

QUARTERLY OF APPLIED MATHEMATICS

EDITED BY

H. W. BODE
J. M. LESSELLS

H. L. DRYDEN
W. PRAGER
J. L. SYNGE

TH. v. KÁRMÁN
I. S. SOKOLNIKOFF

WITH THE COLLABORATION OF

M. A. BIOT
J. P. DEN HARTOG
C. FERRARI
J. N. GOODIER
F. D. MURNAGHAN
S. A. SCHELKUNOFF
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Manuscripts submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Managing Editor, Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. Seventy-five reprints without covers will be furnished free; additional reprints and covers will be supplied at cost.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

Mathematical Work: Only very simple symbols and formulas should be typewritten. All others should be carefully written by hand in ink. Ample space for marking should be allowed above and below all equations. Greek letters used in formulas should be designated by name in the margin.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime ('). between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

All subscripts and exponents should be clearly marked, and dots, bars, tildes, etc. over letters should be avoided.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that reoccurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol *exp* should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fraction, should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t (a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

For further suggestions concerning cuts, bibliography, footnotes, and abbreviations please see the inside back cover of the April issue of 1948.

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ELEMENTS OF THE THEORY OF EQUATIONS. By the late J. V. USPENSKY, Stanford University. 352 pages, \$4.50. An unusually thorough, explicit treatment of the subject, with full development, emphasizing both theory and numerical methods. The sections on computational methods are particularly noteworthy.

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BOOK REVIEWS

Engineering applications of fluid mechanics. By J. C. Hunsaker and B. G. Rightmire. McGraw-Hill Book Company, Inc., New York and London, 1947. ix + 494 pp. \$5.00.

In preparing this book which obviously represents the fruit of a long teaching experience, the authors aimed at converting the "traditional course in hydraulics into a more fundamental treatment of the action of fluids generally. The authors were more anxious that the student understand flow phenomena than that he be familiar with details of many practical devices." Accordingly, the emphasis is on the basic ideas of modern fluid mechanics. The scope of this book is best indicated by the following chapter headings: Introductory survey—Statics—Kinematics and Continuity—Dynamics of an ideal fluid—Energy relations for steady flow—Dimensional analysis and similitude—Incompressible flow in closed circuits—Compressibility phenomena—Drag—Wing theory—Hydrodynamic lubrication—Boundary lubrication—Hydraulic turbines—Pumps, fans and compressors—Propellers and jets—Fluid couplings and torque converters—Hydraulic transmissions and controls. The book is clearly written and constitutes an excellent introduction into fluid mechanics.

W. PRAGER

The physical principles of wave guide transmission and antenna systems. By W. H. Watson. Clarendon Press, Oxford, 1947. xiii + 207 pp., 95 text-figures. \$7.00.

This new book deals extensively with the systems' aspects of microwave transmission, which in other books are treated but briefly if at all. Wave propagation in uniform wave guides is discussed briefly but sufficiently for the main purpose of the book. Expressions for the impedances and admittances of slots in every conceivable position and for some other wave guide elements are given without derivation and the reader is referred to the original sources for the details. Omission of these details is in keeping with the main object of the book, which is to discuss systems of wave guide elements rather than the elements themselves. It also enables the author to give more space to discussion of the results and to concentrate an amazing amount of information into a small book. Wave guide elements, however, are not altogether neglected; the last chapter is devoted to the methods of their analysis.

At those frequencies for which the energy in a wave guide is transmitted to large distances only by the dominant wave, the wave guide discontinuities (probes, slots, etc.) may be represented either as series impedances or shunt susceptances or, more generally, as T or Π passive networks. They give rise to reflected waves which, when superimposed on the original wave system, produce a new wave system. The transformation from one wave system to the other may conveniently be expressed by a matrix. The transformation of a given wave system by any number of discontinuities may thus be expressed by suitable operations on matrices. These basic ideas are presented in Chapter I and are illustrated by their application first to a strip transmission line consisting of two parallel conducting strips and then to waves in free space.

Chapter II is a summary of the essential aspects of dominant waves in rectangular wave guides; and Chapter III is devoted to measurements and to the application to wave guides of the ideas introduced in Chapter I. Higher modes of propagation are discussed briefly in Chapter IV. Chapter V includes a discussion of "Babinet's" principle as applied to wave guides and its applications. Important wave guide discontinuities such as irises, probes, bends and twists are also treated in this chapter. Coupling of a wave guide to free space and to another wave guide by means of slots is studied in the following two chapters. Wave guide arrays are taken up in Chapter VIII. In Chapter IX the author discusses briefly a number of miscellaneous microwave devices. Then, in Chapter X the author suddenly recalls (the responsibility for this opinion is strictly ours) that he is a mathematician and that in accordance with popular beliefs a mathematician's book should contain more formidable mathematical equations; hence he proceeds to introduce some of the mathematical methods which are required in the theoretical studies of wave guide discontinuities.

It is safe to assume that this book will be welcomed by all who are interested either in the theory or in the practice of microwave transmission. It does not attempt to compete with other books in this field, and it has no competitors.

S. A. SCHELKUNOFF