partial y}\right)^2, \tag{9}$$

where

$$\nu_1 = \frac{\mu_1}{\rho_1}; \qquad N = \frac{\gamma - 1}{2} M_1^2 .$$

As in the incompressible case, this is reducible to a total differential equation with a similarity variable,

$$\eta = \frac{y}{2(\nu_1 t)^{1/2}} .$$

Then

$$\frac{d^2\omega}{d\eta^2} - \frac{3}{2} \cdot \frac{N\omega}{1 + N(1 - \omega^2)} \left(\frac{d\omega}{d\eta}\right)^2 + \frac{2\eta}{[1 + N(1 - \omega^2)]^{7/4}} \left(\frac{d\omega}{d\eta}\right) = 0 \tag{10}$$

with boundary conditions $\omega = 0$ at $\eta = 0$; $\omega = 1$ at $\eta = \infty$.

Consistent with the restriction to small Mach number, we assume $N(1 - \omega^2) \ll 1$, and (10) reduces to a relatively simple non-linear total differential equation:

$$\frac{d^2\omega}{d\eta^2} - \frac{3}{2} N\omega \left(\frac{d\omega}{d\eta}\right)^2 + 2\eta \frac{d\omega}{d\eta} = 0. \tag{11}$$

This is easily integrated twice to give, after application of boundary conditions,

$$\operatorname{erf}\,[(\tfrac{3}{4}N)^{1/2} \cdot \omega] = \operatorname{erf}\,[(\tfrac{3}{4}N)^{1/2}] \operatorname{erf}\,(\eta). \tag{12}$$

For $M_1 \to 0$, this reduces to the well known solution of Rayleigh,

$$\frac{u}{U} = 1 - \operatorname{erf}\,(\eta). \tag{13}$$

The skin friction coefficient c_f can be obtained directly from (10), without going to the simpler form, (11). The wall shearing stress is

$$\tau_0 = -\mu_0 \left(\frac{\partial u}{\partial y}\right)_0 = \frac{U}{2} \left(\frac{\mu_1 \rho_1}{t}\right)^{1/2} \cdot (1 + N)^{3/4} \left(\frac{d\omega}{d\eta}\right)_0 \tag{14}$$

To get $(d\omega/d\eta)_0$, we integrate (10) once and let $\eta \to 0$. This gives

$$\left(\frac{d\omega}{d\eta}\right)_0 = \pi^{-1/2}(1 + N)^{-7/8} \tag{15}$$

or

$$c_f \equiv \frac{2\tau_0}{\rho_1 U^2} = \left(\frac{\nu_1}{\pi U^2 t}\right)^{1/2} \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{-1/8} \tag{16}$$

which shows a Mach number behaviour much like that of the boundary layer.

It seems physically clear that when the plate starts, a pressure wave is generated at the wall and moves outward. Professor L. Howarth has mentioned to me that he is working on this more complex aspect of the problem. [Ref. added Oct. 1, 1951: L. Howarth, *Some aspects of Rayleigh's problem for a compressible fluid*, Q. Jour. Mech. and Appl. Math., **4**, pt. 2, 157 (1951)].

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THE QUARTER-INFINITE WING OSCILLATING AT SUPERSONIC SPEEDS*

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Introduction. In the April, 1951 issue of the Quarterly of Applied Mathematics, two papers appeared. One, by H. J. Stewart and Ting-Yi Li[1] purports to extend the method of J. C. Evvard[2] for solving supersonic wing tips to the case of the oscillating wing. The other by J. W. Miles[3] solves the problem of the oscillating rectangular airfoil by the Wiener-Hopf technique. In an application of their method to the rectangular airfoil,[4] Stewart and Li obtain results which disagree with those of Miles. Moreover, Stewartson[5] has treated the problem independently, and his results agree with Miles'.

The *raison d'être* of this note is to add yet another independent solution to the list of those which agree with Stewartson's and Miles'. In view of the fluxive state which this problem is in at the moment, it is felt that this would be of current interest.

The Gardner method. The method presented by C. S. Gardner[6] will be used to obtain the solution. Let x, y, z represent rectilinear space coordinates, x in the direction of the free stream, and y in the "spanwise" direction. Denote by U the freestream velocity, c the velocity of sound, M the Mach number and t the time. Let $z = f(x, y, t)$ describe the surface of the wing, $g = Uf_x + f_t$, $\beta = (M^2 - 1)^{1/2}$, $x' = x/\beta$, $t' = (Mx - \beta^2 ct)/\beta$. Then in terms of x', y, z, t', Gardner's method for the rectangular airfoil consists in

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