# QUARTERLY

OF

# APPLIED MATHEMATICS

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## QUARTERLY

OF

## APPLIED MATHEMATICS

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# QUARTERLY OF APPLIED MATHEMATICS

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### **FOREWORD**

The Quarterly of Applied Mathematics has been founded primarily to meet the needs of certain mathematicians and engineers whose interests extend beyond the accepted boundaries of their respective groups. These mathematicians find their greatest interest in the application of mathematics to physical problems, and these engineers seek solutions of practical problems by advanced mathematical methods. Thus they meet on the common ground of applied mathematics with a stimulating variety of interest.

It is not desirable to attempt too precise a definition of the boundaries of the field to which the Quarterly will be devoted. The mathematical solution of one problem often throws light on another problem in an entirely different field; indeed, the peculiar strength of the mathematical method lies in its power to cut across those lines of demarcation which seem to divide science into separate compartments.

Nevertheless, it is necessary to give an outline of policy for, within fairly wide limits, the pages of the Quarterly should appeal to a common interest. It seems best to start with the common ground of mathematics and engineering as a nucleus, and to build around it a wider circle of interest, embracing mathematical theory related to engineering problems. Thus certain subjects—fluid mechanics, elasticity, plasticity, thermodynamics, and classical mechanics in its engineering applications—are to be regarded as lying within the scope of the Quarterly, and to these must be added electrical engineering, which has been one of the most fruitful fields of mathematical application.

While it is not the purpose of the Quarterly to publish experimental results, we shall welcome mathematical contributions which have an intimate connection with application in industry or practical science. Indeed, the ideal contribution to our pages would be one in which advanced and general mathematical methods lead speedily to results which are in close agreement with experiment, and which are of high importance, either in direct practical application or as an illumination of interesting phenomena hitherto unexplained.

THE EDITORS.

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