

QUARTERLY
OF
APPLIED MATHEMATICS

EDITED BY

H. L. DRYDEN
J. M. LESSELLS

T. C. FRY
W. PRAGER
J. L. SYNGE

TH. v. KÁRMÁN
I. S. SOKOLNIKOFF

WITH THE COLLABORATION OF

H. BATEMAN
J. P. DEN HARTOG
J. N. GOODIER
R. V. SOUTHWELL

M. A. BIOT
H. W. EMMONS
F. D. MURNAGHAN
G. I. TAYLOR

L. N. BRILLOUIN
K. O. FRIEDRICHS
W. R. SEARS
S. P. TIMOSHENKO

VOLUME I

JANUARY · 1944

NUMBER 4

QUARTERLY OF APPLIED MATHEMATICS

This periodical is published under the sponsorship of Brown University. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following industrial companies are gratefully acknowledged:

BELL TELEPHONE LABORATORIES, INC.; NEW YORK, N. Y.,

THE BRISTOL COMPANY; WATERBURY, CONN.,

GENERAL ELECTRIC COMPANY; SCHENECTADY, N. Y.

GULF RESEARCH AND DEVELOPMENT COMPANY, PITTSBURGH, PA.

LEEDS & NORTHRUP COMPANY; PHILADELPHIA PA.

PRATT & WHITNEY, DIVISION NILES-BEMENT-POND COMPANY; WEST HARTFORD, CONN.,

REPUBLIC AVIATION CORPORATION; FARMINGDALE, LONG ISLAND, N. Y.,

UNITED AIRCRAFT CORPORATION; EAST HARTFORD, CONN.

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Managing Editor, Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. Seventy-five reprints without covers will be furnished free; additional reprints and covers will be supplied at cost.

The subscription price for the QUARTERLY is \$6.00 per annual volume, single copies \$2.00. Subscriptions and orders for single copies may be addressed to: Quarterly of Applied Mathematics, 450 Ahnaip St., Menasha, Wisconsin or to Brown University, Providence 12, R. I.

PRINTED BY THE
GEORGE BANTA PUBLISHING COMPANY
MENASHA, WISCONSIN

QUARTERLY
OF
APPLIED MATHEMATICS

EDITED BY

H. L. DRYDEN
J. M. LESSELLS

T. C. FRY
W. PRAGER
J. L. SYNGE

TH. v. KÁRMÁN
I. S. SOKOLNIKOFF

WITH THE COLLABORATION OF

H. BATEMAN
J. P. DEN HARTOG
J. N. GOODIER
R. V. SOUTHWELL

M. A. BIOT
H. W. EMMONS
F. D. MURNAGHAN
G. I. TAYLOR

L. N. BRILLOUIN
K. O. FRIEDRICHS
W. R. SEARS
S. P. TIMOSHENKO

Printed by the
GEORGE BANTA PUBLISHING COMPANY
Menasha, Wisconsin

CONTENTS

W. S. Ament: The lines of principle stress in the plane problem of plasticity	278
H. Bateman: The transformation of partial differential equations	281
L. Bers: Concerning the acceleration potential	93
——— and A. Gelbart: On a class of differential equations in mechanics of continua	168
L. Brillouin: The antenna problem	201
C. L. Brown: The treatment of discontinuities in beam deflection problems	349
W. Z. Chien: The intrinsic theory of thin shells and plates	297
——— and A. Weinstein: On the vibrations of a clamped plate under tension	61
L. H. Donnell: A chart for plotting relations between variables over their entire real range	276
H. L. Dryden: A review of the statistical theory of turbulence	7
W. Feller: On A. C. Aitken's method of interpolation	86
K. O. Friedrichs and J. J. Stoker: Forced vibrations of systems with non-linear restoring force	97
R. E. Gaskell: On moment balancing in structural dynamics	237
A. Gelbart: (<i>See L. Bers</i>)	
G. H. Handelman: A variational principle for a state of combined plastic stress	351
G. E. Hay and W. Prager: On plane rigid frames loaded perpendicularly to their plane	49
M. Herzberger: A direct image error theory	69
N. J. Hoff: A strain energy of derivation of the torsional-flexural buckling loads of straight columns of thin-walled open sections	341
Th. v. Kármán: Tooling up mathematics for engineering	2
P. W. Ketchum: On the discontinuous flow around an airfoil with flap	149
W. M. Kincaid and V. Morkovin: An application of orthogonal moments to problems in statically indeterminate structures	334
Y. H. Kuo: On the force and moment acting on a body in shear flow	273
J. Lehner and C. Mark: An application of the method of the acceleration potential	250
C. C. Lin: On the motion of a pendulum in a turbulent fluid	43
S. Lubkin and J. J. Stoker: Stability of columns and strings under periodically varying forces	215
C. Mark: (<i>See J. Lehner</i>)	
V. Morkovin: On the deflection of anisotropic thin plates	116
——— (<i>See W. M. Kincaid</i>)	
W. Prager: (<i>See G. E. Hay</i>)	
W. C. Randels: A new derivation of Munk's formulae	88
K. Riess: Electromagnetic waves in a bent pipe of rectangular cross section	328
S. A. Schelkunoff: The impedance of a transverse wire in a rectangular wave guide	78
——— On the antenna problem	354
H. J. Stewart: Periodic properties of the semi-permanent atmospheric pressure systems	262

J. J. Stoker: (*See K. O. Friedrichs*)

——— (*See S. Lubkin*)

A. G. Strandhagen: Use of sine transform for non-simply supported beams	346
J. L. Synge: On Herzberger's direct method in geometrical optics	268
H. S. Tsien: Symmetrical Joukowski airfoils in shear flow	130
A. Weinstein: (<i>See W. Z. Chien</i>)	
Book reviews.	189

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The Editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. The papers submitted should be in final form. Only typographical errors may be corrected on proofs; if authors wish to add material, they may do so at their own expense.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical work: Only very simple symbols and formulas should be typewritten. All others should be carefully written by hand in ink. Ample space for marking should be allowed above and below all equations. Greek letters used in formulas should be designated by name in the margin. The difference between capital and lower-case letters should be clearly shown; and care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1) and the letter *l* and the prime ($'$), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*. All subscripts and exponents should be clearly marked, and dots and bars over letters should be avoided as far as possible. Square roots of complicated expressions should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$. Complicated exponents and subscripts should be avoided. Any complicated expression that reoccurs frequently should be represented by a special symbol.

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be given as footnotes. Only in longer expository articles may references be grouped together in a bibliography at the end of the manuscript. The arrangement should be as follows: (for books)—author, title, volume, publisher, place of publication, year, page referred to; (for periodicals)—author, title, name of periodical, volume, page, year. All references should be complete and thoroughly checked.

CONTENTS

H. BATEMAN: The transformation of partial differential equations . . .	281
W. Z. CHIEN: The intrinsic theory of thin shells and plates	297
K. RIESS: Electromagnetic waves in a bent pipe of rectangular cross section	328
W. M. KINCAID and V. MORKOVIN: An application of orthogonal moments to problems in statically indeterminate structures	334
N. J. HOFF: A strain energy derivation of the torsional-flexural buckling loads of straight columns of thin-walled open sections	341
NOTES:	
A. G. Strandhagen: Use of sine transform for non-simply supported beams	346
C. L. Brown: The treatment of discontinuities in beam deflection problems	349
G. H. Handelman: A variational principle for a state of combined plastic stress	351
S. A. Schelkunoff: On the antenna problem	354

Outstanding McGraw-Hill Books

ELECTRIC CIRCUITS AND FIELDS

By HAROLD PENDER and S. REID WARREN, JR., University of Pennsylvania. 534 pages, \$4.00

The fundamental facts and theories of electric circuits, including direct-current circuits, steady-state alternating currents, transients, and non-linear circuits.

FUNDAMENTALS OF OPTICAL ENGINEERING

By DONALD H. JACOBS, National Bureau of Standards. 487 pages. \$5.00

A comprehensive introduction to the methods and principles of optical design.

THE THERMODYNAMICS OF FIREARMS

By CLARK S. ROBINSON, Massachusetts Institute of Technology; Lt. Col., Ordnance Reserve, U. S. Army. 175 pages, \$2.50

Places internal ballistics on a sound basis by means of thermodynamics. An elementary treatment of the behavior of propellant explosives in firearms.

RADIO ENGINEERS' HANDBOOK

By FREDERICK E. TERMAN, Stanford University. 1019 pages, \$6.00

Presents a wealth of sound, technical information especially selected and organized to meet the needs of the radio engineer.

Send for copies on approval

McGRAW-HILL BOOK COMPANY, INC.

330 West 42nd Street

New York 18, N.Y.