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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1) ,the letter l and the prime ($'$), between alpha and a, kappa and k, mu amd u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t\,(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354-372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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is the solution required, where I_0 is the Bessel function of order zero with pure imaginary argument. Thus, returning to the variables, X and T, the dimensionless stress solution is

$$S(X, T) = 1 + CI_0[\tfrac{1}{2}(T^2 - X^2)^{1/2}] \exp(-T/2).$$

The strain and velocity solutions are then obtained from the characteristic conditions by numerical quadratures.

BOOK REVIEWS

Nonlinear vibrations in mechanical and electrical systems. By J. J. Stoker, Interscience Publishers, Inc., New York, 1950. xv + 273. $5.00.

Chapter I contains a concise review of linear vibrations in one degree of freedom. This is followed, in Chapter II, by a discussion of free vibrations of undamped systems with nonlinear restoring forces. One finds here, in addition to the material given in the books by Timoshenko (Vibrations Problems in Engineering. D. Van Nostrand Co., N. Y., 1937) and Den Hartog (Mechanical Vibrations. McGraw-Hill Book Co., N.Y., 1940) the notion of the energy curves in the phase plane. The theory of this chapter is illustrated by interesting examples.

Chapter III is divided into three sections; the first explains Liénhard's graphical construction of trajectories in the phase plane, the second gives the theory of singular points of first order differential equations, and the third shows applications of that theory to nonlinear mechanical and electromechanical systems. A good deal of attention is devoted to a novel treatment of the elastic stability of columns from a dynamical viewpoint, making use of the notion of singular points.

Chapter IV deals essentially with Duffing's equation and its integration by both an iteration and a perturbation method. In this chapter reliance is placed on physical intuition, the more delicate questions of the existence of the perturbation series, and the stability of solutions being left for later treatment. A special case of subharmonic resonance in Duffing's equation with and without damping is discussed, and Rauscher's iteration method is briefly described. Following Rauscher's method, combination tones are introduced, i.e., response frequencies which are linear combinations of the frequencies ω_1 and ω_2 of a forcing function $H_1 \cos \omega_1 t + H_2 \cos \omega_2 t$. The chapter ends with some introductory remarks regarding the stability of harmonic solutions and with a helpful table summarizing the essential differences in the response of Duffing's equation and of the related linear equation.

Chapter V treats, in two sections, Van der Pol's equation without and with forcing function. In the first part the autonomous equation is discussed and a very appealing treatment of relaxation oscillations, and also a higher approximation are included. The second part deals with Van der Pol's method applied to the non-autonomous system and with the stability investigations by Andronow and Witt. It gives altogether a very complete treatment for the two cases of a forcing frequency near the natural frequency of the system, and not near that frequency.

Chapter VI, the last chapter of the book, treats linear equations with periodic coefficients because of their applicability to investigations of the stability of solutions to nonlinear problems. The Floquet theory is well reproduced as is an interesting and simple discussion of the stability of the Mathieu equation $w'' + (\delta + \epsilon \cos z)w = 0$ in the neighborhood of $\epsilon = 0$. Finally, the stability of harmonic solutions of Duffing's equation without damping is investigated by the theory of this chapter.

This final chapter is followed by six appendices giving, in that order, (1) the mathematical justification of the perturbation method, (2) the existence of combination oscillations, (3) the existence of limit cycles in self-excited systems, (4) a proof of a previously introduced and attractive argument regarding the form of the limit cycle of $dv/d\xi = \epsilon^2[F(v) - \xi]/v$ for a special form of $F(v)$, and as $\epsilon \to \infty$ (called the limit of limit cycles), (5) Poincaré's criterion for orbital stability, and (6) a very palatable proof of uniqueness of a limit cycle in the free oscillations of a self-sustained system. The last chapter and the appendices are distinctly not elementary in character.

Last, but far from least, the appendices are followed by a very helpful bibliography which makes reference not only to the source material of the book, but also to many topics only covered sketchily or merely mentioned without description.

The author has, on the one hand, purposely omitted certain important general methods and results achieved in the field of nonlinear vibrations (see the "Introduction") because they can be found elsewhere. On the other hand, he has included a number of refinements which, although interesting, do not add a great deal to an understanding of nonlinear vibration phenomena. The deletion of general theories from a text for the reason that they can be found elsewhere is not conventional in textbooks. The reviewer believes that this book on nonlinear vibrations would have gained from a presentation of the methods of Kryloff and Bogoliuboff (Princeton University Press, 1943), the classical work on stability by Liapounoff (Annales de Toulouse, Paris, 1907), and at least a statement of certain general and useful theorems proved by Levinson (Journal of Math. and Phys., June, December 1943) and by Levinson and Smith (Duke Math. Journal, 1942).

For reasons unclear to the reviewer, and quite unessential as the author observes, he has rendered Duffing's equation so that the amplitude of the forcing function is small with the coefficient of the nonlinear term. This appears to reduce the generality of the results.

Among the sections in the book more difficult to follow one might mention the one on subharmonic resonance. In searching for a solution with frequency ω/n, $n > 1$ being an integer, and ω being the forcing frequency, the author has chosen without further explanation $n = 3$. The reader is left wondering whether there is something magic about $n = 3$, and whether the treatment is the same for n even and n odd. This section suffers also from a lack of correspondence between figures and attending text, and from the use of the term bifurcation without any explanation of its meaning.

As may be expected in a first edition, there are a few errors and many misprints in the book. It is hoped that these will be corrected in later editions.

These remarks are not exhaustive, nor are they intended to detract from the merits of Stoker's "Nonlinear Vibrations". The reader will find this book an excellent addition to the growing literature on nonlinear phenomena. The material selected for presentation is most interesting, and some of it seems to have been published here for the first time. The style is very readable, and the absence of a great mass of unessential mathematical detail is a welcome feature.

R. M. Rosenberg

Matrix analysis of electric networks. By P. Le Corbeiller. Harvard Monographs in Applied Science, Number 1. Harvard University Press, Cambridge, and John Wiley & Sons, Inc., New York, 1950. xi + 112 pp. $3.00.

This is the first of a new series of monographs published by Harvard University under the care of an Editorial Committee (chairman, F. V. Hunt). In a Foreword the Committee states the purpose of the series: it is to be "a medium for publishing the results of University research to a wider audience than would be reached by individual professional journals", and will be concerned primarily with the applied *physical* sciences, with emphasis on "intellectual borrowing among the academic disciplines."

A book must be written primarily for a certain group of readers, and this book has been written, naturally enough, for electrical engineers. But for review it has fallen into the hands of an applied mathematician who certainly does not know all he ought to know about networks in order to appreciate the finesses, and who perhaps is a little too much inclined to try to dig down to mathematical essentials.

The book is pleasantly compact, nicely printed with many diagrams, written in an easy graceful style, and supplied with an index. It is unified in purpose, namely, ". . . to offer to engineering circles a simple approach to Gabriel Kron's method of analysis of stationary electric networks".

Chapter I disposes of preliminary concepts: definitions of terms, elements of matrix algebra against an electrical background, and a page and a half on "The Two Fundamental Topologic Equations". Chapter II, on the mesh method of Kron, contains the kernel of the book. Chapters III and IV, on the node-pair method and the mixed method respectively, take the reader into more complicated technical situations in order to show the versatility of Kron's method.

Since his purpose is to explain a method which has, in Kron's own exposition, been found difficult to understand, the author rightly avoids complexities and deals with d.c. and a.c. currents in steady states. This means that the solution of a network (i.e. the determination of the branch currents in terms of the branch emfs) is nothing more than the solution of a set of linear algebraic equations.

That sounds very simple, and in a sense it is: anyone, given a network and a pair of basic laws, can get all the equations he needs, and anyone with sufficient time and patience can solve a set of linear algebraic equations. Kron's method systematizes this crude procedure, and probably the best way to see what the method consists of is to write down the fundamental relations: (1) $i = Ci'$, $e' = C_t e$; (2) $e' = Z'i'$, $Z' = C_t ZC$; (3) $i = C(C_t ZC)^{-1}C_t e$. Here i is a column matrix of branch currents (B of them), e a column matrix of branch emfs (B of them), i' a column matrix of mesh currents (M of them), e' a column matrix of mesh emfs (M of them), Z a $B \times B$ impedance matrix, and finally C (the central theme) is Kron's rectangular transformation matrix and C_t its transpose.

Suppressing for a moment our curiosity as to the source of these relations, let us look at the last (3). It is precisely what we want, a formula expressing the branch currents in terms of the branch emfs, and so solving our problem. All we need to know are (*i*) the impedance matrix Z and (*ii*) the transformation matrix C. The rest is routine matrix computation.

We do not have to bother about Z, for it is supposed given to us. It is C we must look into. Its meaning is demonstrated with admirable clarity on pp. 27-33 of the book in terms of a simple network with four nodes and six branches. Intuition and reason walk hand in hand in this simple illustration and the reader turns over from p. 33 to p. 34 with his curiosity put on edge by the promise that the fundamental relations will now be proved to hold for all cases.

The problem is clear. In our equation (3) above the meanings of the symbols i, e, Z are clear, and we are, I believe, clear as to what we mean by a network. Then, if you draw a network for me and give me the square matrix Z, can I, or can I not, write down a matrix C such that (3) is the true connection between branch currents and emfs? I accept the challenge—draw me a network with a thousand nodes and ten thousand branches, and I shall find you a matrix C!

Why am I so confident? When I look into the problem, I realise that it has really nothing to do with Z: the equations (1) above are all that matter, and the second is simply the first re-dished, so that attention focuses on $i = Ci'$. Once that is granted, all the rest follows simply in a few lines. And this equation is simply a mathematical statement to the effect that in any given network it is possible to pick out a set of meshes such that if arbitrary currents are assigned to them, then the branch currents obtained from these mesh currents by the ordinary method of superposition satisfy Kirchhoff's first law at the junctions or nodes, and that in this way we can get the most general system of branch currents satisfying the law. The question is one of pure topology, covered by a transparent electric veil.

My belief in the validity of $i = Ci'$ is primarily an act of faith. If it were not generally true, Maxwell's mesh method would be nonsense, and Maxwell was no fool. And there are many electrical engineers, including Kron and Le Corbeiller, for whose intuition I have the highest respect; if there was something wrong here, they would have found it out.

And yet one cannot base theories on acts of faith; one asks for reasons and proofs. Having been too lazy to drill through the iron mask of topology to find the proof of $i = Ci'$ which I know lies there, and having been even too lazy to digest properly the proof given by Ingram and Cramlet in the Journal of Mathematics and Physics, **23**, 134-155 (1944), I turned over from p. 33 to p. 34 expecting to find at least a plausible proof of the fundamental formula on which Kron's method rests.

But I did not find it. Instead I found the problem obscured with the primitive networks and intermediate networks of Kron, and I emerged with my faith unshaken but with my reason as virginal as when I started.

The truth of the matter seems to be that at the basis of network theory there lies an element of topology, perhaps a small matter to a professional topologist, but formidable to an electrical engineer or to an applied mathematician who is repelled by the formal exactitude of topology as it is currently treated. Professor Le Corbeiller disclaims any intention of proving topological formulae, and one can sympathise with that attitude. But it is an impossible attitude to adopt if one is to use the word "proof" at all in connection with what is an essentially topological subject.

However, "proof" is a subjective word and its mathematical meaning is chilling to the rich intuitive understanding which is the essence of engineering. Rigid reasoning is not necessarily the same thing as understanding, and it may well be that Professor Le Corbeiller has chos en the right path to his goal. As far as one reader at least is concerned, he has achieved it; I understand now what Kron's method is, and I owe that understanding to Professor Le Corbeiller's excellent exposition of it, even if I have to look elsewhere for the "proofs".

J. L. SYNGE

Philosophy of mathematics and natural science. By Hermann Weyl. Princeton University Press, 1949. 371 pp. $5.00.

In 1926 Mr. H. Weyl contributed to R. Oldenbourg's *Handbuch der Philosophie* a substantial article in two parts dealing, the first one with the philosophy of mathematics, the second one with that of physics. The present volume is a translation of this article, brought up to date at certain points, and enriched with six essays which represent the author's present thought in a field now extending into chemistry and biology.

This is a book no one should miss who likes science and philosophy. I do not say "philosophy of science", a fashionable topic at the present time. This is a book by a master of mathematical physics who also knows philosophy, classical philosophy, philosophy proper. To read and to reread it is a profit and a delight.

There is a marvelous bibliography, or rather, which is even better, a dozen lists or so, at irregular intervals, of books the author has liked and which he has quoted, or might have quoted, in the last section or chapter. This is an invitation to browse in Mr. Weyl's own study of which every thoughtful reader will be duly appreciative.

And the index is excellent.

P. LECORBEILLER

Analysis and design of experiments. By H. B. Mann. Dover Publications Inc., New York, 1949. 195 pp. $2.95.

An interesting aspect of the development of statistics during the past quarter-century is the growth of a field usually known as the "design of experiments." In the larger centers of statistical teaching, a course in this topic is now a standard part of the curriculum. A word of explanation as to how this arose is, I think, necessary to an understanding of the objectives of Dr. Mann's book.

To quote a description given by Yates, the essence of an experiment is the "imposition of deliberate change with the intention of studying its effects." In most types of experimentation, however, we cannot measure exactly the effects of an imoosed change, because these effects are influenced by extraneous variations, to which the term "experimental errors" is often given. The presence of extraneous influences of this type greatly complicates the problem of drawing sound conclusions from a completed experiment, for the results are found to vary from one part of the experiment to another, so that it may not be clear just what can be inferred with confidence. In this situation experimenters have often consulted statisticians, because of the statistician's knowledge of the theory of errors and of his preoccupation with the drawing of inferences from variable data. That is, the statistician was being consulted about the analysis of the results, not about the conduct of the experiment.

Needless to say, the way in which an experiment is conducted determines the type of conclusion, if any, that can be drawn from its results. Consequently the statistician was led naturally to investigate the logical principles that must govern the conduct of an experiment if it is to permit reasonably clear inferences. This study was initiated about 25 years ago by R. A. Fisher and its fruits form the basis of his book "The Design of Experiments". Two principal developments may be mentioned. The first is an elegant method of computation known as the analysis of variance, by which the calculations needed for performing tests of significance and constructing confidence limits are provided. The analysis of variance is a particular application of the method of least squares, but it can be learned and used competently by experimenters who would disclaim any knowledge of probability or mathematics. The second outcome is a series of ingenious methods for carrying out experiments, discovered mainly by Fisher and Yates, which allow us to eliminate the influence of many extraneous sources of variation from the effects under investigation, with a resultant gain in the precision of the experiment.

These two developments form the content of Dr. Mann's book. As he points out, the analysis of variance is usually presented in the literature for digestion by experimenters rather than by mathematicians, so that the assumptions required and the mathematical justification of the theorems utilized are not fully set forth. The first six chapters give a lucid and adequate exposition of the theory of the analysis of variance as it applies in the common types of experimental design, starting with a knowledge only of calculus and the rudiments of probability. Later chapters present the extensions to nonorthogonal data, including that which arises in incomplete block designs, to factorial experiments, and to the analysis of covariance.

The remainder of the book deals with certain problems, of interest to mathematicians, that have arisen in the construction of experimental designs. Complete sets of $(m - 1)$ orthogonal latin squares of side m have proved useful. Dr. Mann gives a summary of the present state of knowledge on this subject. By use of the properties of Galois fields, he proves the well-known result that a complete orthogonal set can be constructed for any square whose side is the power of a prime. Beyond this, little is known except two results of restricted scope. The first is that if the side of the square is of the form $\Pi p_i^{r^i}$, where the p_i are primes, we can construct a set of r orthogonal squares, where $r = \min. (p_i^{r^i} - 1)$. The second result, on the negative side, is that we cannot construct a set of two orthogonal squares of side 6.

Some account is given of the construction of balanced incomplete block designs. These are arrangements of t different letters in blocks or groups of size k $(k < t)$, such that every pair of letters occurs λ times within the same block. For experimentation, interest centers on arrangements in which λ is as small as possible. Dr. Mann shows how finite geometries have been applied in the construction of these arrangements.

The book is designed for three purposes: for mathematicians with no background in statistics who wish to become acquainted with the theory underlying the design of experiments, for experimenters with the same objective, and for use as a text-book in a course on the subject. The exposition is admirably clear throughout. In a book of limited length, such as this, the topics discussed are necessarily restricted. For the benefit of mathematical readers, some examples of experiments in different fields of research might have been presented in detail, in order to bring out clearly why the analysis of variance is used in the summary of results, and why such arrangements as latin squares and balanced incomplete blocks help to solve the experimenter's problems. As it is, such readers are likely to have a number of unanswered questions after a perusal of the book. For experimenters with the requisite mathematical background, the book should prove both useful and stimulating.

W. G. Cochran

Practical analysis. By Fr. A. Willers. Translated by R. T. Beyer. Dover Publications, Inc., New York, 1948. 422 pp. $6.00.

This German textbook covers very thoroughly the various types of numerical problems which the individual scientist or engineer is apt to encounter in the course of his work. Chapter 1 describes several types of tools such as slide rules and nomograms; the section on calculating machines was entirely re-written for the American edition by T. W. Simpson. Chapter 2 covers Interpolation; Chapter 3, Approximate Integration and Differentiation; Chapter 4, Algebraic Equations; Chapter 5, Analysis of Empirical Functions; Chapter 6, Approximate Integration of Differential Equations.

A feature of this book is that almost as much space is given to graphical methods as to digital computation. This makes the treatment very pleasant and easy to follow. It is all the more regrettable that the numerous and carefully executed diagrams of the German edition should have been, on the whole, very poorly reproduced.

A number of mathematicians are at present servicing very high-powered electronic machines and the kind of problems in numerical analysis which they encounter is not considered in the present book. Treatises on numerical analysis which we hear are in preparation on several sides will probably concentrate on those, to the benefit of the professional, and leave aside the individual worker. This one will still have to execute for himself, at more or less long intervals, some tabulation or integration work with the help of a modest desk-machine; he will find the book by Dr. Willers, with its enormous wealth of examples, a most helpful instructor and guide.

P. LECORBEILLER

Classical mechanics. By Herbert Goldstein. Addison-Wesley Press, Inc., Cambridge 42, Massachusetts, 1950. xii + 399 pp. $6.50.

In this book the author succeeds in a very satisfactory way in showing how classical mechanics leads into the various branches of physics as well as in discussing the mathematical methods.

The chapter headings are: 1. Survey of Elementary Principles. 2. Variational Principles and Lagranges Equations. 3. The Two-Body Central Force Problem. 4. The Kinematics of Rigid Body Motion. 5. The Rigid Body Equations of Motion. 6. Special Relativity in Classical Mechanics. 7. The Hamilton Equations of Motion. 8. Canonical Transformations. 9. Hamilton Jacobi Theory. 10. Small Oscillations. 11. Introduction to the Lagrangian and Hamilton Formulations for Continuous Systems and Fields.

The discussion of canonical transformations, integral invariants, Lagrange and Poisson Brackets, and the Hamilton Jacobi theory are treated in detail; the connection with geometrical optics and wave mechanics is concisely shown.

The discussion of central field problems includes some discussion of collision and scattering processes.

The section on Kinematics of Rigid Body Motion makes use of matrix methods and treats in detail the use of the Cayley-Klein parameters for rigid body rotation; the connection with Pauli-Spin matrices is shown.

The reviewer feels that this book will have a strong appeal to physicists, to physical chemists, and to some applied mathematicians.

ROHN TRUELL

Wave guides. By H. R. L. Lamont. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York. vii + 118 pp. $1.50.

This is the third edition of a Methuen monograph revised in 1950 and first published in 1942. A concise presentation of the essential parts of wave guide theory. It contains a discussion of wave guides as transmission lines, as resonators and as radiators. The book contains a brief outline of the diffraction problem in an aperture together with the use of W. R. Smythe's results. There is a somewhat hasty mention of diaphragms in rectangular guides, change in guide cross section, junctions, slots, and corrugated guides.

ROHN TRUELL

Electromagnetic waves. By F. W. G. White. Fourth edition (revised 1950). Methuen & Co. Ltd., London, and John Wiley & Sons, Inc., New York. viii + 108 pp. $1.25.

The first half of this Methuen monograph is assigned to a review of electromagnetic theory with a discussion of plane wave solutions of the field equations. The Lorentz dispersion theory is reviewed. The second half of the monograph is devoted to application of the dispersion theory to obtain the approximate optical properties of metals, the diffraction of x-rays, and the group velocity in a dispersive medium. The propagation in a dispersive medium with an applied magnetic field and the propagation of radio waves in the upper atmosphere are the topics of the last two chapters. Unfortunately the Bibliography on radio wave propagation contains nothing more recent than 1933.

ROHN TRUELL

Heaviside's electric circuit theory. By H. J. Josephs. John Wiley & Sons, Inc., New York and Methuen & Co. Ltd., London. viii + 115 pp. $1.25.

This is the second edition of a monograph first published in 1946. It is a very concise and well written exposition on the methods of Heaviside. It includes a rather complete discussion of the expansion theorem, operational formulae, and methods leading to integral equations with solutions by operational methods. Borel's theorem, "shifting", and fractional derivatives are reviewed. Impulse functions are dealt with in a chapter entitled "Heaviside's Last Theorem". The application of these methods to transmission lines includes a discussion of the divergent expansions that arise. There is a final chapter on the application of contour integration methods to the solution of circuit problems.

ROHN TRUELL

Radio aerials. By E. B. Moullin. The Clarendon Press, Oxford, 1949. xi + 514 pp. $8.00.

This is the second volume in the series, "International Monographs on Radio," edited by Sir Edward Appleton and Henry G. Booker. Times have changed! A few years ago not a single book could be found devoted entirely to antennas; now there are several excellent books, all coming from Great Britain, which treat this subject broadly, and some, such as the present volume, are concentrating on a single special class of antennas. There may be a difference of opinion as to whether antenna reflectors made up of planes and circular cylinders are of sufficient practical importance to deserve five hundred pages; but there is no question that this book contains much useful information, both theoretical and experimental.

As far as theory is concerned, most problems considered in the book are two-dimensional and highly idealized. The author regards his theoretical solutions as a guide to experimental work and not as predictions of performance of practical reflectors. He chooses those problems which can be solved exactly and counts on the common sense of the engineer to derive the maximum benefit in planning experiments and anticipating the results of experiments. The experimental section takes one-third of the book and contains information on parabolic reflectors in addition to those considered in the theoretical section.

The field of two-dimensional problems admitting exact solutions is covered exhaustively. The author considers infinitely long wires, systems of such wires, infinite gratings, infinite planes, wedges made with semi-infinite planes, wedges with cylindrical backs. The analysis of semi-infinite planes permits him to estimate the edge effects in reflectors of finite size. Of the three-dimensional problems only a few are dealt with: a dipole, a half-wave antenna, arrays of half-wave antennas, a small loop, a large loop carrying uniform current, and a circular ribbon of dipoles. Radiation patterns constitute the principal subject of study.

The author begins the book with a brief discussion of Maxwell's equations, introduces retarded potentials, and obtains the wave equation. Then, he asks the reader to accept the solution of this equation and from it proceeds to develop the solutions for infinite current filaments by integration. Here the author has missed a unique opportunity to avoid the gap in the theoretical development without sacrificing the essentially elementary method of analysis. In the two-dimensional case, retarded potentials do not simplify solutions of problems; on the contrary, they complicate them. It is easy to obtain directly from Maxwell's equations the expressions for the component of E parallel to the current flow—which the author does—and then relate these expressions to the given currents without bringing in the retarded potentials—which the author does not do. This approach would have enabled him to dispense with some tiresome integrations, to avoid difficulties with divergent integrals, and to extricate himself from difficulties in connection with the dihedral reflectors whose angles are not submultiples of 360°.

It is to be regretted that the author is prejudiced against what he terms the "optical approach" to problems of radiation. He admits that this approach is not wrong. Fairly enough he warns the reader that the book is written in the electrical language and that he will insist "on trying to relate fields to currents flowing in conductors." In this he follows a well-established tradition in radio engineering and preserves a consistent point of view throughout the book. But the price of consistency seems high. Very useful aids to thinking and powerful methods for solving radiation problems are thus neglected. An opportunity to compare different approaches is lost. And the time is ripe for future radio engineers to learn the mysteries of languages other than the "electrical language."

This book was intended primarily for reference. Its subject is probably too limited to recommend it as a sole textbook; but students should find the book very instructive in that it shows by easy steps how to construct the solutions of various problems from simple basic solutions.

S. A. Schelkunoff

Physics in Industry. The acceleration of particles to high energies. Based on a session arranged by the Electronics Group at the Institute of Physics Convention in May, 1949. Institute of Physics, London, 1950. x + 58 pp. $1.60.

This concise little book arose as a consequence of a convention at Buxton, England in May 1949. The book should be of real value to the non-specialist who is interested in the progress in the field of high energy accelerators, but who for some reason does not follow the literature. There are five chapters entitled: The Cyclotron (and synchro-cyclotron) (13 pages); Betatrons and Synchrotrons (16 pages); Electrostatic Generators (8 pages); Linear Accelerators (6 pages); Discussion of the Preceding Topics (14 pages).

In general each section discusses briefly principles, design, and performance. Naturally, the discussion of actual design and construction centers on the British work although the bibliographies seem to be rather complete. In fact, the references and bibliographies alone seem to the reviewer to be well worth the price of the book.

Rohn Truell

Theory of oscillations. By A. A. Andronow and C. E. Chaikin. English language edition edited under the direction of Solomon Lefschetz. Princeton University Press, New Jersey, 1949. ix + 358 pp. $6.00.

This volume is a translation and condensation of a much larger one in Russian. The reader should be thankful to Professor Lefschetz for his work of adaptation, pruning, and specific as well as general clarification. The work as it stands reads very easily. It deals mostly with autonomous (or isolated) oscillating systems of the flip-flop type, including the effect of the variation of one of the parameters of the system. Continuous systems are treated in the last chapter; non-linear systems subjected to applied forces, on which information is more easily available, receive passing mention. The reader is carried very progressively through a series of examples, mechanical and electrical. These will take the conscientious reader a great deal of time to go through; however, this study should be very rewarding as it will allow him not only to understand fully the theoretical facts, but to build up his own philosophy of the subject.

A group of "advanced" fields: non-linear oscillations, feedback theory, Laplace and Fourier transforms, noise theory, and a few others, present quite a problem to the instructor and to the graduate student. On the one hand, it is impossible, without a good understanding of them, to analyze some of the most common and important phenomena occurring in practice. On the other hand, once the theory is well understood, its use is in many cases mostly qualitative or else requires much simpler calculations than have been used while learning it. One would like to be able to impart to the students the essentials of these theories "in a nutshell", but personal experience cannot be by-passed.

The solution would seem to be to encourage the better students to self-study of such subjects. The present book by Andronow, Chaikin (and Lefschetz) should be especially recommended for that purpose. The numerous examples treated have been collected in two separate tables at the end of the book, one for mechanical and one for electrical systems, thus making it possible for the student to restrict his introductory work to the type of systems he is most familiar with.

While the specialist will have his quarrels with the original authors about this or that minor point, the book as a whole should be commended as a most helpful contribution to a wider knowledge of non-linear systems.

P. LeCorbeiller