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BOOK REVIEWS

Mathematics of modern engineering. Volume II (Mathematical Engineering). By Ernest G. Keller. (One of a series written in the interest of the Advanced Course in Engineering of the General Electric Company.) John Wiley and Sons, Inc., New York, 1942. xii+309 pp. $4.00.

This book is the second in a series of three, the first of which was written jointly by the present author and R. E. Doherty. Roughly speaking, the present volume treats problems leading to ordinary differential equations while the third volume, apparently, will be devoted to applications of partial differential equations. The material of the book is said to have evolved out of research engineering work and the Advanced Course in Engineering of the General Electric Company. It was prepared for that course and for graduate engineering work in general. It was decidedly not intended either as a textbook in mathematics or in engineering. Its main purpose is to illustrate, by means of many typical examples, the methods of what the author describes as a new phase in engineering and calls Mathematical Engineering.

As in Mathematical Physics, these methods are essentially mathematical in character. Still, mathematical engineering is far from being a part of mathematics. Indeed, solving a practical problem consists of two parts. The formulation of the problem and its reduction to a form in which it lends itself to mathematical treatment constitutes by far the most difficult part. It requires not only knowledge, but also originality and instinct. Once the problem is reduced to a canonical form, its solution may be difficult but can usually be achieved by a well established technique. It should be a comfort to the non-mathematical engineer and to the mathematician alike that, with the numerous engineering examples of the book under review, the second step in each case reduces to routine. In presenting mathematical theories and methods the author has not felt tempted to deviate the least from common usage, thus indirectly paying tribute to the usefulness of "pure" theory.

The great interest, and value, of the book lies in its large collection of examples, taken from the rich experience of the author, and illustrating the application of mathematics to engineering problems. Some of the problems are given as simple exercises (without solutions), others as more or less simple illustrative examples to the mathematical text. The most valuable material is represented by examples where the reader can follow the whole procedure of formulating, and solving, difficult problems of engineering practice. For example, with an intricate problem concerning electric locomotives the factual information is presented in the form in which it once actually confronted the author. The setting up of the problem is done with masterly skill and the final formulation leads to a surprisingly simple mathematical question. This is an example of mathematical engineering at its best.

The style of the book seems to indicate that the author himself was interested only in presenting this new and interesting material. As a matter of fact, the general theory is treated in a rather unsystematic way. The author carelessly mentions here and there what the needs of the moment require, sometimes altogether forgetting the reader. Apparently, the latter is not even expected to be familiar with vectors and matrices. On the other hand, the short summaries of much more advanced topics cannot seriously be meant as a first introduction. Occasionally the intended elementary character of the book is given up, and once (p. 197) the author almost apologizes for mentioning problems which have already been answered by engineers. One example of the haphazard way in which theory is presented may suffice. Ordinary linear differential equations are first presented as they unfortunately used to be presented in elementary textbooks, without matrices. In connection with G. Kron's theories more than twenty pages are devoted to matrices, and here the said equations are put into matrix form (with the danger of producing the false idea that this is an application of the somewhat mystically formulated first "generalization postulate of G. Kron"). Finally, in the last chapter, for reasons unclear to the reviewer, the same linear equations are treated among non-linear problems, this time in the more efficient matrix form. Now the author also gives good advice on how to compute characteristic values while during the first exposition he suggested impracticable methods. More trivial examples of the same nature appear frequently. Thus, treating the two-dimensional automobile (p. 21), the author says: "Evidently, by Koenig's theorem . . . ." For the theorem itself the reader is referred to p. 44, where he finds only a formula and no theorem. For, only when the author needed the theorem for the second time, did he formulate it in the course of the discussion of a special problem.

The book consists of three parts. Chapter I, "Engineering Dynamics and Mechanical Vibrations" treats applications of classical dynamics. The subject of vibrations seems most congenial to the author. Accordingly, special examples are most numerous in the first chapter; and, on the whole, this chapter seems the best. Chapter II is devoted to an "Introduction to Tensor Analysis of Stationary Neworks and Rotating Electrical Machinery." This is actually an introduction to G. Kron's theories. In the reviewer's opinion this chapter is by far the most difficult one and certainly no introduction in the usual meaning of the word. The exposition is less personal, the examples less numerous and less illustrating. Chapter III is entitled "Non-Linearity in Engineering." It treats briefly of various methods of obtaining solutions, or approximate solutions, of ordinary differential equations. Moreover, elliptic and hyperelliptic functions and integrals are treated and examples of their application to vibration problems are given. There is an extensive bibliography.

but altogether too many of the books mentioned will prove valueless (or confusing) to the average reader.

These deficiencies are not serious enough to endanger the usefulness of the book. They are mentioned in the hope that their removal in a future edition will render the book still more enjoyable and still more useful as a textbook. Even in its present shape the book should convince many a sceptical engineer of the power of mathematical reasoning when applied by a man who combines mathematical knowledge with a good engineering mind.

W. FELLER

Microwave transmission. (International Series in Physics.) By J. C. Slater. McGraw-Hill Book Company, Inc., New York and London, 1942. x+309 pp. $3.50.

This new book in the "International Series in Physics" deals very completely with the theory of transmission of microwaves (electromagnetic waves with wave length of the order of magnitude of 1 cm.). Problems concerning generation and reception of microwaves are omitted except for a few brief remarks, because, as the author points out in the introduction, very little is known from a theoretical point of view about their actual operation.

The treatment of the subject is based largely on electromagnetic theory, although the first chapter is devoted to conventional transmission lines for radio waves. The ideas developed there are kept in mind and used for purposes of comparison in the later chapters. Considerable use is also made of analogies with acoustics and optics. The former are particularly pertinent as the wave length of acoustic waves is of the same order of magnitude as that of microwaves. Much of the terminology introduced is taken by direct analogy from the corresponding acoustic case.

The chapter on the fundamentals of electromagnetic theory is fairly elementary but uses vector notation. Meter, Kilogram, Second (Georgi) units are used throughout. A complete discussion of electrodynamics would of course be out of place in a book of this type. This theory is developed in sufficient detail, however, so that the subsequent chapters should be intelligible to any senior in physics or engineering.

After the preliminaries on ordinary transmission lines and electrodynamics, the author goes on to consider in detail the theory of various types of transmission lines. Wave guides and coaxial cables are thoroughly discussed. Particular emphasis is placed on phenomena which occur when the cross section of a wave guide of cable is varied and which take place at junctions of lines of different characteristic impedance. Problems of matching impedances and the applications are included. The various types of waves that may arise are classified and their propagation characteristics are studied. The theory of radiation from antennas is given in detail, with an entire chapter devoted to directive devices for antennas. The final chapter deals with the coupling of coaxial lines and wave guides.

The author's treatment of these subjects is extremely good, and his presentation is clear and logical with all extraneous matter eliminated. The book should find considerable use both as a text for the student and as a reference work for the specialist. It represents, moreover, a real contribution, in that its sets forth in concise and unified form the recent advances in microwaves—an extremely important field at the present time. It is an excellent book, to be recommended to all the scientists interested in this field. Those who wish more detailed theoretical calculations should be reminded of Stratton's *Electromagnetic theory* (McGraw Hill, 1941) and E. V. Condon's *Principles of microwave radio* (Reviews of Modern Physics, 14, 341–389 (1942)) where a number of very interesting examples are discussed in full detail in view of practical applications.

L. BRILLOUIN

Introduction to non-linear mechanics. By N. Kryloff and N. Bogoliuboff. A free translation by Solomon Lefschetz of excerpts from two Russian monographs. (Annals of Mathematics Studies, No. 11.) Princeton University Press, Princeton, 1943. iv+105 pp. $1.65.

The major part of this monograph is a very condensed version of a Russian book by the two authors. By making a part of the work of Kryloff and Bogoliuboff accessible to American scientists, the monograph accomplishes an extremely useful purpose and will, no doubt, be welcomed by any one interested in the subject of Non-Linear Mechanics.

The monograph deals mainly with the equation

$$\frac{d^2x}{dt^2} + \omega^2 x = \epsilon f\left(x, \frac{dx}{dt}\right) \tag{1}$$

(for small values of ϵ) and with the problem of finding approximate solutions of (1), free from secular terms and satisfying (1) to within terms of order ϵ^m with prescribed m. Problems studied in the monograph refer, generally speaking, to non-conservative physical systems and differ in this respect from the corresponding problems in the astronomical theory of perturbations.

The mathematical tools are developed mainly from the viewpoint of applications in Physics and Engineering, and mathematical rigor is not the primary consideration. It should also be observed that the qualitative geometrical methods of Poincaré-Bendixon are not discussed in the monograph which deals exclusively with analytical approximation methods.

The first chapter contains various examples in Mechanics and Electrical Circuit Theory which lead to non-linear equations of type (1). The theory of the first approximation is studied in Chapters II and III. Higher approximations are discussed in Chapter IV. The method of "linearization" described in Chapters V and VI consists in substituting for equation (1) a linear equation which, in the first approximation, has the same solution as (1). The important thing is that this linear equation can usually be set up directly from the physical data of the problem, without reference to the exact equation (1). Chapter VII discusses multiply periodic systems. Periodic disturbances are studied in Chapter VIII. This leads to equations of the type

$$m\frac{d^2x}{dt^2} + kx = \epsilon f\left(t, x, \frac{dx}{dt}\right)$$

where the function f is almost periodic with respect to the variable t. The final chapter contains several excerpts from a monograph by the two authors exhibiting relations between the properties of the exact solutions and those of the approximate solutions.

Throughout the monograph mathematical theories are illustrated by numerous examples of physical systems.

W. HUREWICZ

Heat transmission. By William H. McAdams. (Sponsored by the Committee on Heat Transmission, National Research Council.) Second Edition. McGraw-Hill Book Company, Inc., New York and London, 1942. XIV+459 pp. $4.50.

The knowledge of the processes of the heating and cooling of bodies has gradually accumulated from the dawn of history as a mass of empirical facts. Since the time of Newton who expressed some elementary ideas on the laws of cooling, the thermal aspects of the world have been divided into the equilibrium aspects (thermodynamics) and the non-equilibrium aspects (heat transfer).

All the phenomena of thermodynamics were found to be completely covered by a few sweeping general laws which serve to correlate precisely seemingly unrelated phenomena. No corresponding simple generalizations have been found for the non-equilibrium thermal phenomena. The basic physical laws of conduction and radiation were discovered and applied during the last century and it was clearly understood then that the great majority of heat transfer problems encountered in practice could be adequately treated by these laws only if the attendant convective movements of a fluid were properly accounted for simultaneously. It is the presence of these convective movements which has and still does retard the analytical development of heat transfer knowledge as compared with other aspects of science.

The first edition of *Heat Transmission* by Wm. H. McAdams, Prof. of Chemical Engineering, Mass. Inst. of Tech., appeared in 1933. The emphasis of the book was on the presentation, for ready reference, of all the best empirical data and computational methods required for the solution of practical heat transfer problems. That the time was ripe for the general collection and presentation of these heat transfer facts is indicated by the fact that the book was sponsored by the Committee on Heat Transfer of the National Research Council. That the book was successful as a text and reference work is attested by the fact that in the 10 years between the first and second editions 9 impressions were needed to satisfy the demand and approximately eight thousand copies were sold.

One quarter of the first edition was devoted to conduction and radiation heat transfer, the physical laws, the physical constants of materials, the empirical and mathematical methods for the solution of problems. The remaining 3/4 of the book presented information needed for the solution of problems involving convection. A chapter on Dimensional Analysis presented the basis of the correlation of empirical data. A chapter on Flow of Fluids presented laminar and turbulent friction and pressure loss data. Succeeding chapters dealt in turn with, the concept of thermal resistance, convection heat transfer by fluids inside pipes, fluids outside pipes, condensing vapors, and boiling liquids. The book closed with some 75 pages of Appendix dealing with thermal properties of materials, Bibliography and Subject Index.

Like most branches of science, the field of heat transfer is one in which developments are being made at an ever more rapid pace. Ten years of work made a considerable revision of the first edition necessary if the book was to maintain its position of excellence in the field of heat transfer literature. During these years, as before the first edition, Prof. McAdams followed carefully the developments as they were made and indeed was himself responsible for some of them.

The new edition has again been sponsored by the Committee on Heat Transfer of the National Research Council.

The book is written in the same spirit as the first edition and follows the same general outline described above. The subject matter of every chapter has however been completely revised. The thoroughness with which the new work in heat transfer has been reported is well indicated by the fact that the former Bibliography of 414 references has been expanded to 789, i.e. almost doubled. These are not useless references either, as each one is followed by a page number of the book where it is mentioned or where its subject matter is pertinent. The new data has been so well assimulated into the old that the book has been increased by only 100 pages.

The opening chapter of the new edition is a four page introduction in which the definitions of conduction, radiation and convection are given together with the basic equation for each. A most important addition is made on page 4 where a table of "Approximate Range of Values of Heat Transfer Coefficients Ordinarily Encountered" is given.

Heat transfer by conduction is covered in 20 pages and as in the earlier edition presents only the most directly useful facts, in good part through the ideas of a mean area or a thermal resistance. Analytical methods of solution are hardly more than mentioned in passing although some analytical results in the form of resistance formulas are tabulated.

The 17 page chapter on the Heating and Cooling of Solids remains much the same as before. Analytical results, a single sketchy derivation of one case is given, are presented as dimensionless charts, several new ones being included. A graphical method of solving transient heat conduction problems has been added and illustrated by an example.

Radiant Heat Transmission has again been written by H. C. Hottel, a colleague of McAdams on the Chemical Engineering Staff of Mass. Inst. of Tech. The subject matter, 40 pages of it, has been completely revised. The matter of spectral distribution of energy and its effect on heat transfer, hardly more than mentioned in the first edition, is treated in as much detail as the accumulated data on absorptivity and emissivity of bodies will permit. Following the derivation of the general differential equation and a sample integration is a discussion of geometrical and "non-black body" factors. Considerable new results have been added here in the form of what might have been called theorems regarding interrelations between the factors under various circumstances. The radiant heat transfer from gases has likewise been completely rewritten.

The chapter on Dimensional Analysis in the first edition was easily the weakest in the whole book. Considering the fundamental position which this type of analysis holds in the correlation of all convection data, it is important that the fundamental ideas as well as its method of use be made very clear to the reader. The new writing of this chapter is enormously better, in spite of what the reviewer regards as incorrect statements in connection with the "dimensional constants." It is indicated that dimensional constants must be added if more than four dimensions (say M, L, θ, T) are used. Why four? Why not three ($T = ML^2\theta^{-2}$ by kinetic theory)? The answer is simple, kinetic theory as such does not play any part in the heat transfer correlations contemplated in the book. For a similar reason and with the same advantage it is sometimes proper and desirable to use five or six dimensions without adding dimensional constants.

The discussion of Flow of Fluids remains much the same as before except for the addition of some of the dimensionless correlations of velocity distribution in turbulent flow in pipes. The sameness, by chance, even extends to the page numbers for Chapter V; first edition pages 99–134, second edition pages 99–132.

Once the idea of thermal resistance (or conductance) is introduced to permit calculation of series and parallel thermal circuits, the most difficult problem of analysis of heat exchanger performance is the question of "mean temperature difference." The latest pertinent facts are presented in Chapter VI together with a brief discussion of the important experimental problem of the measurement of surface temperature.

The next four chapters deal with the details of the correlated heat transfer data on fluids inside tubes, outside tubes, condensation and boiling. The revisions and additions are extensive and too numerous to mention in detail. They involve new data used to improve the accuracy of old correlations, new improved methods of correlating old and new data. New theoretical results are mentioned where of practical importance as, for example, in the analogy between heat transfer and friction. Even some new types of heat transfer problems have found an important place, as for example film boiling.

A new last chapter has been added entitled "Applications to Design." In this chapter, structural details, economic considerations and the choice of "optimum" operating conditions are discussed. The criterion of "optimum" is the fixed charges plus operating costs which depend upon the power required to pass the fluids through the heat exchanger. Differentiation gives the conditions for minimum cost which are solved for the optimum conditions sought.

Throughout the book a generous number of illustrative examples are given. As a further aid to the reader, each chapter is concluded with a page of problems on which to test one's real understanding of the subject matter.

The new edition of *Heat Transmission* assures the continuation of the high reputation of the earlier work for another decade or so.

HOWARD W. EMMONS

Empirical equations and nomography. By Dale S. Davis. McGraw-Hill Book Company, Inc., New York and London, 1943. ix+200. $2.50.

The first part of the present book will prove useful to engineers and scientists confronted with the task of finding "a simple mathematical expression fitting some experimental data." The first two chapters are devoted to a discussion of frequently used functions which contain a certain number of constants. Methods of determining the numerical values of these constants are given. The third chapter is concerned with the analysis of experimental data involving two independent variables. It is obvious that this problem cannot be treated comprehensively in the space of some twenty pages allotted to it; the reviewer believes, however, that the very useful graphical method developed by A. Lafay [Génie Civil, **40**, 298 (1902)] should have been discussed.

The second part contains an introduction to Nomography. Chapters IV to IX bring the most important types of alignment charts for three and more variables and Chapter X is devoted to special slide rules.

Throughout the book the mathematical argument is of an elementary character; the discussion is illustrated by numerous examples, a large portion of which are concerned with problems of chemical engineering.

W. PRAGER

Electromagnetic Waves. By S. A. Schelkunoff. D. van Nostrand Inc., New York, 1943. xv+530. $7.50.

This new addition to a well-known series has been awaited with much interest by all those acquainted with Dr. Schelkunoff's contributions to propagation theory, and it will be found that their expectations have been entirely fulfilled. This monumental piece of work is equally remarkable for the originality and consistency of its approach as for the wealth of information contained in its five hundred densely packed pages.

The author's systematic use of the harmonic oscillation, with complex variables and coefficients, is in line with the marvelous development which has occurred in the communication field during the last fifty years. Alternating current theory, then acoustics, then vibrational mechanics successively dropped the differential equations which physics offered as a basis and systematically restricted themselves to harmonic oscillations. This has resulted in the replacement of the differential operator by $i\omega$, leading to a tremendous simplification of steady-state analysis, which has been reduced to the calculation of amplitude ratios and phase differences. The genuinely difficult problems have not disappeared for all that but are now relegated to Fourier or Laplace transform theory, and it has become apparent that an enormous field of application can be covered by purely algebraic processes.

Not the least advantage of this method has been the unification brought into the three chapters of technical science mentioned above. Electrical impedances gave the model after which acoustical and mechanical impedances were fashioned; and mixed mutual impedances, thereafter, made it possible to write the equations of electro-mechanical or acoustico-mechanical transducers. There was an exciting era of intense development in this field during the twenties; and it was amusing to hear at that time, and even a good deal later, irate die-hards denouncing "impedances" with bitter irony or viewing with alarm the spread of "analogies."

Dr. Schelkunoff has set about to carry this point of view into Electromagnetic Theory, and it may well be that his will be the honor of having brought into the fold of harmonic oscillation theory the last chapter of Physics which still had to be incorporated. (One might think of Optics, but of course half of the book is really Optics.) Having given, in the first pages of his fourth chapter, a short and quite personal derivation of Maxwell's equations (1–15, p. 69), Dr. Schelkunoff without taking breath adds immediately: "Since we are concerned primarily with fields varying harmonically with time, we replace the instantaneous field intensities and current densities by the corresponding complex variables and write Maxwell's equations as follows:

$$\int E_n ds = -\int\int i\omega\mu H_n dS - \int\int M_n dS,$$
$$\int H_n ds = \int\int (g + i\omega\epsilon) E_n dS + \int\int J_n dS." \qquad (1-16)$$

Thus the sacrosanct Maxwell equations are swept away with movie-like swiftness, and instead we have the steady-state equations of a medium characterized by a distributed series impedance $i\omega\mu$ and a distributed shunt admittance $g+i\omega\epsilon$ (p. 81).

The analogy with a transmission line whose series inductance is μ, shunt conductance g and shunt capacitance ϵ, all taken per unit length, is inescapable (p. 243). In particular the above primary constants simply beg to be transformed into the familiar secondary constants of transmission line theory; here the intrinsic propagation constant σ and the intrinsic impedance η are defined by

$$\sigma = \sqrt{i\omega\mu(g + i\omega\epsilon)}, \qquad \eta = \sqrt{\frac{i\omega\mu}{g + i\omega\epsilon}} \tag{9–1}$$

(p. 81) (σ is in neper/meter, η in ohms; the book is written in MKS—p. 60). For free space we shall have $g=0$, and the following numerical values of the fundamental constants (p. 82):

$$\begin{aligned} &\text{impedance of free space } \eta_0 \approx 120\pi \text{ ohms,} \\ &\text{characteristic velocity } \quad v_0 \approx 3.10^8 \text{ meters/second.} \end{aligned} \tag{9-4}$$

* * * * * * * *

Surprising as it may appear to transmission engineers and sound engineers, who daily handle their respective characteristic impedances Z_0 or ρc, there still are very competent physicists who balk at the idea of free space having a characteristic impedance of about 377 ohms. Yet, in the words of Professor Ronold W. P. King:[1] "The existence of such a characteristic resistance for electromagnetic effects is just as mysterious, but not more so, than the existence of the finite velocity v_0." Dr. Schelkunoff explains very well how this constant could have been overlooked by the builders of the classical theory: "The physicist concentrates his attention on one particular wave: a wave of force or a wave of velocity or a wave of displacement. His original differential equations may be of the first order and may involve both force and velocity; but by tradition he eliminates one of these variables, obtains a second order differential equation in the other and calls it the "wave equation." Thus he loses sight of the interdependence of force and velocity waves . . . " (p. vii). Still, it is surprising to see that one has started with two constants ϵ_0 and μ_0, recognize the fundamental importance of their product, and not enquire about their ratio.

Then, the reader will ask, how can the Theory of Relativity give a leading role to the velocity of light and not mention the impedance of free space. Has Einstein no use for η_0? Well, he has, and he has not. First, an essential point in Special Relativity is the merging of the magnetic and the electric

[1] Mimeographed "Notes on Antennas" for the course of Electronics and Cathode Ray Tubes (Eng. 270), Harvard University.

fields into one skew-symmetrical tensor. When doing this in the MKS system, homogeneity requires the use of the components of E and of $\eta_0 H$; but the factor η_0 is not apparent, for instance, in the equations on p. 44 of "The Meaning of Relativity" (by A. Einstein, Princeton Univ. Press, 1923) which uses a system of units in which $\eta_0 = 1$. Secondly, if we try to connect the universal constant η_0 with other members of this interesting family, we find that η_0 times a (charge)[2] has the dimensions of "action," and more precisely that

$$\eta_0 e^2 = \frac{2h}{137}$$

(e = charge of the electron, h = Planck's constant). We see from this that there is more to η_0 than appears in Special Relativity, the first step in the successive Einsteinian extensions of Maxwell's theory.

* * * * * * * *

We have dealt at length with this question of the "impedance of free space" because it exemplifies the spirit of the whole work. It occurs in the course of a short but apt presentation of the "Fundamental Electromagnetic Equations" (Chapter IV), immediately applied to harmonic oscillations. The book as a whole is devoted not to Electromagnetism in general but, as specified in the title, to *Electromagnetic Waves.*

Three preliminary chapters introduce the more advanced mathematical tools which will be used, but sparingly, in what follows: such topics as contour integration, Bessel and Legendre functions. Chapter V is a short and original presentation of Network Theory.

The central part of the book begins with Chapter VI, "About Waves in General," a sort of preview of the questions which will be treated in detail later, during which we are introduced to radiation from given currents, propagation along wave guides, and to such general tools as electric and magnetic current sheets, the method of images and conformal representation.

In the following four chapters, we meet the most thorough treatment available of the propagation of waves, guided or bounded, in one, two and three dimensions. It is impossible to do justice here to the richness of the material, which must have cost tremendous labor and which is in great part taken from the author's own publications. We find in Chapter IX, however, classical problems of Fresnel optics, adroitly adapted to contemporary radio needs. Chapter XI is a relatively short treatment of antenna theory, principally of conical antennas, and in the last chapter we return to wave guides and solve various problems involving discontinuities, even to an iris or a transversal wire. This subject is still under development by the author, and the readers of the Quarterly have had the benefit of one of its recent extensions.[2]

[2] See Quarterly of Applied Mathematics, 1, 78 (1943).

The specialist in wave propagation has no need to be told of the value of this book; but the reviewer would like to explain to his fellow non-specialists why it is particularly important that they should not miss it. When the results of much present-day research will suddenly be made available, it will be a hard task to catch up, not only with the new knowledge, but still more with the new modes of attack. The borderland between radio and optics is one of the fields from which great things can confidently be expected. Dr. Schelkunoff's book is a great opportunity for those not at present engaged in research to get familiar with methods which they will want to use tomorrow.

P. Le Corbeiller