

$$F_z^2 = \frac{2}{aL^2} \tanh \frac{aL^2}{2} \quad (12)$$

Computations show that Gross and Lehr's values of  $F_y$  have a constantly increasing error which deviates about 4% from our results when  $aL^2 = 1$ .

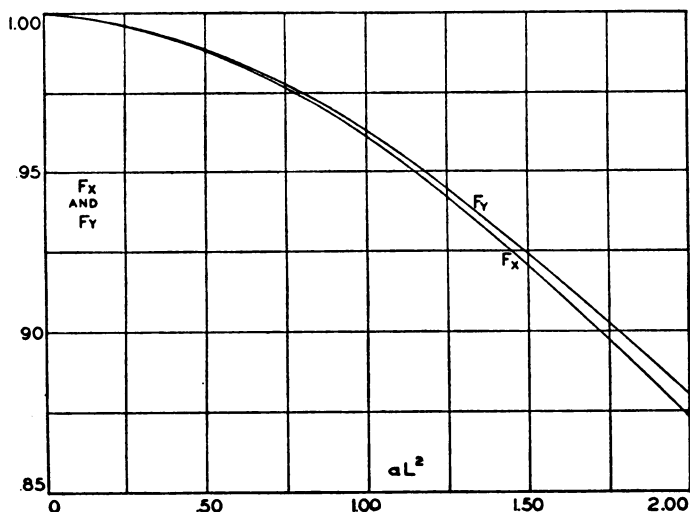


FIG. 1.

The two factors  $F_x$  and  $F_y$  are very important to the designer. For this reason curves of these two factors with  $aL^2$  as the independent variable are given in Fig. 1. The values of  $F_y$  were computed from Jahnke and Emde.

## ON WAVES IN BENT PIPES\*

By S. A. SCHELKUNOFF (*Bell Telephone Laboratories*)

In a recent issue of this *QUARTERLY*,<sup>1</sup> Karlem Riess obtained expressions for the fields of electromagnetic waves in bent pipes of rectangular cross section by the perturbation method. While it is true that in a bent pipe the waves cannot be classified into transverse electric and transverse magnetic types because in general both  $E$  and  $H$  have components in the direction of wave propagation, a different classification into two types is possible. This permits another method which yields the general solution in terms of Bessel functions.

In the one wave type, the plane of the electric ellipse is normal to the axis of bending (the  $Y$ -axis in Figure 1, p. 329 of Riess' paper); these waves have been called *electrically oriented* ( $EO_{m,n}$  wave type) and the fields of these waves are obtainable from  $H_y$  which may be expressed as the product of Bessel and sine (or cosine) functions.

\* Received Feb. 18, 1944.

<sup>1</sup> Vol. 1, No. 4, pp. 328-333.

In the other wave type, the plane of the magnetic ellipse is normal to the axis of bending; these waves are *magnetically oriented* ( $MO_{m,n}$  wave type) and their fields are obtainable from  $E_y$ . In each case the order of Bessel functions is equal to the angular phase constant.

For a bent pipe formed by the intersection of two concentric spheres and two coaxial cones emerging from the center there is also a solution in terms of known functions. In one wave type,  $EO_{m,n}$  type, the plane of the electric ellipse is normal to the radius; in the other,  $MO_{m,n}$  type, the plane of the magnetic ellipse is normal to the radius. The fields of  $EO$ -waves are calculable from  $H_r$  and the fields of  $MO$ -waves from  $E_r$ ;  $H_r$  and  $E_r$  themselves can be expressed in terms of Bessel and Legendre functions. These waves may be called *spherically oriented* in order to distinguish them from the *plane oriented waves* described earlier. The letters  $S$  and  $P$  in front of  $EO$  and  $MO$  may be conveniently used in the abbreviations.

#### CORRECTIONS TO MY PAPER

### A STRAIN ENERGY DERIVATION OF THE TORSIONAL-FLEXURAL BUCKLING LOADS OF STRAIGHT COLUMNS OF THIN-WALLED OPEN SECTIONS

QUARTERLY OF APPLIED MATHEMATICS, 1, 341-345 (1944).

By

N. J. HOFF

In the last term of the right hand side member of Eq. (3) on page 343,  $n$  should be raised to the second power and not to the fourth power.

The following equation defining  $T$  should be added:

$$T = (1/\rho^2)(n^2R + GC).$$